

A MOMENT METHOD SIMULATION OF ELECTROMAGNETIC SCATTERING FROM CONDUCTING BODIES

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Abstract—In this paper a moment method simulation of electromagnetic scattering problem is presented. An effective numerical method for solving this problem based on the method of moments and using block-pulse basis functions is proposed. Some examples of engineering interest are included to illustrate the procedure. The scattering problem is treated in detail, and illustrative computations are given for some cases. This method can be generalized to apply to objects of arbitrary geometry and arbitrary material.

1. INTRODUCTION

The development of numerical methods for solving integral equations in Electromagnetics has attracted intensive researches for more than four decades [1, 2]. The use of high-speed computers allows one to

make more computations than ever before. During these years, careful analysis has paved the way for the development of efficient and effective numerical methods and, of equal importance, has provided a solid foundation for a thorough understanding of the techniques.

Over several decades, electromagnetic scattering problems have been the subject of extensive researches (see [3–41]). Scattering from arbitrary surfaces such as square, cylindrical, circular, spherical [3–9] are commonly used as test cases in computational Electromagnetics, because analytical solutions for scattered fields can be derived for these geometries [3].

Simulating the problem of electromagnetic scattering from conducting bodies leads to solve the integral equations of the first kind with complex kernels. For solving integral equations of the first kind, several numerical approaches have been proposed. These numerical methods often use the basis functions and transform the integral equation to a linear system that can be solved by direct or iterative methods [42]. It is important in these methods to select an appropriate set of basis functions so that the approximate solution of integral equation has a good accuracy.

It is the purpose of this paper to use a set of orthogonal basis functions called block-pulse functions (BPFs) and to apply them to the method of moments for solving the problem of electromagnetic scattering. Using this method, the first kind integral equation reduces to a linear system of algebraic equations. Solving this system gives an approximate solution for these problems.

First of all, the electric field integral equation is introduced. Some characteristics of BPFs are described in Section 3. Then, the method of moments is proposed for solving integral equations of the first kind using block-pulse basis functions which is presented in Section 4. Finally, the problem of electromagnetic scattering from conducting bodies is described in detail and solved by the presented method in Section 5, and illustrative computations are given to complete the procedure.

2. ELECTRIC AND MAGNETIC FIELD INTEGRAL EQUATIONS

The key to the solution of any scattering problem is a knowledge of the physical or equivalent current density distributions on the volume or surface of the scatterer. Once these are known then the radiated or scattered fields can be found using the standard radiation integrals. A main objective then of any solution method is to be able to predict accurately the current densities over the scatterer. This can

be accomplished by the integral equation (IE) method [43].

In general there are many forms of integral equations. Two of the most popular for time-harmonic Electromagnetics are the *electric field integral equation* (EFIE) and the *magnetic field integral equation* (MFIE). The EFIE enforces the boundary condition on the tangential electric field and the MFIE enforces the boundary condition on the tangential components of the magnetic field. The electric field integral equation will be discussed here.

2.1. Electric Field Integral Equation

The electric field integral equation (EFIE) is based on the boundary condition that the total tangential electric field on a perfectly electric conducting (PEC) surface of scatterer is zero [43]. This can be expressed as:

$$\mathbf{E}_t^t(\mathbf{r} = \mathbf{r}_s) = \mathbf{E}_t^{inc}(\mathbf{r} = \mathbf{r}_s) + \mathbf{E}_t^{scat}(\mathbf{r} = \mathbf{r}_s) = 0 \quad \text{on } S \quad (1)$$

or

$$\mathbf{E}_t^{scat}(\mathbf{r} = \mathbf{r}_s) = -\mathbf{E}_t^{inc}(\mathbf{r} = \mathbf{r}_s) \quad \text{on } S \quad (2)$$

where S is the conducting surface of the scatterer and $\mathbf{r} = \mathbf{r}_s$ is the position vector of any point on the surface of the scatterer. The subscript t indicates tangential components.

The incident field that impinges on the surface of the scatterer induces on it an electric current density \mathbf{J}_s which in turn radiates the scattered field. The scattered field everywhere can be found using the following equation [43]:

$$\mathbf{E}^{scat}(\mathbf{r}) = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{A}) = -j\frac{1}{\omega\mu\epsilon} [\omega^2\mu\epsilon\mathbf{A} + \nabla(\nabla \cdot \mathbf{A})] \quad (3)$$

where

ϵ is the permittivity of the medium.

μ is the permeability of the medium.

ω is the angular frequency of the incident field.

∇ is the gradient operator.

