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TOWARDS THE DISPERSION RELATIONS FOR DIELECTRIC OPTICAL FIBERS WITH HELICAL WINDINGS UNDER SLOW- AND FAST-WAVE CONSIDERATIONS — A COMPARATIVE ANALYSIS

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Abstract—The paper presents the electromagnetic (EM) wave propagation in cylindrical optical fibers with helical windings under slow- and fast-wave considerations. Field components are deduced for both the cases, and also, the dispersion relations are obtained by applying the boundary conditions, as modified by the presence of conducting helical windings. Two special cases are considered corresponding to the values of the helical pitch angle as 0° and 90°. A comparison of the dispersion relations is presented.

1. INTRODUCTION

Optical waveguides have been investigated extensively during the past four decades, and such guides with various forms of geometrical cross-sections have been explored in the literature [1–9]. Fibers with helical structures fall under the category of complex waveguides, and these have drawn considerable interest among the R&D community owing to the much use of helical structures in all low and medium power traveling wave tubes (TWTs) [10]. The analysis of helical structures generally includes waveguides under slow-wave consideration with conducting sheath and tape helixes. The implementation of this concept in the case of optical fibers has been discussed before by the investigators [11–16].

The use of helical windings in the case of optical fibers essentially makes the analysis much rigorous. However, such a winding is purposely introduced as it can control the dispersion characteristics of the guide. For example, it has been investigated before that, under fast-wave consideration, elliptical fibers with helical windings yield the existence of band gap for 0° helix pitch angle, which is attributed to the existence of periodicity in the structure. However, such band gaps were not observed corresponding to 90° pitch angle, which is owing to the elimination of periodicity [12] in this case. Further, under the fast-wave consideration, the number of propagating modes depends much on the helix pitch angle. The aim of the present communication is to compare the dispersion relations of circular step-index fiber having a conducting sheath helix [10] between the core and the cladding regions under slow- and fast-wave considerations.

2. THEORY

We consider the case of a fiber with circular cross-section wrapped with a sheath helix at the core-clad boundary, as shown in Fig. 1. The description of a sheath helix is in Ref. [10]. In practice, a sheath helix can be approximated by winding a very thin conducting wire around the cylindrical surface so that the spacing between the adjacent windings is very small and yet they are insulated from each another. In our structure, the helical windings are made at a constant angle ψ — the helix pitch angle. The structure has high conductivity in a preferential direction. The pitch angle can be effectively used to control the propagation behavior of such fibers, and serves as an additional controlling parameter [12–16]. We assume that the core and the cladding regions have the respective real refractive indices n_1 and n_2. In the case of slow-wave consideration, which essentially have $n_1 = n_2 = 1$, and the phase velocity $v_p < c$, the speed of light in free-space.

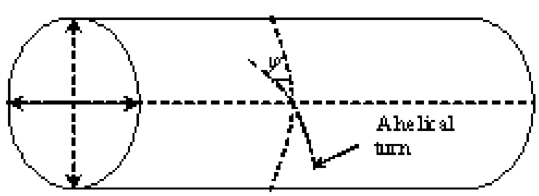

Figure 1. The sheath helix.

A fiber with helical windings is more explicitly illustrated in Fig. 2. An alternative way of realizing the sheath is to have a thin planer sheet made of alternate conducting thin strips and non-conducting gaps in an oblique fashion, and then wrapping it along the cylindrical core without any overlap (Fig. 3).

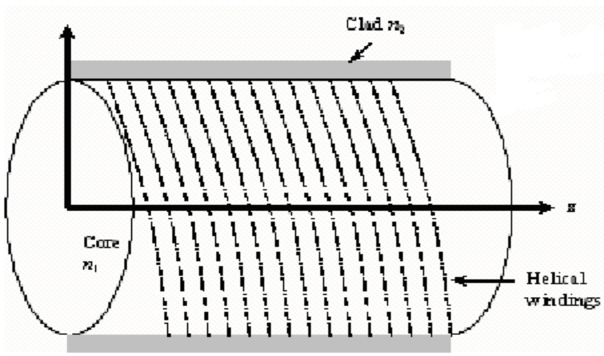

Figure 2. The fiber structure.

