Progress In Electromagnetics Research, PIER 81, 225–235, 2008

DESIGN OF PHOTONIC BAND GAP FILTER

R. Srivastava

Department of Physics
Udai Pratap Autonomous College
Varanasi 221007, India

K. B. Thapa

Department of Physics
U. I. E. T., C. S. J. M. University
Kanpur-208024, India

S. Pati

Department of Physics
Udai Pratap Autonomous College
Varanasi 221007, India

S. P. Ojha

Choudhary Charan Singh University
Meerut 200005, India

Abstract—In this paper a new type of optical filter using photonic band gap materials has been suggested. A detailed mathematical analysis is presented to predict allowed and forbidden bands of wavelengths with variation of angle of incidence. It is possible to get desired ranges of the electromagnetic spectrum filtered with this structure by changing the incidence angle of light.

1. INTRODUCTION

Photonic crystals PCs have drawn much attention as a new kind of optical materials [1–6]. PCs made of periodic dielectric materials in one, two or three spatial directions that exhibit electromagnetic stop bands or photonic band gaps (PBGs). PBGs have been investigated intensively relating to their ability of controlling the propagation of light [6–9]. The absence of electromagnetic wave or light wave inside PBGs will lead to some unusual properties, which can be used for Bragg mirrors or narrow-band filters [10–16]. An optical filter is a device, which has the property of adding or dropping a particular wavelength channels from the multi wavelength network.

A great deal of work has been done by technologists for the development of methods for designing multi-layer films with prescribed characteristics [17–21]. Tunable optical filters have received much attention due to their application in fibre optic communications and other optical fields. Several configurations have been proposed, including tunable multiple electrode asymmetric directional couplers [23], tunable Mach Zehnder interferometers [24, 25], fibre Fabry-Perot filters [26, 27], tunable waveguide arrays [28, 29], liquid crystal Fabry-Perot filters [30, 31], tunable multi grating filters [32], and acousto-optic tunable filters [33]. Another class of most popular filter based on the phenomenon of multi-beam interference and based on waveguides [34–37].

Fabrication of optical filters in the near infrared region of the wavelength was suggested by Ojha et al. [38] in 1992. Chen et al. [39] in 1996 suggested the design of optical filters by photonic band gap air bridges and calculated the important results and some aspects of filter properties. Recently D'Orazio et al. [40] have fabricated the photonic band gap filter for wavelength division multiplexing. In another investigation Villar et al. [41] have analyzed the one-dimensional photonic band gap structures with a liquid crystal defect for the development of fiber-optic tunable wavelength filters.

In this paper a new type of optical filter using photonic band gap materials has been suggested. The working principle as well as the theoretical analysis of this filter is based on the Kronig and Penney model.

2. THEORY

It is well known that when electrons move through a periodic lattice, allowed and forbidden energy bands are obtained. The same idea may be applicable to the case of optical radiation if the electron waves are replaced by optical waves and the lattice periodicity structure is replaced by a periodic refractive index pattern. One expects allowed and forbidden bands of frequencies instead of energies. By choosing a linearly periodic refractive index profile in the filter material one obtains a given set of wavelength ranges that are allowed or forbidden to pass through the filter material. Selecting a particular x-axis through the material, we shall assume a periodic step function for the index of the form [22, 42]

$$n\left(x\right)=\begin{cases} n_1, & 0\le x\le a;\\ n_2, & -b\le x\le 0;\end{cases} \tag{1}$$

where $n(x) = n\left(x + md\right)$ and m is the translation factor, which takes the values $m = 0, \pm 1, \pm 2, \pm 3, \ldots,$ and $d = a + b$ is the period of the lattice with a and b being the width of the two regions having refractive indices (n_1) and (n_2) respectively. The refractive index profile of the materials in the form of rectangular symmetry is shown in the Figure 1.

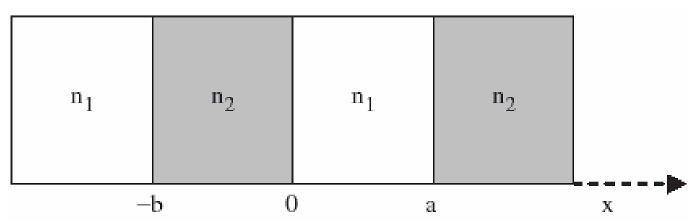

Figure 1. Periodic refractive index profile of material.