\mathbf{A} is the magnetic vector potential, so that:

$$\mathbf{A}(\mathbf{r}) = \mu \int \int_S \mathbf{J}_s(\mathbf{r}') \frac{e^{-j\beta R}}{4\pi R} ds' \quad (4)$$

where, R is the distance from source point to the observation point.

Equations (3) and (4) can also be expressed as [43]:

$$\begin{aligned} & \mathbf{E}^{scat}(\mathbf{r}) \\ &= -j\frac{\eta}{\beta} \left[\beta^2 \int \int_S \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' + \nabla \int \int_S \nabla' \cdot \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' \right] \end{aligned} \quad (5)$$

where, η is the intrinsic impedance of the medium and β is the phase constant; \mathbf{r} and \mathbf{r}' are the position vectors of the observation point and source point respectively. also:

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-j\beta R}}{4\pi R} \quad (6)$$

$$R = |\mathbf{r} - \mathbf{r}'| \quad (7)$$

In Eq. (5), ∇ and ∇' are, respectively, the gradients with respect to the observation and source coordinates and $G(\mathbf{r}, \mathbf{r}')$ is referred to as Green's function for a three-dimensional scatterer.

If the observations are restricted on the surface of the scatterer ($\mathbf{r} = \mathbf{r}_s$), then Eq. (5) through Eq. (6) can be expressed using Eq. (2) as:

$$\begin{aligned} j\frac{\eta}{\beta} \left[\beta^2 \int \int_S \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}_s, \mathbf{r}') ds' + \nabla \int \int_S \nabla' \cdot \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}_s, \mathbf{r}') ds' \right] \\ = \mathbf{E}_t^{inc}(\mathbf{r} = \mathbf{r}_s) \end{aligned} \quad (8)$$

Because the right side of Eq. (8) is expressed in terms of the known incident electric field, it is referred to as the *electric field integral equation* (EFIE). It can be used to find the current density $\mathbf{J}_s(\mathbf{r}')$ at any point $\mathbf{r} = \mathbf{r}'$ on the scatterer. It should be noted that Eq. (8) is actually an integro-differential equation, but usually it is referred to as an integral equation.

Equation (8) is a general surface EFIE for three-dimensional problems and its form can be simplified for two-dimensional geometries. Note that this equation gives the EFIE for conducting surfaces. EFIE for the resistive surfaces will be described in detail in Section 5.

3. BLOCK-PULSE FUNCTIONS

One very important step in any numerical solution is the choice of basis functions.

Block-pulse functions (BPFs) have been studied by many authors and applied for solving different problems; for example, see [44, 45].

3.1. Definition

An m -set of BPFs is defined over the interval $[0, T)$ as [44]:

$$\phi_i(t) = \begin{cases} 1, & \frac{iT}{m} \leq t < \frac{(i+1)T}{m}, \\ 0, & \text{otherwise,} \end{cases} \quad (9)$$

where $i = 0, 1, \dots, m-1$ with a positive integer value for m . Also, consider $h = T/m$, and ϕ_i is the i th BPF.

There are some properties for BPFs, the most important properties are disjointness, orthogonality, and completeness.

The disjointness property can be clearly obtained from the definition of BPFs:

$$\phi_i(t)\phi_j(t) = \begin{cases} \phi_i(t), & i = j, \\ 0, & i \neq j, \end{cases} \quad (10)$$

where $i, j = 0, 1, \dots, m-1$.

The other property is orthogonality. It is clear that

$$\int_0^1 \phi_i(t)\phi_j(t)dt = h\delta_{ij}, \quad (11)$$

where δ_{ij} is the Kronecker delta.

The third property is completeness. For every $f \in \mathcal{L}^2([0, 1))$ when m approaches to the infinity, Parseval's identity holds [44]:

$$\int_0^1 f^2(t)dt = \sum_{i=0}^{\infty} f_i^2 \|\phi_i(t)\|^2, \quad (12)$$

where,

$$f_i = \frac{1}{h} \int_0^1 f(t)\phi_i(t)dt. \quad (13)$$

3.2. Vector Forms

Consider the first m terms of BPFs and write them concisely as m -vector:

$$\Phi(t) = [\phi_0(t), \phi_1(t), \dots, \phi_{m-1}(t)]^T, \quad t \in [0, 1) \quad (14)$$

above representation and disjointness property, follows:

$$\Phi(t)\Phi^T(t) = \begin{pmatrix} \phi_0(t) & 0 & \dots & 0 \\ 0 & \phi_1(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \phi_{m-1}(t) \end{pmatrix}, \quad (15)$$

$$\Phi^T(t)\Phi(t) = 1, \quad (16)$$

$$\Phi(t)\Phi^T(t)V = \tilde{V}\Phi(t), \quad (17)$$

where V is an m -vector and $\tilde{V} = \text{diag}(V)$. Moreover, it can be clearly concluded that for every $m \times m$ matrix B :

$$\Phi^T(t)B\Phi(t) = \hat{B}^T\Phi(t), \quad (18)$$

where \hat{B} is an m -vector with elements equal to the diagonal entries of matrix B .