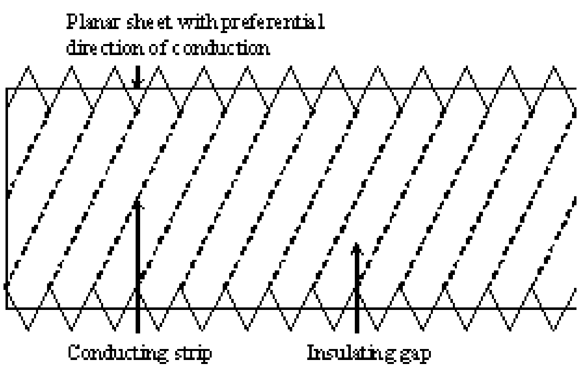

Figure 3. Planer sheet made of alternate conducting thin strips and non-conducting gaps.

We start the analysis with a sheath helix which is perfectly conducting in a direction making an angle ψ with the plane perpendicular to the axis, and vanishing conductivity in a direction normal to the direction of conduction. Although we present the analysis for the general case when there is no restriction on the pitch angle ψ, but for simplicity we consider only two particular values of ψ, viz. $0°$ and $90°$. The analysis requires the use of cylindrical coordinate system (r, ϕ, z) [17] with the z-axis being the direction of wave propagation.

The expressions for field components corresponding to circular step-index fiber [11] for slow and fast wave structures can be given as follows.

2.1. Field Components for Slow-wave Structure

$$E_{Z_1} = AI_\nu(\tau r)e^{-i\beta z}\cos\nu\phi \tag{1}$$

$$H_{Z_1} = BI_\nu(\tau r)e^{-i\beta z}\cos\nu\phi \tag{2}$$

$$E_{\phi_1} = -\left[\frac{i\nu\beta}{\tau^2 a}AI_\nu(\tau r)\sin\nu\phi + \frac{i\omega\mu}{\tau}BI'_\nu(\tau r)\cos\nu\phi\right]e^{-i\beta z} \tag{3}$$

$$H_{\phi_1} = \left[-\frac{i\nu\beta}{\tau^2 a}BI_\nu(\tau r)\sin\nu\phi + \frac{i\omega\varepsilon}{\tau}AI'_\nu(\tau r)\cos\nu\phi\right]e^{-i\beta z} \tag{4}$$

$$E_{Z_2} = CK_\nu(\tau r)e^{-i\beta z}\cos\nu\phi \tag{5}$$

$$H_{Z_2} = DK_\nu(\tau r)e^{-i\beta z}\cos\nu\phi \tag{6}$$

$$E_{\phi_2} = -\left[\frac{i\nu\beta}{\tau^2 a}CK_\nu(\tau r)\sin\nu\phi + \frac{i\omega\mu}{\tau}DK'_\nu(\tau r)\cos\nu\phi\right]e^{-i\beta z} \tag{7}$$

$$H_{\phi_2} = \left[-\frac{i\nu\beta}{\tau^2 a}DK_\nu(\tau r)\sin\nu\phi + \frac{i\omega\varepsilon}{\tau}CK'_\nu(\tau r)\cos\nu\phi\right]e^{-i\beta z} \tag{8}$$

