

## **DESIGN OF BAND-PASS WAVEGUIDE FILTER USING FREQUENCY SELECTIVE SURFACES LOADED WITH SURFACE MOUNT CAPACITORS BASED ON SPLIT-FIELD UPDATE FDTD METHOD**

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**Abstract**—A new band-pass narrow-band, miniaturized and single resonance within a wide range of frequency band frequency selective surface structure with lumped capacitors suitable for low frequency and narrowband waveguide filter applications is presented. The filter structure consists of five unit cells in one direction with notch square ring elements each consists of two lumped capacitors placed on the transverse plane of the rectangular waveguide. To reduce the simulation time in design procedure, split-field update FDTD method is used for the analysis of the unit cell of the analogous infinite frequency selective surface at different oblique incidence of plane waves to predict the FSS performance in the waveguide. As an application, a waveguide filter has been designed to be used in an easy to fabricate and inexpensive S-band band-pass filter. By using lumped capacitors, several undesirable higher order resonance frequencies near the dominant resonance frequency are eliminated and the waveguide filter dimensions are reduced considerably in one direction compared with the analogous waveguide filter without lumped capacitors.

### **1. INTRODUCTION**

In the past years, waveguide filters by using periodic structures with periodicity in the longitudinal direction of the waveguide, to attain a band-pass or band-stop frequency characteristic have been introduced [1,2]. Frequency selective surfaces (FSSs) have also been used for their state of being special to act as metallic-dielectric spatial filters in order to design dual-frequency waveguide filters [3,4]. Recently, filter designing by placing FSSs transversally into the waveguide to achieve more advantages including miniaturization, being

lightweight and easy to be fabricated, have been presented [5,6]. Making multiple images of the FSS, the metallic walls of the waveguide cause the FSS unit cell to be infinite [4,6]. The fundamental mode of a rectangular waveguide can be considered as two transverse electric uniform plane waves (in the longitudinal direction of the waveguide) bouncing back and forth at oblique incidence between the waveguide smaller size side walls having  $180^\circ$  phase reversal with the propagation angle that depends on frequency and can be expressed as [7]:

$$\theta_{inc}(Degree) = \frac{180}{\pi} \cos^{-1} \left( \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} / \left(\frac{2\pi f}{c}\right) \right) \quad (1)$$

In this equation,  $c$ ,  $a$  and  $f$  are the free-space velocity of light, the widest dimension of the rectangular waveguide, and the operating frequency respectively. Therefore, if the FSS exhibits stable resonance with respect to polarization and incidence angle of plane waves, the waveguide filter characteristics will be more symmetric around the resonance frequency and more similar to the frequency response of the analogous infinite FSS. When used as a band-pass filter in the waveguide, a conventional FSS may pass some higher order resonance frequencies that is not favorable. Furthermore the unit cell size in a conventional FSS is large for low frequency bands. Lumped elements including inductors and capacitors at microwave and millimeter wave frequencies are now available and can be used in high frequency band periodic structures such as frequency selective surfaces [8] and artificial magnetic conductors. In this paper, lumped capacitors are used in the FSS structure as an alternative approach to achieve all these features simultaneously. In order to reduce the simulation time in the waveguide filter design procedure, first the unit cell size of the analogous infinite FSS is analyzed. Because these structures are needed to be analyzed over a wide range of frequency band, fast time domain methods are preferred rather than the frequency domain methods. Several time domain methods have been presented for the analysis of periodic structures. Split-field update FDTD method [9] is one of the most powerful methods used for the analysis of periodic structures at oblique incidence. Here, lumped elements formulations are incorporated in this method formulation based on [10] and the proposed structure is analyzed using this method. Finally a band-pass waveguide filter is designed using the presented FSS structure.

