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FRACTIONALLY SPACED CONSTANT MODULUS ALGORITHM FOR WIRELESS CHANNEL EQUALIZATION

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Abstract—Wireless channel identification and equalization is one of the most challenging tasks because broadcast channels are often subject to frequency selective, time varying fading and there are several bandwidth limitations. Furthermore, each receiver channel has vastly different types of channel characteristics and signals to noise ratio. Here in this paper we consider channel equalization and estimation problem from trans-receiver perspective, specifically we try to estimate blind equalization schemes particularly using constant modulus Algorithm (CMA). We try to estimate a linear channel model driven by a QAM source and adapt a FSE ($T/2$) using CMA. It has been shown CMA-FSE successfully reduces the cluster variance so that transfer to a decision directed mode is possible and simultaneously error is reduced.

1. INTRODUCTION

One approach to remove inter-symbol interference in a communication channel is to employ adaptive blind equalization [12, 13]. The most popular class of algorithms used for blind equalization is those that minimize the constant modulus criteria. Here we have used CMA algorithm [4] to estimate the channel variation and compare the performance of the Constant Modulus Algorithm with a more computationally efficient signed-error version of CMA (SE-CMA) [1, 3, 5, 6]. For channel estimation we used to take a FIR channel with fractionally spaced equalizer (FSE) with sampling interval $T/2$ where T is taken as symbol period. The source symbols are chosen from a finite M array real alphabet of zero mean means all symbols are equally probable.

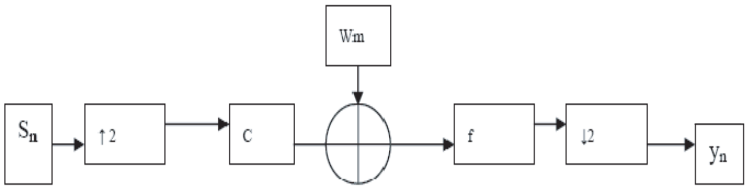

Figure 1. $T/2$ spaced multirate system model with added interference.

Here in Fig. 1, S_n implies baud spaced source symbol at sample index n, c is the vector representing the fractionally spaced channel impulse response. w is the additive white Gaussian channel noise, f represents vector containing the fractionally spaced equalizer coefficients. y_n is the baud spaced equalizer output. No of coefficients in the channel and equalizer response vectors are N_c and N_f respectively. The system output can be expressed as

$$y_n = s^T(n)Cf + w^T(m)f \tag{1}$$

where $s(n) = [s_n, s_{n-1}, \ldots, s_{n-N_s+1}]^T$ is the length $N_s = [(N_c + N_f - 1)/2]$ vector of baud spaced source symbols $w(m) = [w_m, w_{m-1}, \ldots, w_{m-N_f+1}]^T$ is the vector of additive zero mean white Gaussian noise with variance σ_ω and C is the $N_s * N_f$ decimated channel convolution matrix given by

$$C = \begin{bmatrix} c_1 \ldots\ldots\ldots c_0 \\ \ldots\ldots\ldots\ldots\ldots\ldots c_1 \ldots\ldots c_0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ c_{N_c-1} c_{N_c-2} \ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots c_{N_c-1} \cdot c_{N_c-2} \end{bmatrix}$$

2. CONSTANT MODULUS ALGORITHM

The CMA criterion may be expressed by the non negative cost function $J_{CMA_{p,q}}$ parameterized by positive integer p and q.

$$J_{CMA_{p,q}} = \frac{1}{pq} E\left\{ ||y_a|^p - \gamma|^q \right\} \tag{2}$$

where γ is a fixed constant. This is a gradient based algorithm [19] and work on the premise that the existing interference causes fluctuation in the amplitude of output that otherwise has a constant modulus. For simplest case we put $p = 2$ & $q = 2$. It updates weights by minimizing the cost function. The steepest gradient descent algorithm [13–15, 19] is obtained by taking the instantaneous gradient of $J_{CMA_{p,\,q}}$ which results equation which updates the system.