If θ is the angle of incident on this periodic structure the one-dimensional wave equation for the spatial part of the electromagnetic eigen mode $\psi_k\left(x\right)$ is given by

$$\frac{d^2\psi_k\left(x\right)}{dx^2}+\frac{n^2\left(x\right)\cos^2\theta\cdot\omega_k^2}{c^2}\psi_k\left(x\right)=0, \tag{2}$$

where $n(x)$ is given by Equation (1). Therefore, Equation (2) for wave equation may be written as

$$\frac{d^2\psi_k\left(x\right)}{dx^2}+\frac{n_1^2\cdot\cos^2\theta_1\cdot\omega_k^2}{c^2}\psi_k\left(x\right)=0;\quad 0\le x\le a \tag{3a}$$

$$\frac{d^2\psi_k\left(x\right)}{dx^2}+\frac{n_2^2\cdot\cos^2\theta_2\cdot\omega_k^2}{c^2}\psi_k\left(x\right)=0;\quad -b\le x\le 0 \tag{3b}$$

where θ_1 and θ_2 are ray angle in the layer of refractive index n_1 and n_2 respectively.

The periodic nature of the problem allows the application of Bloch's theorem which solution can be written as $\psi_K = u_K\left(x\right)e^{iKx}$ where K is known as Bloch wave number and $u_K\left(x\right)$ is the value of the eigen function. Thus using this theorem Equations (3a) and (3b) can be written as

$$\frac{d^2u_1}{dx^2}+2iK\frac{du_1}{dx}+\left(\alpha^2-K^2\right)u_1=0;\quad 0\le x\le a \tag{4a}$$

$$\frac{d^2u_2}{dx^2} + 2iK\frac{du_2}{dx} + \left(\beta^2 - K^2\right)u_2 = 0; \quad -b \le x \le 0 \tag{4b}$$

where $\alpha = (\frac{n_1\omega}{c}\cos\theta_1)$, $\beta = (\frac{n_2\omega}{c}\cos\theta_2)$, $\theta_1 = \cos^{-1}[1 - \frac{\sin^2\theta}{n_1}]^{1/2}$, $\theta_2 = \cos^{-1}[1 - \frac{\sin^2\theta}{n_2}]^{1/2}$ and u_1 represents the value of $u_K(x)$ in the interval (0, a) and u_2 in the interval ($-b$, 0) respectively. The solution of differential Equations (4a) and (4b) can be written as

$$u_1 = Ae^{i(\alpha-K)x} + Be^{-i(\alpha+K)x} \tag{5a}$$

$$u_2 = Ce^{i(\beta-K)x} + De^{-i(\beta+K)x} \tag{5b}$$

Now applying the boundary conditions as given below

$$u_1(x)|_{x=0} = u_2(x)|_{x=0} \tag{6a}$$

$$u_1'(x)|_{x=0} = u_2'(x)|_{x=0} \tag{6b}$$

$$u_1(x)|_{x=a} = u_2(x)|_{x=-b} \tag{6c}$$

$$u_1'(x)|_{x=a} = u_2'(x)|_{x=-b} \tag{6d}$$

we get four equations having four unknown constants. To obtain a non-trivial solution for the equations, the determinant of the coefficients of the unknown constants must be zero, which is given as

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix} = 0, \tag{7}$$

where

$A_{11}=A_{12} = A_{13} = A_{14} = 1;$
$A_{21}=i(\alpha - K)$, $A_{22}=-i(\alpha + K)$, $A_{23}=i(\beta - K)$, $A_{24}=-i(\beta + K)$;
$A_{31}=e^{ia(\alpha-K)}$, $A_{32} = e^{-ia(\alpha+K)}$, $A_{33} = e^{-ib(\beta-K)}$, $A_{34} = e^{ib(\beta+K)}$;
$A_{41}=i\,(\alpha - K)\,e^{ia(\alpha-K)}$, $A_{42} = -i\,(\alpha + K)\,e^{-ia(\alpha+K)}$,
$A_{43}=i\,(\beta - K)\,e^{-ib(\beta-K)}$, $A_{44} - i\,(\beta + K)\,e^{ib(\beta+K)}$.