## 2. METHOD FORMULATION

The proposed method for including lumped elements in the split-field method is presented in this section for 3D case. Here a parallel  $RLC$  is considered. When a parallel  $RLC$  is inserted into the FDTD space lattice, the Maxwell's curl  $\vec{H}$  equation becomes [10]:

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{Res} + \vec{J}_{Cap} + \vec{J}_{Induc} \quad (2)$$

$$J_{z-Res} = \frac{dz}{R dx dy} E_z \quad (3)$$

$$J_{z-Cap} = \frac{C dz}{dx dy} \left( \frac{\partial E_z}{\partial t} \right) \quad (4)$$

$$J_{z-Induc} = \frac{dz}{L dx dy} \int_0^t E_z dt \quad (5)$$

A periodic structure in  $y$  and  $z$  directions is considered and lumped elements are  $x$ ,  $y$  or  $z$ -directed that is defined by an index. Using (2)–(5), the 3D transformed field equations with lumped elements for  $P$  components are given by:

$$\begin{aligned} \frac{\varepsilon_r}{c} \frac{\partial P_x}{\partial t} + \sigma \eta_0 P_x = & \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} - \frac{\bar{k}_y}{c} \frac{\partial Q_z}{\partial t} + \frac{\bar{k}_z}{c} \frac{\partial Q_y}{\partial t} - \frac{dx}{c \varepsilon_0 R_x dy dz} P_x \\ & - \frac{C_x dx}{c \varepsilon_0 dy dz} \left( \frac{\partial P_x}{\partial t} \right) - \frac{c \mu_0 dx}{L_x dy dz} \int_0^t P_x dt \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\varepsilon_r}{c} \frac{\partial P_y}{\partial t} + \sigma \eta_0 P_y = & -\frac{\partial Q_z}{\partial x} + \frac{\partial Q_x}{\partial z} - \frac{\bar{k}_z}{c} \frac{\partial Q_x}{\partial t} - \frac{dy}{c \varepsilon_0 R_y dx dz} P_y \\ & - \frac{C_y dy}{c \varepsilon_0 dx dz} \left( \frac{\partial P_y}{\partial t} \right) - \frac{c \mu_0 dy}{L_y dx dz} \int_0^t P_y dt \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\varepsilon_r}{c} \frac{\partial P_z}{\partial t} + \sigma \eta_0 P_z = & \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} + \frac{\bar{k}_y}{c} \frac{\partial Q_x}{\partial t} - \frac{dz}{c \varepsilon_0 R_z dx dy} P_z \\ & - \frac{C_z dz}{c \varepsilon_0 dx dy} \left( \frac{\partial P_z}{\partial t} \right) - \frac{c \mu_0 dz}{L_z dx dy} \int_0^t P_z dt \end{aligned} \quad (8)$$

$$\frac{\mu_r}{c} \frac{\partial Q_x}{\partial t} + \frac{\sigma}{\eta_0} Q_x = -\frac{\partial P_z}{\partial y} + \frac{\partial P_y}{\partial z} + \frac{\bar{k}_y}{c} \frac{\partial P_z}{\partial t} - \frac{\bar{k}_z}{c} \frac{\partial P_y}{\partial t} \quad (9)$$

$$\frac{\mu_r}{c} \frac{\partial Q_y}{\partial t} + \frac{\sigma}{\eta_0} Q_y = \frac{\partial P_z}{\partial x} + \frac{\partial P_x}{\partial z} + \frac{\bar{k}_z}{c} \frac{\partial P_x}{\partial t} \quad (10)$$

$$\frac{\mu_r}{c} \frac{\partial Q_z}{\partial t} + \frac{\sigma}{\eta_0} Q_z = -\frac{\partial P_y}{\partial x} + \frac{\partial P_x}{\partial y} - \frac{\bar{k}_y}{c} \frac{\partial P_x}{\partial t} \quad (11)$$

By defining new variables, the extra time derivatives on the right hand side can be eliminated. In [9] the fields in the directions of the structure periodicities are split into two components. Here, UPML absorbing boundary condition is used. For similar periodic boundary conditions for “a” portions of the fields in the UPML region and non-UPML region, the field components in the direction of the structure periodicities are split into three components as for the UPML region [11]. New variables must be defined as follows:

$$P_x = P_{xa} + \frac{\bar{k}_z}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} Q_y - \frac{\bar{k}_y}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} Q_z; \quad Q_x = Q_{xa} - \frac{\bar{k}_z}{\mu_r} P_y + \frac{\bar{k}_y}{\mu_r} P_z \quad (12)$$

$$P_y = P_{ya} + P_{yb} - \frac{\bar{k}_z}{\varepsilon_r + \frac{C_y dy}{\varepsilon_0 dx dz}} Q_x; \quad Q_y = Q_{ya} + Q_{yb} + \frac{\bar{k}_z}{\mu_r} P_x \quad (13)$$

$$P_z = P_{za} + P_{zb} + \frac{\bar{k}_y}{\varepsilon_r + \frac{C_z dz}{\varepsilon_0 dx dy}} Q_x; \quad Q_z = Q_{za} + Q_{zb} - \frac{\bar{k}_y}{\mu_r} P_x \quad (14)$$