3.3. BPFs Expansion

The expansion of a function $f(t)$ over $[0, 1)$ with respect to $\phi_i(t)$, $i = 0, 1, \dots, m-1$ may be compactly written as [44]:

$$f(t) \simeq \sum_{i=0}^{m-1} f_i \phi_i(t) = F^T \Phi(t) = \Phi^T(t)F, \quad (19)$$

where $F = [f_0, f_1, \dots, f_{m-1}]^T$ and f_i s are defined by (13).

Now, assume $k(t, s)$ is a function of two variables in $\mathcal{L}^2([0, 1) \times [0, 1))$. It can be similarly expanded with respect to BPFs such as:

$$k(t, s) \simeq \Phi^T(t)K\Psi(s), \quad (20)$$

where $\Phi(t)$ and $\Psi(s)$ are m_1 and m_2 dimensional BPF vectors respectively, and K is the $m_1 \times m_2$ block-pulse coefficient matrix with k_{ij} , $i = 0, 1, \dots, m-1$, $j = 0, 1, \dots, m-1$, as follows:

$$k_{ij} = m_1 m_2 \int_0^1 \int_0^1 k(t, s) \phi_i(t) \psi_j(s) dt ds. \quad (21)$$

For convenience, we can put $m_1 = m_2 = m$.

4. IMPLEMENTING THE METHOD OF MOMENTS USING BLOCK-PULSE BASIS FUNCTIONS

In this section, we extend the definition of BPFs over any interval $[a, b]$. Then, we apply them to solve the integral equations of the first kind by moments method.

Consider the following Fredholm integral equation of the first kind:

$$\int_a^b k(s, t)x(t)dt = y(s) \quad (22)$$

where, $k(s, t)$ and $y(s)$ are known functions but $x(t)$ is unknown. Moreover, $k(s, t) \in \mathcal{L}^2([a, b] \times [a, b])$ and $y(s) \in \mathcal{L}^2([a, b])$. Approximating the function $x(s)$ with respect to BPFs by (19) gives:

$$x(s) \simeq F^T \Phi(s) \quad (23)$$

such that the m -vector F is BPFs coefficients of $x(s)$ that should be determined.

Substituting Eq. (23) into (22) follows:

$$F^T \int_a^b k(s, t)\Phi(t)dt \simeq y(s) \quad (24)$$

Now, let $s_i, i = 0, 1, \dots, m-1$ be m appropriate points in interval $[a, b]$; putting $s = s_i$ in Eq. (24) follows:

$$F^T \int_a^b k(s_i, t)\Phi(t)dt \simeq y(s_i), \quad i = 0, 1, \dots, m-1 \quad (25)$$

or:

$$\sum_{j=0}^{m-1} \left[f_j \int_a^b k(s_i, t)\phi_j(t)dt \right] \simeq y(s_i), \quad i = 0, 1, \dots, m-1 \quad (26)$$

Now, replace \simeq with $=$, hence Eq. (26) is a linear system of m algebraic equations for m unknown components f_0, f_1, \dots, f_{m-1} . So, an approximate solution $x(s) \simeq F^T \Phi(s)$, is obtained for Eq. (22).

5. ELECTROMAGNETIC SCATTERING FROM CONDUCTING BODIES

Now, the problem of electromagnetic scattering from conducting bodies is solved using the presented approach. We consider two cases for implementing our method.

5.1. Conducting Strip

In Fig. 1, there is a metallic strip that is very long in the $\pm z$ direction.

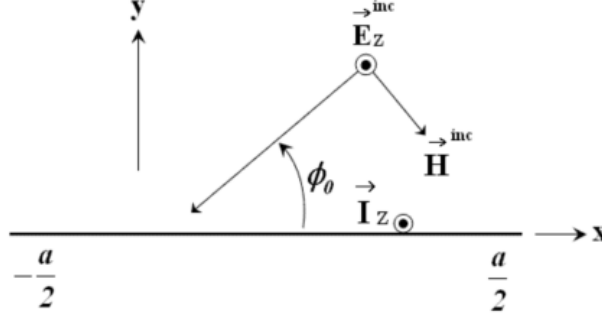


Figure 1. A metal strip of width a is encountered by an incoming TM-polarized plane wave.