On solving Equation (7) we obtain

$$\cos(K \cdot d) = \cos(\alpha a)\cdot\cos(\beta b) - \frac{1}{2}\left(\frac{n_1\cdot\cos\theta_1}{n_2\cdot\cos\theta_2} + \frac{n_2\cdot\cos\theta_2}{n_1\cdot\cos\theta_1}\right)\cdot\sin(\alpha a)\cdot\sin(\beta b) \tag{8}$$

Now, abbreviating the L.H.S. as L_λ, Equation (8) may be written as

$$L_\lambda = \cos(K \cdot d) \tag{9}$$

3. RESULT AND DISCUSSION

For the proposed filter, we have chosen the dielectric materials as Na_3AlF_6/ZnS for ultraviolet region with low and high index contrast. The refractive index for Na_3AlF_6 is $n_1 = 1.34$ and for ZnS is $n_2 = 2.2$. The thickness of the layers is a and b respectively. Taking the values of the a and b as Yablonovite structure $a = 85\%$ of d and $b = 15\%$ of d where $d = a + b$. Using these values, Equation (9) is plotted against the wavelength λ and the curves are depicted in the Figures 2 to 5 respectively. The photonic bands obtained in this manner are shown in the Tables 1 to 4 respectively. Because of the existence of the cosine function on the right-hand side of the Equation (9), the upper and lower limiting values will obviously be $+1$ and -1 respectively. The

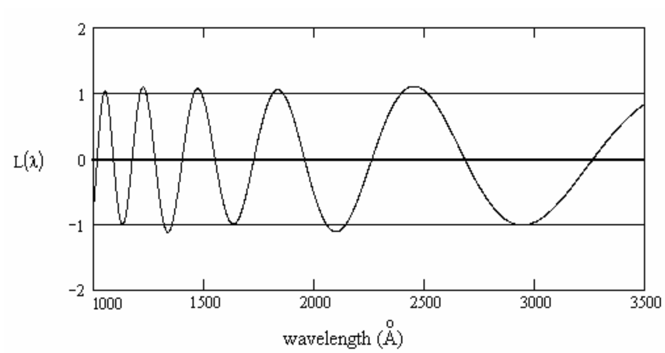

Figure 2. Variation of $L(\lambda)$ with wavelength (λ) for $n_1 = 1.34$, $n_2 = 2.2$, $d = 500\,\text{nm}$, $a = 0.85d$ and $b = 0.15d$ and $\theta = 0°$.

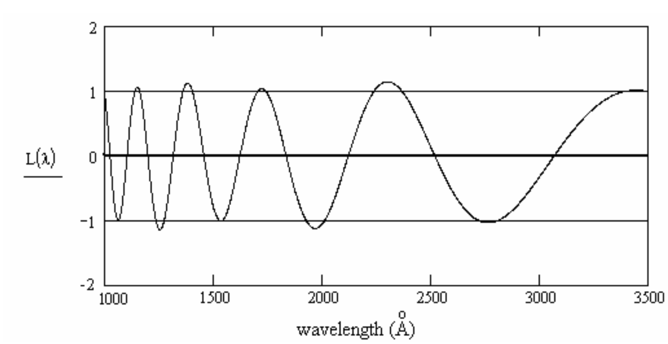

Figure 3. Variation of $L(\lambda)$ with wavelength (λ) for $n_1 = 1.34$, $n_2 = 2.2$, $d = 500\,\text{nm}$, $a = 0.85d$ and $b = 0.15d$ and $\theta = 30°$.

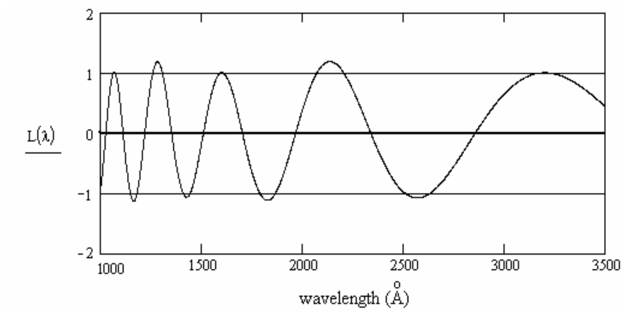

Figure 4. Variation of $L(\lambda)$ with wavelength (λ) for $n_1 = 1.34$, $n_2 = 2.2$, $d = 500$ nm, $a = 0.85d$ and $b = 0.15d$ and $\theta = 45°$.