$$f(n+1) = f(n) - \mu g(w(n)) \tag{3}$$

$$g(w(n)) = r^*(n)\psi(y_n) \tag{4}$$

$$\psi(y_n) = -\nabla_{y_n}\frac{1}{4}\left(|y_n|^2 - \gamma\right)^2 = y_n\left(\gamma - |y_n|^2\right) \tag{5}$$

where f is the length N_f is equalizer coefficient vector, $r(n)$ is the length-N_f receiver input vector, μ is small step size and $\psi(y_n)$ is the CM error function for $\mathrm{CMA}_{2,\,2}$. Where γ is taken as

$$\gamma = E\left\{|S_n|^4\right\}/E\left\{|S_n|^2\right\} \tag{6}$$

To improve computational efficiency a signed error algorithm used to modify the update equation of the channel equalizer. Here we took only the sign of the error function there simplified by eliminating multiply operation. The Equation (3) modified as

$$f(n+1) = f(n) - \mu r(n)\mathrm{sgn}(\psi(y_n)) \tag{7}$$

This modified CMA known as SE-CMA is equivalent to $\mathrm{CMA}_{1,\,1}$. Now we consider $\mathrm{CMA}_{1,\,1}$ cost function

$$J_{CMA_{1,\,1}} = E\left\{||y_n| - \beta|\right\} \tag{8}$$

and corresponding update equation

$$f(n+1) = f(n) - \mu r(n)\mathrm{sgn}(y_n(\beta - |y_n|)) \tag{9}$$

When $\beta = \sqrt{\gamma}$, the $\mathrm{CMA}_{1,\,1}$ update equation is identical to SE-CMA so (8) becomes

$$J_{SE-CMA} = E\left\{||y_n| - \sqrt{\gamma}|\right\} \tag{10}$$

Selection of γ is important because of the systems convergence. The dissipation constant γ should be chosen such that the $\gamma = a_v^2$ where a_v is the vth positive member of the source alphabet and integer v satisfies

$$v = \arg\min_k \frac{\left(k - 1/2\left(1 + \sqrt{M^2}/2 - 1\right)\right)^2}{k - 1/2} \tag{11}$$

For BPSK γ is taken as 1 and for 8 PAM γ is taken as 9/5 for satisfactory result. The cost function of SE-CMA depends on the following terms s, C, f, γ, M, N_s, σ_ω. S is the set of all $M^N s$ source symbol possibilities. For a system with BPSK source, a channel length of 6 and an equalization length of 2 ($M = 2$; $N_c = 6$; $N_f = 2$; $N_s = 3$). The set of possible symbol combination is

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right\}$$

We know

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt \tag{12}$$

and

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt \tag{13}$$

$$J_{SE-CMA} = E\{||y_n| - \sqrt{\gamma}|\} = E\left\{\left|\left|s^T C f + w^T f\right| - \sqrt{\gamma}\right|\right\} \tag{14}$$

After full expansion the expression comes as

$$\begin{aligned} J_{SE-CMA} = {} & M^{-N_s} \sum_{n \in s} -\sqrt{\gamma} - 4\left(\sqrt{\gamma} + s^T C f\right) Q\left(\frac{\sqrt{\gamma} + s^T C f}{\sigma_\omega \|f\|_2}\right) \\ & + 2 s^T C f Q\left(\frac{s^T C f}{\sigma_\omega \|f\|_2}\right) + \sqrt{\frac{2}{\pi}} \sigma_\omega \|f\|_2 \\ & \left[2 \exp\left(-\frac{\left(\sqrt{\gamma} + s^T C f\right)^2}{2\sigma_\omega^2 \|f\|_2^2}\right) - \exp\left(-\frac{\left(s^T C f\right)^2}{2\sigma_\omega^2 \|f\|_2^2}\right) \right. \end{aligned} \tag{15}$$

It can be seen that the cost function is complex enough that from general conceptual study it is not possible the rigorous analysis. Convergence analysis of fractional spaced equalizers draws two important conclusions, 1. a finite length channel satisfying a length & zero condition allows CMA-FSE to be globally convergent [2, 5–7] and 2. The linear FSE filter length need not be longer than the channel delay spread. For some channel with deep spectral nulls, the CMA FSE doesn't require a large number of parameters so that it can converge faster.