The tangential electric field at the surface of a perfect conductor must vanish (i.e, equal zero) [46]. So, at the surface of the strip, the incident electromagnetic field (E^{inc}) must be canceled by a scattered electromagnetic field (E^{scat}). To meet this boundary condition, a current must flow on the surface of the strip to produce an opposing electric field. Therefore, the incident and scattered field exactly cancel at the surface of the perfect conductor. Expressed mathematically:

$$E^{inc} + E^{scat} = 0 \quad (27)$$

or

$$E^{inc} = -E^{scat} \quad (28)$$

According to Fig. 1, the incoming electromagnetic wave is polarized with an electric field parallel to the z -axis. This polarization therefore produces a current on the strip that follows along the z -axis. The magnetic field of this wave is entirely in the x - y plane, and is therefore transverse to the z -axis. It is referred as a transverse magnetic (TM) polarized wave.

The magnetic vector potential of the current flowing along the strip is given by [46]:

$$A_z = \frac{\mu_0}{4j} \int_{-a/2}^{a/2} I_z(x') H_0^{(2)}(k|x-x'|) dx' \quad (29)$$

where:

$k = \frac{2\pi}{\lambda}$, free space wave number.

λ is the wave length.

$\mu_0 = 4\pi \times 10^{-7}$ H/m, free space permeability.

$G(x, x') = \frac{1}{4j} H_0^{(2)}(k|x - x'|)$, 2D free space Green's function.

$H_0^{(2)}(x)$ is a Hankel function of the second kind 0th order.

So, the electric field is given by:

$$E_z(x) = j\omega A_z(x) \quad (30)$$

or

$$E(x) = \frac{\omega\mu_0}{4} \int_{-a/2}^{a/2} I_z(x') H_0^{(2)}(k|x - x'|) dx' \quad (31)$$

where, ω is the frequency of the wave.

According to boundary condition (28), the Eq. (31) becomes:

$$\int_{-a/2}^{a/2} I_z(x') H_0^{(2)}(k|x - x'|) dx' = -\frac{4}{\omega\mu_0} E^{inc}(x) \quad (32)$$

Choosing $E^{inc}(x) = e^{jkx \cos \phi_0}$:

$$\int_{-a/2}^{a/2} I_z(x') H_0^{(2)}(k|x - x'|) dx' = -\frac{4}{\omega\mu_0} e^{jkx \cos \phi_0} \quad (33)$$

It is a Fredholm integral equation of the first kind with complex kernel of the following form:

$$\int_a^b h(x') G(x, x') dx' = g(x) \quad (34)$$

where:

$$\begin{aligned} h(x') &= I_z(x') \\ G(x, x') &= H_0^{(2)}(k|x - x'|) \\ g(x) &= -\frac{4}{\omega\mu_0} e^{jkx \cos \phi_0} \end{aligned}$$

In Figs. 2–4, the approximate solutions of this equation for $a = 2\lambda$ and $\phi_0 = 0, \frac{\pi}{4}, \frac{\pi}{2}$, and $f = 0.3$ GHz are given. Also the current distributions for $a = 4\lambda$ are shown in Figs. 5–7.

The radar cross section (RCS) in two dimensions is defined mathematically as [46]:

$$\sigma(\phi) = \lim_{r \rightarrow \infty} 2\pi r \frac{|\mathbf{E}^{scat}|^2}{|\mathbf{E}^{inc}|^2} \quad (35)$$

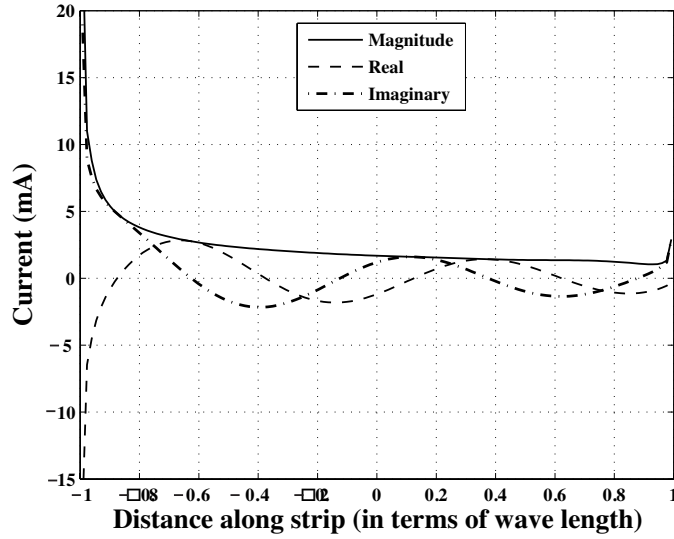


Figure 2. Current distribution across a 2λ -wide conducting strip created by a TM-polarized plane wave for $\phi_0 = 0$.