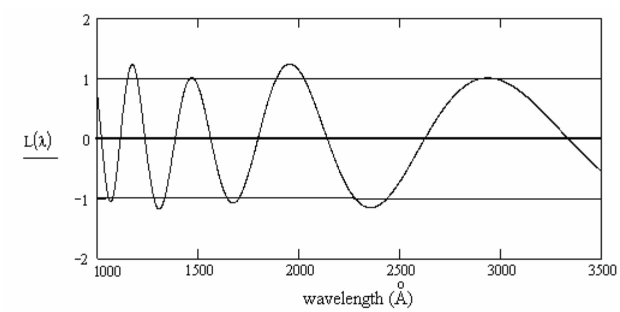

Figure 5. Variation of $L(\lambda)$ with wavelength (λ) for $n_1 = 1.34$, $n_2 = 2.2$, $d = 500$ nm, $a = 0.85d$ and $b = 0.15d$ and $\theta = 60°$.

Table 1. Photonic bands for ($n_1 = 1.34$, $n_2 = 2.2$, $d = 500$ nm, $a = 0.85d$ and $b = 0.15d$ and $\theta = 0°$.

Allowed Bands	Allowed Ranges (in A^0)	Band Width (in A^0)
1.	1000–1044	44
2.	1055–1127	72
3.	1133–1210	77
4.	1237–1318	81
5.	1355–1455	100
6.	1487–1631	144
7.	1634–1813	179
8.	1855–2054	199
9.	2143–2397	254
10.	2509–2910	401
11.	2978–3500	522

Table 2. Photonic bands for ($n_1 = 1.34$, $n_2 = 2.2$, $d = 500\,\text{nm}$, $a = 0.85d$ and $b = 0.15d$ and $\theta = 30°$.

Allowed Bands	Allowed Ranges (in A^0)	Band Width (in A^0)
1.	1000–1059	59
2.	1062–1137	75
3.	1157–1233	76
4.	1271–1359	88
5.	1401–1524	123
6.	1542–1706	164
7.	1736–1923	187
8.	2011–2239	228
9.	2363–2715	352
10.	2810–3413	603

Table 3. Photonic bands for ($n_1 = 1.34$, $n_2 = 2.2$, $d = 500\,\text{nm}$, $a = 0.85d$ and $b = 0.15d$ and $\theta = 45°$.

Allowed Bands	Allowed Ranges (in A^0)	Band Width (in A^0)
1.	1000–1062	62
2.	1070–1145	75
3.	1179–1257	78
4.	1304–1407	103
5.	1442–1594	152
6.	1604–1787	183
7.	1863–2070	234
8.	2202–2506	304
9.	2631–3190	559

portion of the curve lying between these limiting values will yield the allowed ranges of λ and those outsides will show the forbidden ranges of transmission.

From the study of these figures it is found that the width of the allowed photonic bands increases as the wavelength increases, for a fixed values of a, b, n_1 and n_2. Actually these allowed bands give the different ranges of wavelengths that can be transmitted through the filter structure. The ranges of transmission depend on the values of controlling parameters a, b, n_1 and n_2. So by choosing suitable values

Table 4. Photonic bands for ($n_1 = 1.34$, $n_2 = 2.2$, $d = 500\,\text{nm}$, $a = 0.85d$ and $b = 0.15d$ and $\theta = 60°$.

Allowed Bands	Allowed Ranges (in A^0)	Band Width (in A^0)
1.	1000–1055	55
2.	1075–1148	73
3.	1196–1279	83
4.	1332–1459	127
5.	1476–1643	167
6.	1700–1887	187
7.	2023–2275	252
8.	2400–2921	531

of these parameters one can get the desired range of transmission (or reflection). Furthermore, the overall transmission of the filter generally decreases as the value of $(\frac{n_2-n_1}{n_2})$ increases for the fixed value of a and b.

This type of filter is used in fiber optic communications and other optical fields.