3. CHANNEL MODEL FOR A QAM SOURCE

Channel output of a QAM communication system described by equation

$$x(t) = \sum_{n=0}^{\infty} a_n h(t - nT - t_0) + w(t) \tag{16}$$

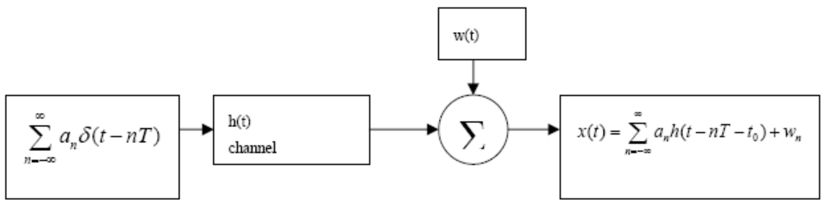

Figure 2. Channel model for a QAM.

Figure 2 shows a sequence of independent identically distributed complex data $\{a_n\}$ is sent by the transmitter over a LTI channel with impulse response $h(t)$. The receiver attempts to recover the input data sequence $\{a_n\}$ for measurable channel output $x(t)$ in which T is the symbol period. The channel output may be corrupted by $w(t)$ channel noise which is zero mean stationary, white and complex Gaussian with variance σ^2 and is independent of the channel input a_n. Assume the complex data and noise both satisfy symmetric property $E\left\{a_n^2\right\} = E\left\{w_t^2\right\} = 0$ in addition $E\left\{|a_n|^4\right\} - 2E^2\left\{|a_n|^2\right\} < 0$, i.e., the kurtosis $K(a_n)$ of a_n is negative as is often the case of QAM system. When the distortion caused by channel (LTI [10, 11], non ideal) is significant, equalization is needed to remove the ISI [20] at the sampling instants ($t = nT$). Due to the presence of ISI, the recovery of input signal sequence a_n requires that the channel impulse response $h(t - T_0)$ be identified either explicitly or in decision feedback equalization (DFE) [9, 20] or implicitly as in linear equalization. In traditional blind equalization system the channel output sampled at the known baud rate $1/T$. The sampled channel output

$$\begin{aligned} x(nT) &= \sum_{k=0}^{\infty} a_k h(nT - kT - t_0) + w(nT) \\ &= a_n \odot h(nT - t_0) + w(nT) \end{aligned} \tag{17}$$

is a stationary process, using this notations $x_n \triangleq x(nT)$; $w_n \triangleq w(nT)$; $h_n \triangleq h(nT - t_0)$ the previous equation can be written as discrete convolution with noise

$$x_n = \sum_{k=-\infty}^{\infty} a_k h_{n-k} + w_n \tag{18}$$

Based on this relationship traditional linear TSE are designed as FIR filter $\theta(z^{-1}) = \sum_{k=0}^{N} \theta_k z^{-k}$ to be applied on x_n to remove the ISI from the equalizer output $y_n = \sum_{k=0}^{N} \theta_k x_{n-k}$. Most of the equalizer algorithm proposes to adjust the parameter $\{\theta_k\}_{k=0}^{N}$. CMA aims to minimize the cost function given in (19).

$$\begin{aligned} J_2(\theta) =& \frac{1}{4} E\left\{\left(|y_n|^2 - R_2\right)^2\right\} \\ R_2 =& \frac{E\left\{|a_n|^4\right\}}{E\left\{|a_n|^2\right\}} \end{aligned} \tag{19}$$

If we wish to maximize $|K(y_n)|$ it requires $E\left|y_n^2\right| = E\left|a_n^2\right|$ where $K(y_n)$ is kurtosis of signal y_n.