2.2. Field Components for Fast-wave Structure

$$E_{Z_1} = A_1J_\nu(ur)e^{i(\omega t-\beta z+\nu\phi)} \tag{9}$$

$$H_{Z_1} = B_1J_\nu(ur)e^{i(\omega t-\beta z+\nu\phi)} \tag{10}$$

$$E_{\phi_1} = -\frac{i}{u^2}\left[\frac{i\nu\beta}{a}A_1J_\nu(ur) + \omega\mu_0 uB_1J'_\nu(ur)\right]e^{i(\omega t-\beta z+\nu\phi)} \tag{11}$$

$$H_{\phi_1} = -\frac{i}{u^2}\left[\frac{i\nu\beta}{a}B_1J_\nu(ur) + \omega\varepsilon_1 uA_1J'_\nu(ur)\right]e^{i(\omega t-\beta z+\nu\phi)} \tag{12}$$

$$E_{Z_2} = C_1K_\nu(wr)e^{i(\omega t-\beta z+\nu\phi)} \tag{13}$$

$$H_{Z_2} = B_1J_\nu(ur)e^{i(\omega t-\beta z+\nu\phi)} \tag{14}$$

$$E_{\phi_2} = -\frac{i}{w^2}\left[\frac{i\nu\beta}{a}C_1K_\nu(wr) - \omega\mu_0 wD_1K'_\nu(wr)\right]e^{i(\omega t-\beta z+\nu\phi)} \tag{15}$$

$$H_{\phi_2} = -\frac{i}{w^2}\left[\frac{i\nu\beta}{a}D_1K_\nu(wr) + \omega\varepsilon_2 wC_1K'_\nu(wr)\right]e^{i(\omega t-\beta z+\nu\phi)} \tag{16}$$

In above equations, the subscripts 1 and 2 correspond to the situations in the core and the cladding sections, respectively. Also, for the fast-wave structure

$$k^2 n_1^2 - \beta^2 = u^2 \quad \text{and} \quad \beta^2 - k^2 n_2^2 = w^2,$$

and for the slow wave structure

$$k^2 - \beta^2 = u^2 = \tau^2 \quad \text{and} \quad \beta^2 - k^2 = w^2 = -\tau^2.$$

2.3. Boundary Conditions

Remembering that the tangential component of the electric field in the direction of the conducting helix should be zero, and in the direction perpendicular to the helical winding, the tangential component of both the electric and magnetic field [18, 19] must be continuous, we can have the following boundary conditions for slow- as well as fast-wave structures:

$$E_{Z_1} \sin\psi + E_{\phi_1} \cos\psi = 0 \tag{17}$$

$$E_{Z_2} \sin\psi + E_{\phi_2} \cos\psi = 0 \tag{18}$$

$$(E_{Z_1} - E_{Z_2}) \cos\psi - (E_{\phi_1} - E_{\phi_2}) \sin\psi = 0 \tag{19}$$

$$(H_{Z_1} - H_{Z_2}) \sin\psi + (H_{\phi_1} - H_{\phi_2}) \cos\psi = 0 \tag{20}$$

2.4. Dispersion Relation under Slow-wave Consideration

Using Eqs. (1)–(8) and Eqs. (17)–(20), we finally get

$$\begin{aligned} &A\left[I_\nu(\tau a)\cos\nu\phi\sin\psi - \frac{i\nu\beta}{\tau^2 a} I_\nu(\tau a)\sin\nu\phi\cos\psi\right] \\ &+B\left[-\frac{i\omega\mu}{\tau} I'_\nu(\tau a)\cos\nu\phi\cos\psi\right] = 0 \end{aligned} \tag{21}$$

$$\begin{aligned} &C\left[K_\nu(\tau a)\cos\nu\phi\sin\psi - \frac{i\nu\beta}{\tau^2 a} K_\nu(\tau a)\sin\nu\phi\cos\psi\right] \\ &-D\left[\frac{i\omega\mu}{\tau} K'_\nu(\tau a)\cos\nu\phi\cos\psi\right] = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} &A\left[I_\nu(\tau a)\cos\nu\phi\cos\psi + \frac{i\nu\beta}{\tau^2 a} I_\nu(\tau a)\sin\nu\phi\sin\psi\right] \\ &+B\left[\frac{i\omega\mu}{\tau} I'_\nu(\tau a)\cos\nu\phi\sin\psi\right] \\ &-C\left[K_\nu(\tau a)\cos\nu\phi\cos\psi + \frac{i\nu\beta}{\tau^2 a} K_\nu(\tau a)\sin\nu\phi\sin\psi\right] \\ &-D\left[\frac{i\omega\mu}{\tau} K'_\nu(\tau a)\cos\nu\phi\sin\psi\right] = 0 \end{aligned} \tag{23}$$