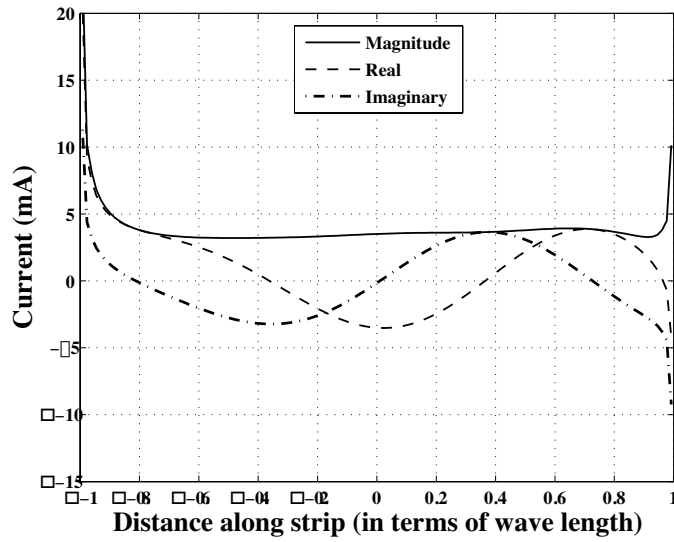


Figure 3. Current distribution across the 2λ -wide conducting strip for $\phi_0 = \frac{\pi}{4}$.

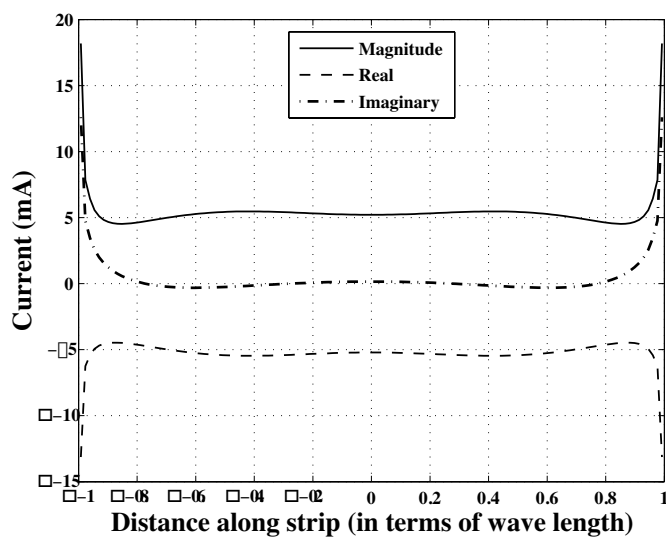


Figure 4. Current distribution across the 2λ -wide conducting strip for $\phi_0 = \frac{\pi}{2}$.

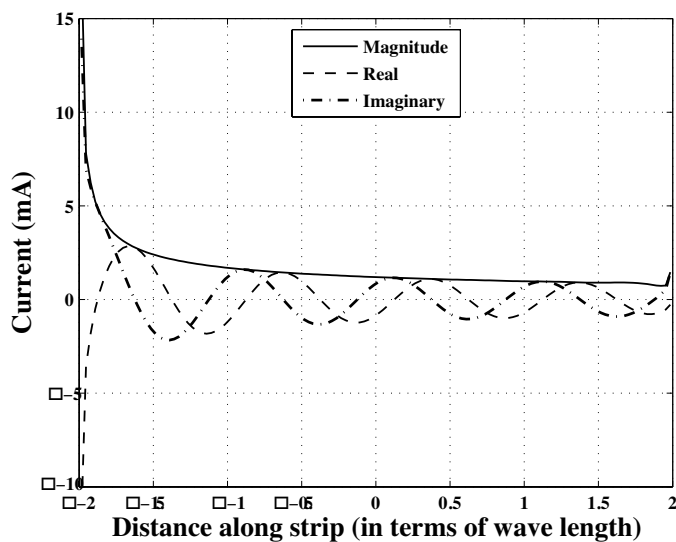


Figure 5. Current distribution across a 4λ -wide conducting strip created by a TM-polarized plane wave for $\phi_0 = 0$.