REFERENCES

1. Yablonovitch, E., "Inhibited spontaneous emission in solid state physics and electronics," *Phys. Rev. Lett.*, Vol. 58, 2059–2062, 1987.
2. Jonopoulos, J. D., P. Villeneuve, and S. Fan, "Photonic crystals: Putting a new twist on light," *Nature*, Vol. 386, 143–149, 1997.
3. Zheng, Q. R., Y. Q. Fu, and N. C. Yuan, "Characteristics of planar PBG structures with a cover layer," *Journal of Electromagnetic Waves and Applications*, Vol. 20, 1439–1453, 2006.
4. Fink, Y., J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos, and E. L. Thomas, "A dielectric omnidirectional reflector," *Science*, Vol. 282, 1679–1682, 1998.
5. Rojas, J. A. M., J. Alpuente, J. Piñeiro, and R. Sánchez, "Rigorous full vectorial analysis of electromagnetic wave propagation in 1D," *Progress In Electromagnetics Research*, PIER 63, 89–105, 2006.
6. Chigrin, D. N., A. V. Lavrinenko, D. A. Yarotsky, and S. V. Gaponenko, "Observation of total omnidirectional reflection

from a one-dimensional dielectric lattice," *Appl. Phys. A: Mater. Sci. Process.*, Vol. 68, 25–28 1999.

7. Joannopoulos, J. D., R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light*, Princeton University Press, Princeton, NJ, 1995.
8. Burstein, E. and C. Weisbuch, *Confined Electron and Photon: New Physics and Applications*, Plenum Press, New York, 1995.
9. Soukoulis, C., *Photonic Band Gap Materials*, 71–79, Kluwer Academic, Dordrecht, 1996.
10. Macleod, H., *Thin-film Optical Filters*, Adam Hilger, Bristol, 1986.
11. Yeh, P., *Optical Waves in Layered Media*, John Wiley & Sons, New York, 1988.
12. Guida, G., "Numerical studies of disordered photonic crystals," *Progress In Electromagnetics Research*, PIER 41, 107–131, 2003.
13. Birks, T., T. Knight, and J. Russel, "Endlessly single-mode photonic crystal fiber," *Opt. Lett.*, Vol. 22, 961–963, 1997.
14. Hattori, H. T., A. Kazmierczak, V. M. Schneider, and C. L. Barbosa "Photonic crystal micro-cavity based radiation filter," *Journal of Electromagnetic Waves and Applications*, Vol. 19, 1525–1534, 2005.
15. Khalaj-Amirhosseini, M., "Microwave filters using waveguides filled by multi-layer dielectric," *Progress In Electromagnetics Research*, PIER 66, 105–110, 2006.
16. Guenneu, S., A. Nicolet, F. Zolla, and S. Lasquellec, "Numerical and theoretical study of photonic crystal fibers," *Progress In Electromagnetics Research*, PIER 41, 271–305, 2003.
17. Zandi, O., Z. Atlasbaf, and K. Forooraghi, "Flat multilayer dielectric reflector antennas," *Progress In Electromagnetics Research*, PIER 72, 1–19, 2007.
18. Aissaoui, M., J. Zaghdoudi, M. Kanzari, and B. Rezig, "Optical properties of the quasi-periodic one-dimensional generalized multilayer Fibonacci structures," *Progress In Electromagnetics Research*, PIER 59, 69–83, 2006.
19. Maka, T., D. N. Chigrin, S. G. Romanov, and C. M. S. Torres, "Three dimensional photonic crystals in the visible regime," *Progress In Electromagnetics Research*, PIER 41, 307–335, 2003.
20. Wu, C.-J., "Transmission and reflection in a periodic superconductor/dielectric film multilayer structure," *Journal of Electromagnetic Waves and Applications*, Vol. 19, 1991–1996, 2006.