$$
\begin{aligned}
&A\left[\frac{i\omega\varepsilon}{\tau}I'_\nu(\tau a)\cos\nu\phi\cos\psi\right]\\
&+B\left[I_\nu(\tau a)\cos\nu\phi\sin\psi-\frac{i\nu\beta}{\tau^2 a}I_\nu(\tau a)\sin\nu\phi\cos\psi\right]\\
&-C\left[\frac{i\omega\varepsilon}{\tau}K'_\nu(\tau a)\cos\nu\phi\cos\psi\right]\\
&D\left[K_\nu(\tau a)\cos\nu\phi\sin\psi-\frac{i\nu\beta}{\tau^2 a}K_\nu(\tau a)\sin\nu\phi\cos\psi\right]=0 \quad (24)
\end{aligned}
$$

In Eqs. (21)–(24), a is the radius of the fiber core. Eliminating the constants A, B, C and D from the above set of Eqs. (21)–(24), we finally get, corresponding to $\psi = 0°$, the dispersion relation as

$$
\begin{aligned}
&\frac{\nu^2\beta^2}{\tau^4a^2}I_\nu^2(\tau a)K_\nu(\tau a)K'_\nu(\tau a)\sin^2\nu\phi+\frac{\nu^2\beta^2}{\tau^4a^2}I_\nu(\tau a)I'_\nu(\tau a)K_\nu^2(\tau a)\sin^2\nu\phi\\
&+\frac{\omega^2\mu\varepsilon}{\tau^2}I_\nu(\tau a)I'_\nu(\tau a)K'^{2}_\nu(\tau a)\cos^2\nu\phi-\frac{\omega^2\mu\varepsilon}{\tau^2}I'_\nu(\tau a)K_\nu(\tau a)K'_\nu(\tau a)\cos^2\nu\phi\\
&=0 \qquad (25)
\end{aligned}
$$

Considering a special case corresponding to $\nu = 1$ and $\phi = 0°$, we can have

$$I_1(\tau a)K'_1(\tau a)-K_1(\tau a)=0 \qquad (26)$$

On the other hand, $\nu = 1$ and $\phi = 90°$ yield

$$I_1(\tau a)K'_1(\tau a)-I'_1(\tau a)K_1(\tau a)=0 \qquad (27)$$

Following the above procedure, corresponding to $\psi = 90°$, we get the dispersion relation as

$$I'_\nu(\tau a)K_\nu(\tau a)-I_\nu(\tau a)K'_\nu(\tau a)=0 \qquad (28)$$

which, for $\nu = 1$, gives

$$I'_1(\tau a)K_1(\tau a)-I_1(\tau a)K'_1(\tau a)=0 \qquad (29)$$

This is to be pointed out here that, in order to avoid mathematical complexity, we consider the low order azimuthal mode index (i.e., $\nu = 1$). However, the analysis is valid for any order of the mode index.

2.5. Dispersion Relation under Fast-wave Consideration

Using Eqs. (9)–(16) and Eqs. (17)–(20), we finally obtain

$$\left(\sin\psi+\frac{\nu\beta}{u^2a}\cos\psi\right)A_1J_\nu(ua)+\frac{i\omega\mu_0}{u}B_1J'_\nu(ua)\cos\psi=0 \tag{30}$$

$$\left(\sin\psi+\frac{\nu\beta}{w^2a}\cos\psi\right)C_1K_\nu(wa)+\frac{i\omega\mu_0}{w}D_1K'_\nu(wa)\cos\psi=0 \tag{31}$$