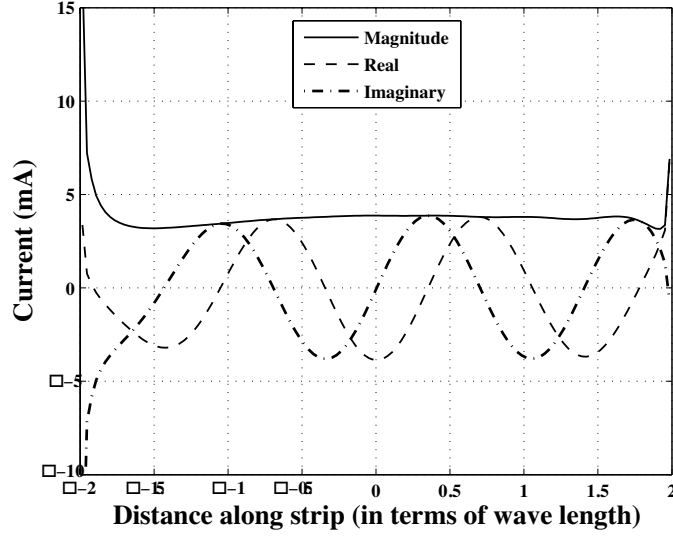


Figure 6. Current distribution across the 4λ -wide conducting strip for $\phi_0 = \frac{\pi}{4}$.

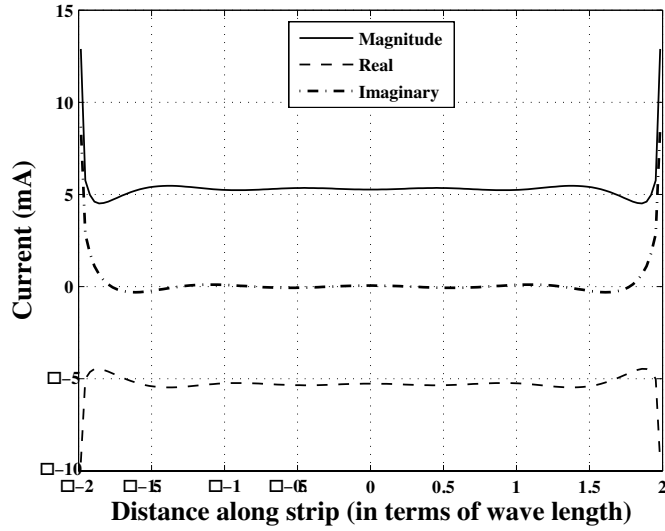


Figure 7. Current distribution across the 4λ -wide conducting strip for $\phi_0 = \frac{\pi}{2}$.

In the presented case, the RCS is obtained of the following equation [46]:

$$\sigma(\phi) = \frac{k\eta^2}{4} \left| \int_{-a/2}^{a/2} I(x') e^{jkx' \cos \phi} dx' \right|^2 \quad (36)$$

Also, it is possible to define a logarithmic quantity with respect to the RCS, so that:

$$\sigma_{dBlm} = 10 \log_{10} \sigma \quad (37)$$

The bistatic radar cross section of the strip is given in Fig. 8 for $\phi_0 = \frac{\pi}{2}$ and $a = 8\lambda$.

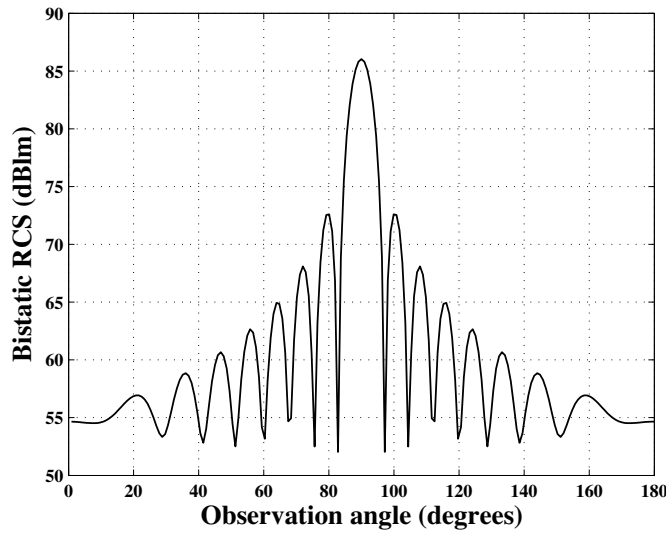


Figure 8. The bistatic RCS of a 8λ -wide conducting strip for $\phi_0 = \frac{\pi}{2}$.