Substituting these variables into the systems (6)–(11) results in:

$$\begin{aligned} & \left( \frac{\varepsilon_r}{c} + \frac{C_x dx}{c \varepsilon_0 dy dz} \right) \frac{\partial P_{xa}}{\partial t} + \left( \sigma \eta_0 + \frac{dx}{c \varepsilon_0 R_x dy dz} \right) P_{xa} \\ &= \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} + \frac{\sigma \eta_0 + \frac{dx}{c \varepsilon_0 R_x dy dz}}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} \bar{k}_y Q_z - \frac{\sigma \eta_0 + \frac{dx}{c \varepsilon_0 R_x dy dz}}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} \bar{k}_z Q_y \\ & - \frac{c \mu_0 dx}{L_x dy dz} \int_0^t P_{xa} dt - \frac{\bar{k}_z}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} \frac{c \mu_0 dx}{L_x dy dz} \int_0^t Q_y dt \\ & - \frac{\bar{k}_y}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} \frac{c \mu_0 dx}{L_x dy dz} \int_0^t Q_z dt \end{aligned} \quad (15)$$

$$\begin{aligned} & \left( \frac{\varepsilon_r}{c} + \frac{C_y dy}{c \varepsilon_0 dx dz} \right) \frac{\partial P_{ya}}{\partial t} + \left( \sigma \eta_0 + \frac{dy}{c \varepsilon_0 R_y dx dz} \right) P_{ya} \\ &= -\frac{\partial Q_z}{\partial x} - \frac{c \mu_0 dy}{L_y dx dz} \int_0^t P_{ya} dt \end{aligned} \quad (16)$$

$$\begin{aligned}
& \left( \frac{\varepsilon_r}{c} + \frac{C_y dy}{c\varepsilon_0 dx dz} \right) \frac{\partial P_{yb}}{\partial t} + \left( \sigma\eta_0 + \frac{dy}{c\varepsilon_0 R_y dx dz} \right) P_{yb} \\
&= \frac{\partial Q_x}{\partial z} + \frac{\sigma\eta_0 + \frac{dy}{c\varepsilon_0 R_y dx dz}}{\varepsilon_r + \frac{C_y dy}{\varepsilon_0 dx dz}} \bar{k}_z Q_x - \frac{c\mu_0 dy}{L_y dx dz} \int_0^t P_{yb} dt \\
&\quad - \frac{\bar{k}_z}{\varepsilon_r + \frac{C_y dy}{\varepsilon_0 dx dz}} \frac{c\mu_0 dy}{L_y dx dz} \int_0^t Q_x dt
\end{aligned} \tag{17}$$

$$\begin{aligned}
& \left( \frac{\varepsilon_r}{c} + \frac{C_z dz}{c\varepsilon_0 dx dy} \right) \frac{\partial P_{za}}{\partial t} + \left( \sigma\eta_0 + \frac{dz}{c\varepsilon_0 R_z dx dy} \right) P_{za} \\
&= \frac{\partial Q_y}{\partial x} - \frac{c\mu_0 dz}{L_z dx dy} \int_0^t P_{za} dt
\end{aligned} \tag{18}$$

$$\begin{aligned}
& \left( \frac{\varepsilon_r}{c} + \frac{C_z dz}{c\varepsilon_0 dx dy} \right) \frac{\partial P_{zb}}{\partial t} + \left( \sigma\eta_0 + \frac{dz}{c\varepsilon_0 R_z dx dy} \right) P_{zb} \\
&= -\frac{\partial Q_x}{\partial y} - \frac{\sigma\eta_0 + \frac{dz}{c\varepsilon_0 R_z dx dy}}{\varepsilon_r + \frac{C_z dz}{\varepsilon_0 dx dy}} \bar{k}_y Q_x - \frac{c\mu_0 dz}{L_z dx dy} \int_0^t P_{zb} dt \\
&\quad - \frac{\bar{k}_y}{\varepsilon_r + \frac{C_z dz}{\varepsilon_0 dx dy}} \frac{c\mu_0 dz}{L_z dx dy} \int_0^t Q_x dt
\end{aligned} \tag{19}$$