$$|K(y_n)| \triangleq E\left\{|y_n|^4\right\} - 2\left(E\left\{|y_n|^2\right\}\right) - \left|E\left\{y_n^2\right\}\right|^2 \tag{20}$$

From the Fig. 3 it is apparent that $x(t)$ is a continuous time cyclostationary process with period T as long as the channel bandwidth is greater than the minimum BW $1/2T$. Let sampling interval $\Delta = \frac{T}{P}$; sampled channel output $x(k\Delta) = \sum_{n=0}^{\infty} a_n h(k\Delta - np\Delta - t_0)$ for $p > 1$, the over sampled channel output $x(k\Delta)$ can be divided into p sub-sequences. $x_k^{(i)} \triangleq x[(kp+i)\Delta] = x(kT+i\Delta)$, $i = 1, \dots, p$. By defining the sub-channel impulse response as $h_k^{(i)} \triangleq h(pk\Delta + i\Delta - t_0) = h(kT + i\Delta - t_0)$; the p sub-sequence can be written as $x_k^{(i)} = \sum_{n=0}^{+\infty} a_n h_{k-n}^{i} + w_k^{(i)}$, these p sub-sequences can be viewed as stationary outputs of p discrete FIR channels. $H_i(z) = \sum_{k=0}^{K} h_k^i z^{-k}$ with a common input sequence a_k.

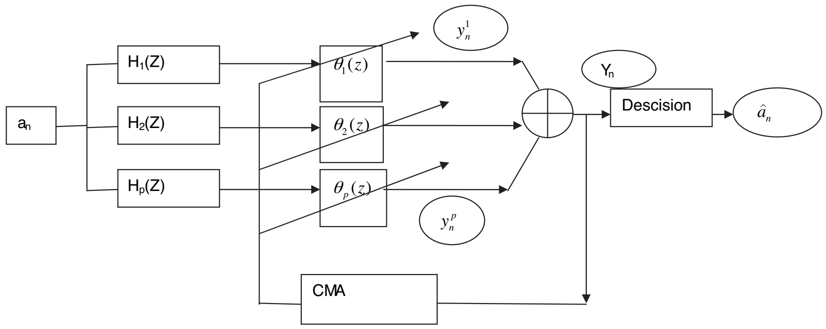

Figure 3. Multichannel vector representation of blind adaptive FSE.

One adjustable filter [8] is provided for each sub-sequences $x_k^{(i)}$, thus the actual equalizer is a vector of filters. $\theta_i(z) = \sum_{k=0}^{N} \theta_k^{(i)} z^{-k}$, $i = 1, \ldots, p$. The p stationary filter output $\left\{ y_n^{(i)} \right\}$ are summed to form the stationary equalizer output

$$y_n = \sum_{i=1}^{p} y_n^{(i)} \tag{21}$$

Define FSE parameter as $\theta \triangleq [\theta_0^{(1)}, \theta_N^{(1)}, \theta_0^{(p)}, \ldots, \theta_N^{(p)}]$. To adaptively adjust θ without a training sequence, CMA can be implemented to jointly update the p filters to minimize cost function. From cost function we obtain stochastic gradient algorithm as follows

$$\theta_k^{(i)}(n+1) = \theta_k^{(i)}(n) - \mu x_{n-k}^{(i)} y_n \left(|y_n|^2 - R_2 \right), \; i = 0, 1, \ldots, p-1; \tag{22}$$

where μ is small step size and $\theta_k^{(i)}$ is the kth coefficient of the ith filter at the nth iteration. This combination of CMA & FSE used to implement blind adaptive algorithm for equalizer [18].

4. BLIND ADAPTIVE FSE

Figure 4 shows the frequency response of the microwave channel taken for simulation [16, 17]. Fig. 5 shows noisy received signals from a linear channel model driven by a QAM [15] source constellation and adapts a FSE ($T/2$) using CMA. User selectable variables are in the Input

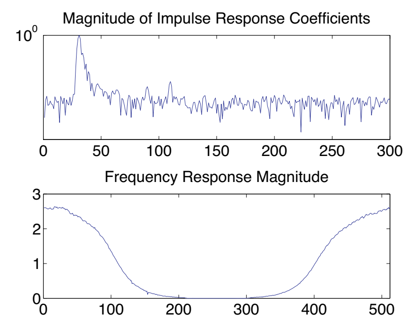

Figure 4. Frequency response characteristics of channel.