5.2. Thin Wire

In Fig. 9, an electromagnetic wave traveling from the right encounters a wire at angle α .

According to boundary condition on the conductive surface:

$$E^{inc} + E^{scat} = 0 \quad (38)$$

An expression is required to relate the current induced on a wire by an incident electric field to the scattered field it produces. For a

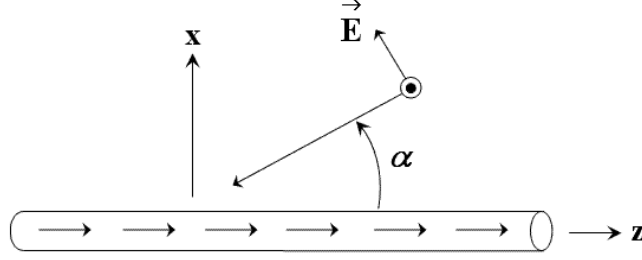


Figure 9. An electromagnetic wave encounters a wire of radius a and length L at an angle α .

wire along z -axis with a radius a of length L , the relationship is [46, 47]:

$$\frac{d^2 A(z)}{dz^2} + k^2 A(z) = j4\pi\omega\epsilon_0 E_z(z) \quad (39)$$

$$A(z) = \int_{-L/2}^{L/2} I_z(z') G(z, z') dz' \quad (40)$$

$$G(z, z') = \int_0^{2\pi} \frac{e^{-jkR}}{R} d\phi' \quad (41)$$

$$R = \sqrt{(z - z')^2 + \left(2a \sin \frac{\phi'}{2}\right)^2} \quad (42)$$

In many applications, the wire radius is very small compared with a wavelength. So, R is often approximated using the following form:

$$R \approx \sqrt{(z - z')^2 + a^2} \quad (43)$$

The incident electric field along the conductor from a plane wave at angle α is:

$$E_z = e^{jk \cos \alpha} \sin \alpha \quad (44)$$

Consider a special case of a plane wave at broadside ($\alpha = \frac{\pi}{2}$). This plane wave will drive the current in a symmetrical manner. This implies that $I(z) = I(-z)$, which means $A(z) = A(-z)$. For these conditions, the final form of the integral equation of the wire current is [46]:

$$\int_{-L/2}^{L/2} I_z(z') G(z, z') dz' = C_1 \cos kz + j \frac{4\pi\omega\epsilon_0}{k^2 \sin \alpha} e^{jkz \cos \alpha} \quad (45)$$

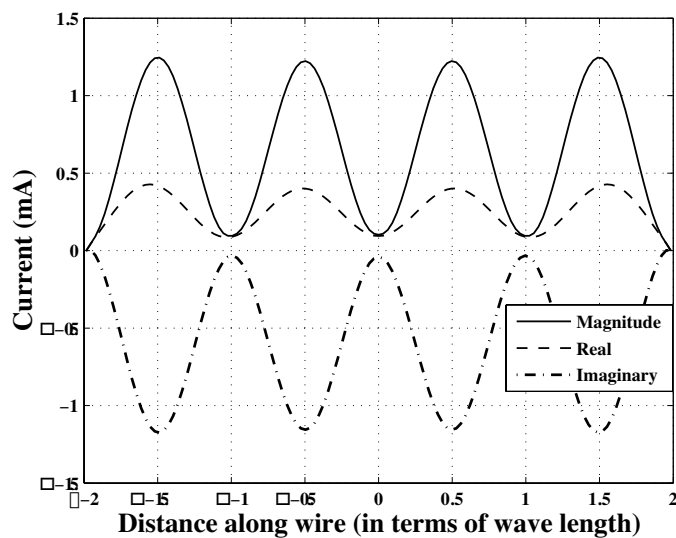


Figure 10. Current distribution along the thin wire of length 4λ for $\alpha = \frac{\pi}{2}$.

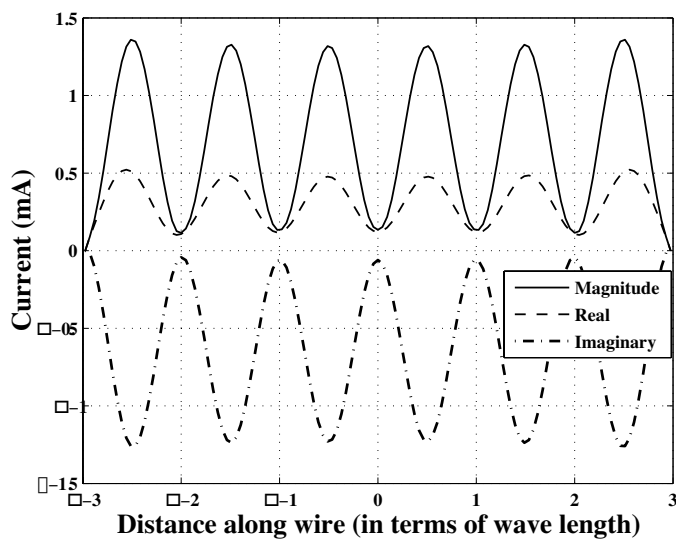


Figure 11. Current distribution along the thin wire of length 6λ for $\alpha = \frac{\pi}{2}$.