$$\frac{\mu_r}{c} \frac{\partial Q_{xa}}{\partial t} + \frac{\sigma}{\eta_0} Q_{xa} = -\frac{\partial P_z}{\partial y} + \frac{\partial P_y}{\partial z} - \frac{\sigma \bar{k}_y}{\eta_0 \mu_r} P_z + \frac{\sigma \bar{k}_z}{\eta_0 \mu_r} P_y \tag{20}$$

$$\frac{\mu_r}{c} \frac{\partial Q_{ya}}{\partial t} + \frac{\sigma}{\eta_0} Q_{ya} = -\frac{\partial P_z}{\partial x} \tag{21}$$

$$\frac{\mu_r}{c} \frac{\partial Q_{yb}}{\partial t} + \frac{\sigma}{\eta_0} Q_{yb} = -\frac{\partial P_x}{\partial z} - \frac{\sigma \bar{k}_y}{\eta_0 \mu_r} P_z - \frac{\sigma \bar{k}_z}{\eta_0 \mu_r} P_x \tag{22}$$

$$\frac{\mu_r}{c} \frac{\partial Q_{za}}{\partial t} + \frac{\sigma}{\eta_0} Q_{za} = -\frac{\partial P_y}{\partial x} \tag{23}$$

$$\frac{\mu_r}{c} \frac{\partial Q_{zb}}{\partial t} + \frac{\sigma}{\eta_0} Q_{zb} = \frac{\partial P_x}{\partial y} + \frac{\sigma \bar{k}_y}{\eta_0 \mu_r} P_x \tag{24}$$

Once “a” and “b” portions are updated, the new values of the  $x$ -

components are found by:

$$\begin{aligned} & \left( 1 - \frac{\bar{k}_y^2}{\mu_r \left( \varepsilon_r + \frac{C_z dz}{\varepsilon_0 dx dy} \right)} - \frac{\bar{k}_z^2}{\mu_r \left( \varepsilon_r + \frac{C_y dy}{\varepsilon_0 dx dz} \right)} \right) Q_x \\ &= Q_{xa} - \frac{\bar{k}_z}{\mu_r} (P_{ya} + P_{yb}) + \frac{\bar{k}_y}{\mu_r} (P_{za} + P_{zb}) \end{aligned} \quad (25)$$

$$\begin{aligned} & \left( 1 - \frac{\bar{k}_y^2}{\mu_r \left( \varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz} \right)} - \frac{\bar{k}_z^2}{\mu_r \left( \varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz} \right)} \right) P_x \\ &= P_{xa} + \frac{\bar{k}_z}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} (Q_{ya} + Q_{yb}) - \frac{\bar{k}_y}{\varepsilon_r + \frac{C_x dx}{\varepsilon_0 dy dz}} (Q_{za} + Q_{zb}) \end{aligned} \quad (26)$$

The remaining field components are given by:

$$P_y = P_{ya} + P_{yb} - \frac{\bar{k}_z}{\left( \varepsilon_r + \frac{C_y dy}{\varepsilon_0 dx dz} \right)} Q_x; \quad P_z = P_{za} + P_{zb} + \frac{\bar{k}_y}{\left( \varepsilon_r + \frac{C_z dz}{\varepsilon_0 dx dy} \right)} Q_x \quad (27)$$

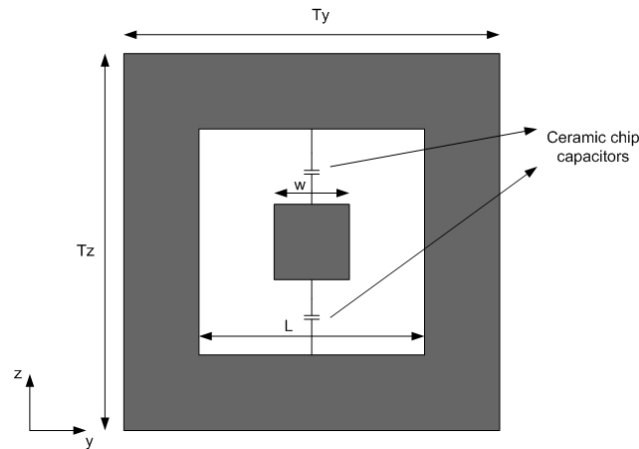
$$Q_y = Q_{ya} + Q_{yb} + \frac{\bar{k}_z}{\mu_r} P_x; \quad Q_z = Q_{za} + Q_{zb} - \frac{\bar{k}_y}{\mu_r} P_x \quad (28)$$

The stability factor condition does not change and is explained in [9].