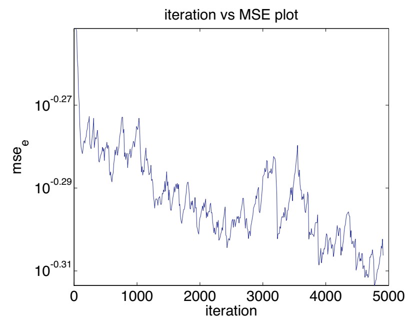

Figure 5. CMA blind equalizer performance with 16 QAM, step size = 0.001, SNR at equalizer input 35 dB.

Variables section, and the channel is selected in the Channel section of the code. The CMA-FSE successfully reduces the cluster variance so that transfer to a Decision Directed mode is possible, further error rate reduction is desired. Simulated results demonstrate CMAs ability to adapt a FSE blindly from a received sequence synthesized from a shortened version (length-16). The source is 16 QAM, white and equiprobable. The equalizer is length-16 and initialized with a unity center spike and all other taps zero, and the step-size is 0.001. The

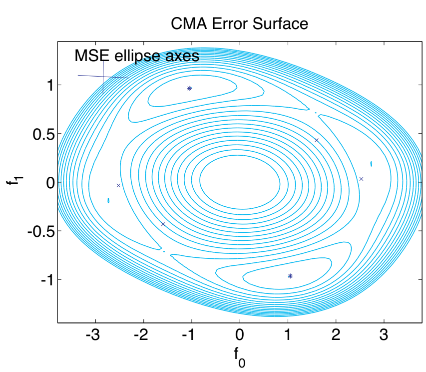

Figure 6. CM Cost Surface plots over equalizer plane for length-2 real-valued channels.

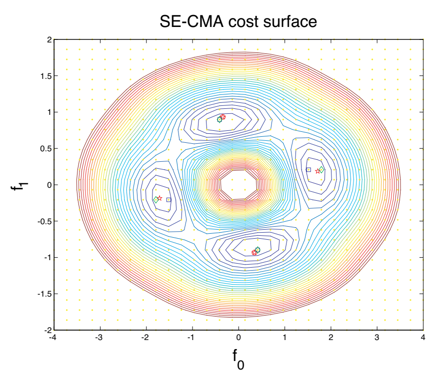

Figure 7. Cost contours for SE-CMA of 2-tap fractionally spaced equalizers given the following Input parameters: SNR = 15, alphabet size = 8 source variance = 1.

SNR at the equalizer input is 35 dB. The received signal is normalized to (near) unity power.

The convergence of equalizer parameter vector under CMA can be viewed as a transverse of the CMA cost surface [2, 5, 6, 8] with average movement in the direction of steepest descent, so from the Fig. 6 dynamical behavior of CMA can be estimated.

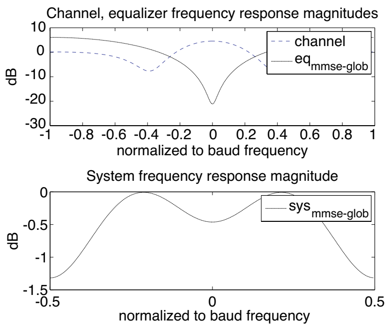

Figure 8. Frequency response analysis of 6 tap, $T/2$ fractionally-spaced, 16 QAM source, equalizer length 2, SNR 50 dB, step size $5e^{-3}$, no of iteration 5000.