$$\begin{aligned}&\left(\cos\psi-\frac{\nu\beta}{u^2a}\sin\psi\right)A_1J_\nu(ua)-\frac{i\omega\mu_0}{u}B_1J'_\nu(ua)\\&-\left(\cos\psi-\frac{\nu\beta}{w^2a}\sin\psi\right)C_1K_\nu(wa)+\frac{i\omega\mu_0}{w}\sin\psi D_1K'_\nu(wa)=0\end{aligned} \tag{32}$$

$$\begin{aligned}&-\frac{i\omega\varepsilon_1}{u}A_1J'_\nu(ua)\cos\psi+\left(\sin\psi+\frac{\nu\beta}{u^2a}\cos\psi\right)B_1J_\nu(ua)\\&+\frac{i\omega\varepsilon_2}{w}C_1K'_\nu(wa)\cos\psi-\left(\sin\psi+\frac{\nu\beta}{w^2a}\cos\psi\right)D_1K_\nu(wa)=0\end{aligned} \tag{33}$$

Eliminating the constants A_1, B_1, C_1 and D_1 from the above Eqs. (30)–(33), we finally get the dispersion relation, corresponding to $\psi=0^\circ$ and $\nu=1$, as

$$\begin{aligned}&\frac{\beta^2}{u^3a^2}J_1^2(ua)K_1(wa)\left\{-\frac{K_1(wa)}{w}-K_0(wa)\right\}\\&-\frac{\beta^2}{w^3a^2}\left\{-\frac{J_1^2(ua)K_1^2(wa)}{u}+J_0(ua)J_1(ua)K_1^2(wa)\right\}\\&-\frac{4(\pi/\lambda)^2n_2^2}{w}\left\{-\frac{J_1^2(ua)}{u}+J_0(ua)J_1(ua)\right\}\left\{\frac{K_1^2(wa)}{w^2}+K_0^2(wa)\right.\\&\left.+2K_0(wa)\frac{K_1(wa)}{w}\right\}-\frac{4(\pi/\lambda)^2n_1^2}{u}\left\{\frac{J_1^2(ua)}{u^2}+J_0^2(ua)-\frac{2J_1(ua)}{u}J_0(ua)\right\}\\&\left\{-\frac{K_1^2(wa)}{w}-K_0(wa)K_1(wa)\right\}=0\end{aligned} \tag{34}$$

The dispersion relation, corresponding to $\psi=90^\circ$ and $\nu=1$, becomes

$$\begin{aligned}&\frac{1}{u^2}\left\{J_1(ua)K_1(wa)+uJ_0(ua)K_1(wa)\right\}\\=\;&\frac{1}{w^2}\left\{J_1(ua)K_1(wa)+wJ_1(ua)K_0(wa)\right\}\end{aligned} \tag{35}$$

3. RESULTS AND DISCUSSION

In this communication, we focus our analysis on the variation of the dispersion behavior of the fiber under consideration. In order to plot the dispersion relations, we plot the normalized frequency parameter V against the normalized propagation constant b_{nor}, given as

$$b_{nor} = \left\{ \frac{\beta^2 - k^2 n_2^2}{k^2(n_1^2 - n_2^2)} \right\}^{1/2}. \tag{36}$$

In our Illustrative case, we consider $n_1 = 1.5$, $n_2 = 1.46$, and the operating wavelength $\lambda = 1.55\,\mu$m. As stated earlier, we considered two special cases corresponding to the values of the pitch angle ψ as $0°$ and $90°$.

The dispersion curves corresponding to Eqs. (34) and (35) are shown in Figs. 4 and 5, respectively. We observe in Fig. 4 that, corresponding to the case of fast-wave structure with $\psi = 0°$ and $\nu = 1$, the dispersion curves have the usual trend, as observed in the case of other general type of fiber. However, in the present case, we notice the strange feature that there exists one band gap which falls within the limiting range $V = 27$ to $V = 29$. Also, we find that the first modal cutoff exists at $V \approx 4$. From the features of the dispersion curves, it may the inferred that an additional effect of the use of conducting helical winding is to split a mode into a pair of adjacent modes, which is essentially equivalent to removing the mode degeneracy.