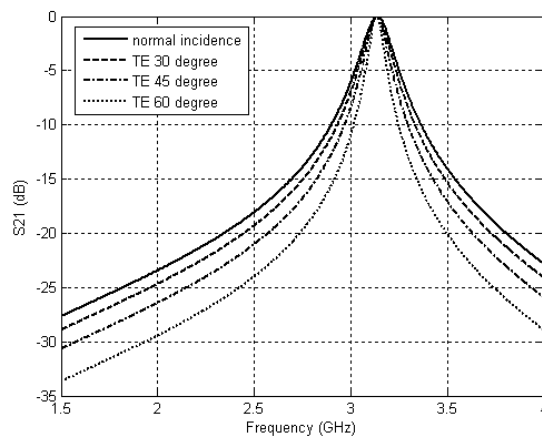
### 3. FILTER DESIGN AND NUMERICAL RESULTS

In order to design the waveguide filter, first the analogous infinite band-pass FSS used in the waveguide filter is analyzed using split-field method to predict the waveguide filter performance. The unit cell of the FSS structure designed at the 3.15 GHz with four  $2 \text{ mm} \times 1 \text{ mm}$  ceramic chip capacitors is shown in Fig. 1. All capacitors are 1 pf. A dielectric substrate with  $\varepsilon_r = 2.5$  and  $h = 1 \text{ mm}$  is used. The transmission coefficient of this structure for different incidence angles of  $TE_x$  plane wave with  $z$ -directed electric field component is illustrated in Fig. 2. As it shows, the structure exhibits stable resonance in different incidence angles. The maximum shift in the resonance frequency is less than 0.1%. The transmission coefficient of this structure at normal incidence from 1GHz to 14GHz is shown in Fig. 3. The only resonance frequency in this frequency band is 3.15GHz. Therefore, several undesirable higher order resonance frequencies near the dominant resonance frequency are eliminated. In fact, not only the unit cell size of a conventional FSS with notch square ring elements without

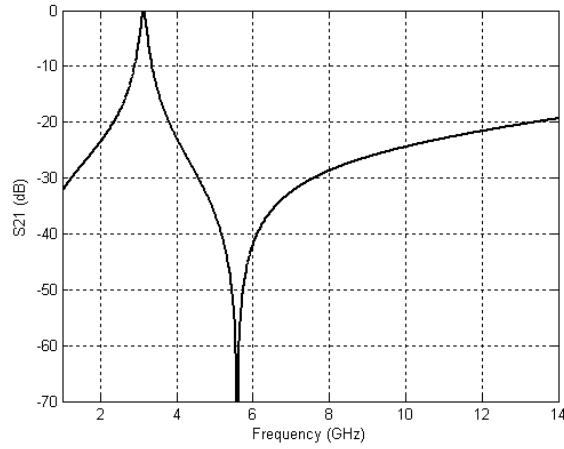
lumped capacitors at this frequency band is larger by a factor of about four in comparison with the structure shown in Fig. 1, but also a conventional notch square ring FSS is transparent at some higher order resonance frequencies near the dominant resonance frequency that is not favourable. Furthermore, the maximum resonance frequency shift for different incidence angles for this structure is much less than the resonance frequency shift of a conventional FSS with notch square ring.



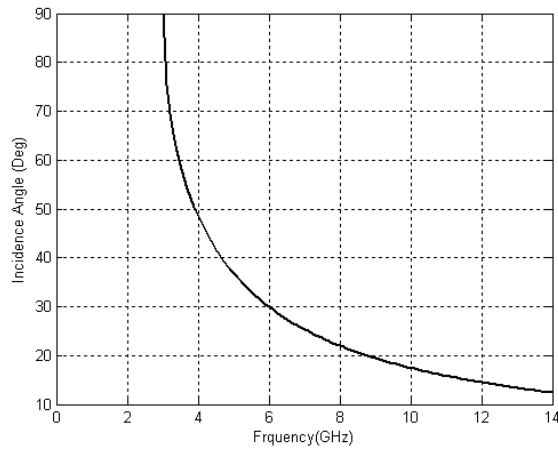
**Figure 1.** The Geometry of notch square ring FSS with ceramic chip capacitors.  $(Ty, Tz, L, W) = (10, 10, 6, 2)$  mm.