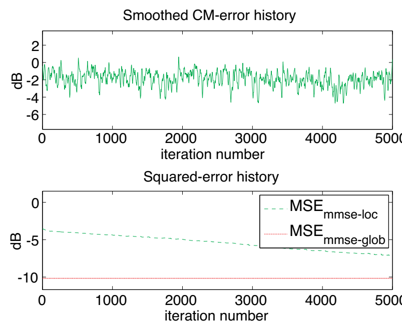

Figure 9. Plot of smoothed CM error history and variation of square error with iteration.

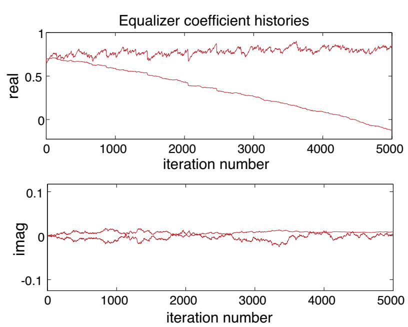

Figure 10. Plot of CMA equalizer coefficient history with iteration.

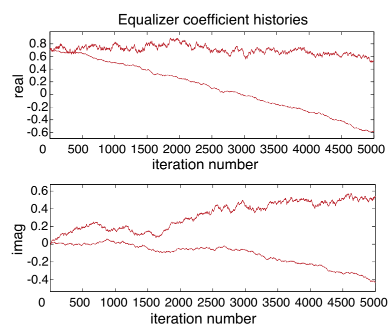

Figure 11. SE-CMA equalizer coefficient history with iteration.

Figure 7 shows FSE-CMA cost surface which is multimodal, FSE is able to perfectly equalize the channel. Here multiple minima exist. It can be shown that multiple minima occur very near to the wiener solutions corresponding to particular combination of system polarity and system delay. CM minima with better MSE stay closer to corresponding wiener solutions since CM criterion operates purely on the magnitude of the equalizer output not the phase shift of adaptive filter.

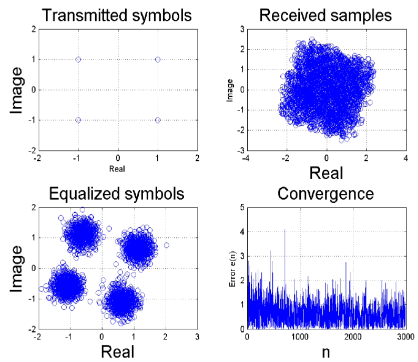

Figure 12. Transmitted symbol, received and equalized symbol, convergence plot with iteration for CMA.

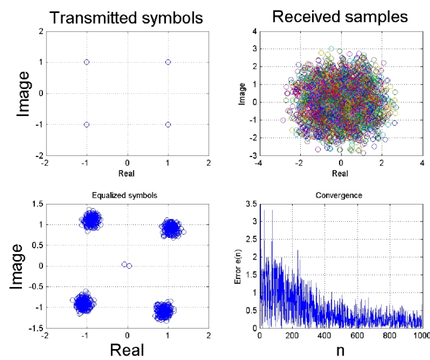

Figure 13. Transmitted symbol, received and equalized symbol, convergence plot with iteration for FSE-CMA.

5. CONCLUSION

Study of convergence of blind equalizer CMA-FSE is based on simple length and zero condition. The requirement of no common zero among sub-channels may sometime be restrictive. When common zeros exist CMA-FSE may not be able to estimate the common factor among sub-channels, in this case an additional linear filter may be added after the vector equalizer to minimize the remaining ISI. Another option is to simply increase the length of all filters. The vector blind equalizer structure of FSE can be simply applied to spatial channel diversity because the sub channels of Fig. 3 can be replaced by physical channels in an array of wide band sensors. Hence $x_n(t)$ can be treated as the output of each channel sampled at baud rate as long as the length and zero condition is satisfied, the convergence will hold for spatial and spectral diversity channel. Finally conclusion can be drawn that a finite length channel which satisfy the 'length and zero' conditions will be convergent and the linear FSE filter length need not be longer than the channel delay spread for perfect equalization. Constant modulus cost function describing the deformation of the CM error surface. Figs. 10 and 11 estimate the proximity of equalizer coefficient history with iteration number.

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