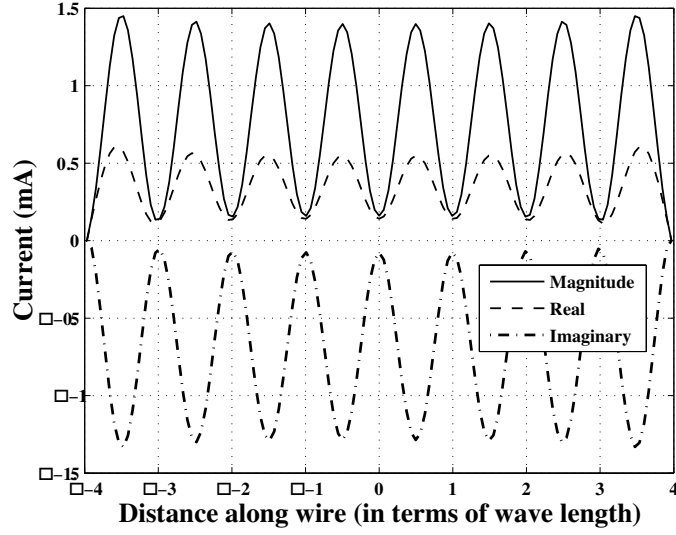


Figure 12. Current distribution along the thin wire of length 8λ for $\alpha = \frac{\pi}{2}$.

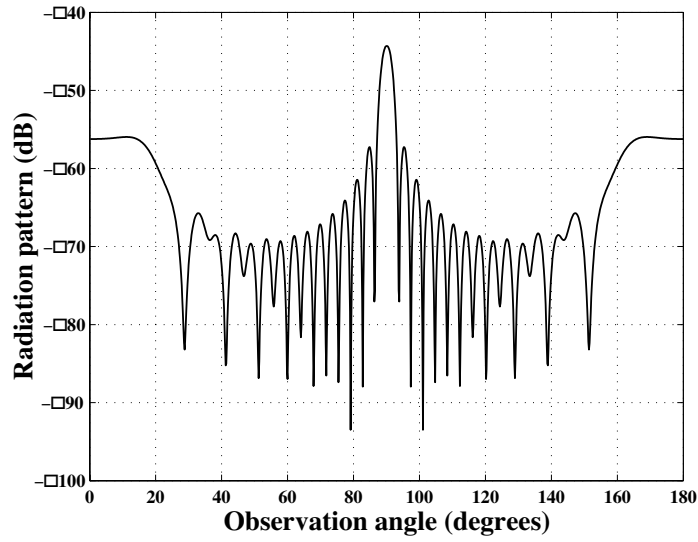


Figure 13. Radiation pattern of a thin wire of length 8λ .

This is a Fredholm integral equation of the first kind and C_1 is an unknown coefficient that must be determined. For determining C_1 , the number of match points must be $m+1$ instead of m . The approximate solution of this equation gives the current distribution along the wire. Considering $a = 0.001L$, the current distributions for $L = 4\lambda$, 6λ and 8λ , and for $\alpha = \frac{\pi}{2}$ are shown in Figs. 10–12 respectively.

The radiation pattern of this wire is obtained of the following equation [48]:

$$f(\alpha) = \int_{-L/2}^{L/2} I_z(z') e^{jkz' \cos \alpha} dz' \quad (46)$$

Also, it is possible to define a logarithmic quantity with respect to f , so that:

$$F = 20 \log_{10} |f| \text{ (dB)} \quad (47)$$

Figure 13 gives the radiation pattern F for $L = 8\lambda$.

6. CONCLUSION

The presented method in this paper is applied to solve the problem of electromagnetic scattering from conducting bodies.

As the numerical results showed, this method reduces an integral equation of the first kind to a linear system of algebraic equations.

The problem of electromagnetic scattering from conducting bodies was described in detail, and illustrative computations were given for some cases. The presented approach can be generalized to apply to objects of arbitrary geometry and arbitrary material.

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