**Figure 2.** Transmission coefficient of the FSS structure shown in Fig. 1 for different plan waves.

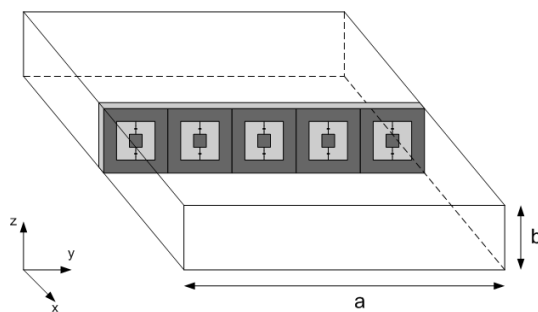


**Figure 3.** Transmission coefficient of the FSS structure shown in Fig. 1 for normal incidence.

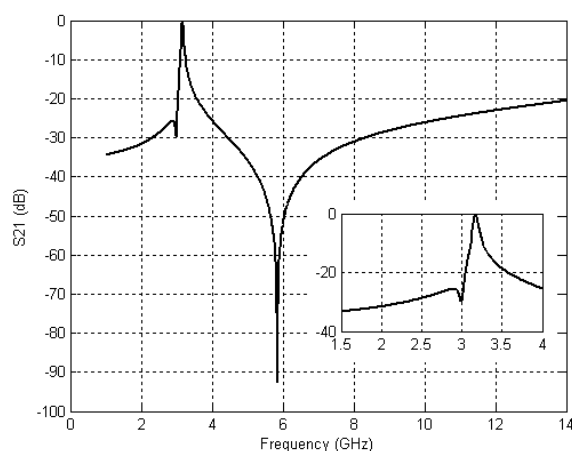


**Figure 4.** The incidence angle of the corresponding plane wave versus frequency for the dominant mode of the waveguide shown in Fig. 5.

The waveguide filter setup is shown in Fig. 5. The widest side of the waveguide size is chosen so that the waveguide cut-off frequency is below the FSS resonance frequency. Therefore, five FSS unit cells can be placed in the transverse plane of the rectangular waveguide. Fig. 4 illustrates the incidence angle of the corresponding plane wave bouncing at oblique incidence between the waveguide smaller size lateral walls versus frequency for the dominant mode of this waveguide



**Figure 5.** Setup of the filter within the waveguide. ( $a = 50$  mm,  $b = 10$  mm).



**Figure 6.** Transmission coefficient for the dominant mode of the waveguide filter shown in Fig. 5.

obtained by (1). The transmission coefficient of the waveguide filter from 1GHz to 14GHz is shown in Fig. 6. As it shows, the resonance occurs at 3.17GHz and there is no other resonance frequency in this frequency band. The frequency response of the waveguide filter at each frequency is similar to the frequency response of the analogous infinite FSS excited by a plane wave with the corresponding incidence angle showed in Fig. 4. If a conventional notch square ring FSS without lumped capacitors is used for the design of a waveguide filter at this frequency, the size of the narrower waveguide side increased about 80% in comparison with the waveguide shown in Fig. 5, because the FSS unit cell size is increased about 80%, while the wider side size is

the same as the waveguide shown in Fig. 5 because of the waveguide cut-off frequency considerations. Therefore, the waveguide dimension in one direction is reduced considerably and undesirable higher order resonance frequencies near the dominant resonance frequency are eliminated by using lumped capacitors in the FSS structure. The filter bandwidth is less than 2%. Therefore, it is suitable for narrow band applications.

#### 4. CONCLUSION

A new band-pass FSS structure with lumped capacitors suitable for the design of the waveguide filters has been presented. By using lumped capacitors, the waveguide filter dimension is reduced about 80% in one direction compared with the analogous waveguide filter without lumped capacitors. Furthermore several undesirable higher order resonance frequencies near the dominant resonance frequency are eliminated. The future work is designing wideband waveguide filters by using multiple FSSs placed on the transverse plane of the rectangular waveguide.

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