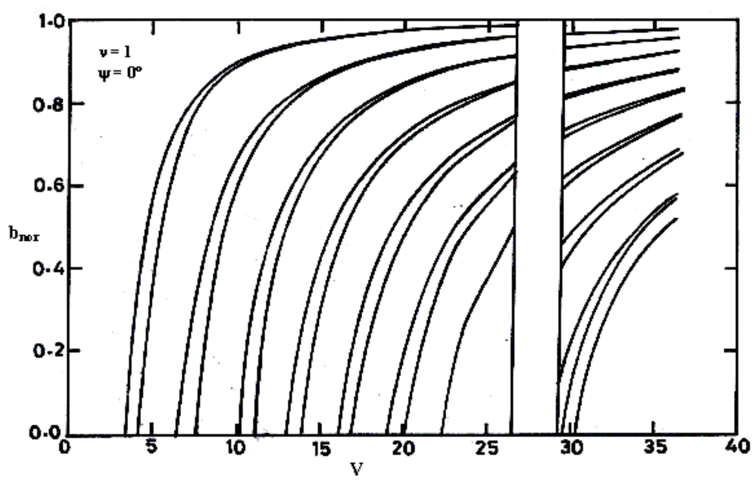

Figure 4. Dispersion curve corresponding to $\psi = 0°$.

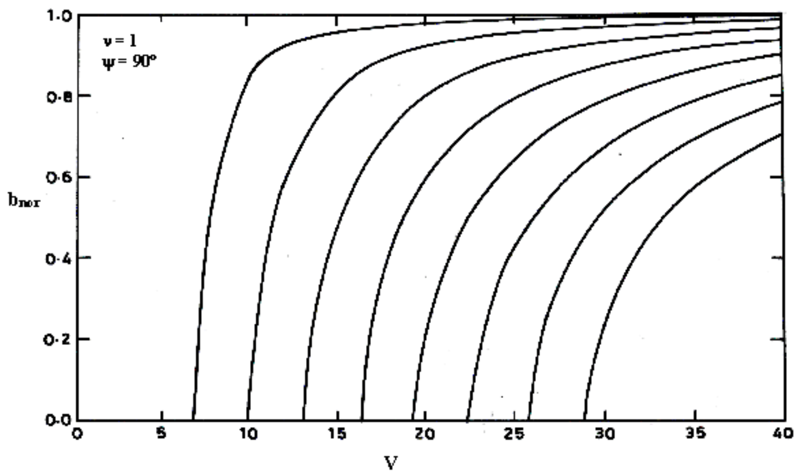

Figure 5. Dispersion curve corresponding to $\psi = 90°$.

Corresponding to the case of $\psi = 90°$ and $\nu = 1$, as illustrated in Fig. 5, we observe that the degeneracy of modes is again sustained, which is owing to the reason that the helical windings are now only parallel to the optical axis of the fiber. In other words, in this case, the sheath helix essentially degenerates into a sheath made of conducting lines parallel to the optical axis of the fiber. Further, since there is no periodicity observed in the direction of wave propagation, the existence of band gap is strictly eliminated. We also observe that the cutoff of the first mode [20–29] exists approximately at $V \approx 7$. As such, we see that the introduction of helical winding reduces the modal cutoff.

The descriptive analysis of the fiber under the slow-wave consideration is still in progress, and will be taken up in a future communication. The authors expect that all the results stated in the paper are of much technical significance owing to the features of the guide governed by the helixes.

4. CONCLUSION

Form the above analytical investigation, conclusion may be drawn that the helical windings play a vital role in determining the propagation characteristics of the fiber. The introduction of helix along the direction perpendicular to the propagation axis brings in a kind of band gap in fibers, and also, shifts the modal cutoff to a lower value as compared to the case when the helical turns are only parallel to the optical axis.

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