21. Delano, E. and R. J. Pegis, *Progress in Optics*, E. Wolf (ed.), Chap. 7, 69, North-Holland, Amsterdam, 1969.
22. John, S. and Wang, "Quantum optics of localized light in a photonic band gap," *Phys. Rev. B*, Vol. 43, 12772–12789, 1991.
23. Brooks, D. and S. Ruschin, "Integrated electrooptic multielectrode tunable filter," *J. Lightwave Technol.*, Vol. 13, 1508–1513, 1995.
24. Wooten, E. L., R. L. Stone, E. W. Miles, and E. M. Bradely, "Rapidly tunable narrowband wavelength filter using $LiNbO_3$," *J. Lightwave Technol.*, Vol. 14, 2530–2536, 1996.
25. Oda, K., N. Yakato, T. Kominato, and H. Toba, "A 16-channel frequency selection switch for optical FDM distribution systems," *IEEE J. Sel. Areas Commun.*, Vol. 8, 1132–1140, 1990.
26. Stone, J. and L. W. Stulz, "High-performance fibre Fabry-Perot filters," *Electron. Lett.*, Vol. 27, 2239–2240, 1991.
27. Born, M. and E. Wolf, *Principles of Optics*, Cambridge University Press, Cambridge, 1998.
28. Zirngibl, M., C. H. Joyner, and B. Glance, "Digitally tunable channel-dropping filter/equalizer based on waveguide grating router and optical amplifier integration," *IEEE Photonics Technol. Lett.*, Vol. 6, 513–515, 1994.
29. Ishida, O., H. Takahashi, and Y. Inoue, "Digitally tunable optical filters using arrayed-waveguide grating (AWG) multiplexers and optical switches," *J. Light Wave Technol.*, Vol. 15, 321–327, 1997.
30. Sneh, A. and K. M. Johnson, "High-speed tunable liquid crystal filter for WDM networks," *J. Lightwave Technol.*, Vol. 14, 1067–1080, 1996.
31. Chen, P. L., K. C. Lin, W. C. Chuang, Y. C. Tzeng, K. Y. Lee, and W. Y. Lee, "Analysis of a liquid crystal FabryPerot etalon filter: A novel model," *IEEE Photonics Technol. Lett.*, Vol. 9, 467–469, 1997.
32. Fujii, Y., "High-isolation polarization-independent optical circulator coupled with single-mode fibers," *J. Lightwave Technol.*, Vol. 9, 456–460, 1991.
33. Smith, D. A., J. E. Baran, J. J. Johnson, and K. W. Cheung, "Integrated-optic acoustically tunable filters for WDM networks," *IEEE J. Sel. Areas Commun.*, Vol. 8, 1151–1159, 1990.
34. Zhao, L. P., X. Zhai, B. Wu, T. Su, W. Xue, and C.-H. Liang, "Novel design of dual-mode bandpass filter using rectangle structure," *Progress In Electromagnetics Research B*, Vol. 3, 131–141, 2008.

35. Jopson, R. M., J. Jtone, and L. W. Stulz, "Nonreciprocal transmission in a fiber Fabry-Perot resonatorcontaining a magnetooptic material," *Photonics Tech. Lett.*, Vol. 2, 702–704, 1990.
36. Morishita, K., "Optical fiber devices using dispersive materials," *IEEE J. Lightwave Technol.*, Vol. 7, 198–201, 1989.
37. Reid, D. C. J., C. M. Rogdale, I. Robbins, D. J. Robbins, J. Buus, and W. J. Stewart, "Phase-shifted Moire grating fibre resonators," *Electron. Lett.*, Vol. 26, 10–12, 1990.
38. Ojha, S. P., P. K. Chaudhary, P. Khastgir, and O. N. Singh, "Operating characteristics of an optical filter with a linearly periodic refractive index pattern in the filter material," *Jpn. J. Appl. Phys.*, Vol. 31, 281–285, 1992.
39. Chen, J. C., A. Haus, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, "Optical filters from photonic band gap air bridges," *IEEE J. Lightwave Technol.*, Vol. 14, 2575–2580, 1996.
40. D'Orazio, A., M. De Sario, V. Petruzzelli, and F. Prudenzano, "Photonic band gap filter for wavelength division multiplexer," *Opt. Express*, Vol. 11, 3, 230–239, 2003.
41. Del Villar I., I. R. Matias, F. J. Arregui, and R. O. Claus, "Analysis of one-dimensional photonic bandgap structures with a liquid crystal defect toward development of fiber-optic tunable wavelength filters," *Opt. Express*, Vol. 11, 5, 430–436, 2003.
42. Dowling, J. P. and C. M. Bowden, *J. Mod. Optics*, Vol. 41, 345, 1994.