Progress In Electromagnetics Research, PIER 81, 267–277, 2008

TRANSITION BEHAVIOR OF k-SURFACE: FROM HYPERBOLA TO ELLIPSE

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Abstract—The transition behavior of the k-surface of a lossy anisotropic indefinite slab is investigated. It is found that, if the material loss is taken into account, the k-surface does not show a sudden change from hyperbola to the ellipse when one principle element of the permittivity tensor changes from negative to positive. In fact, after introducing a small material loss, the shape of the k-surface can be a combination of a hyperbola and an ellipse, and a selective high directional transmission can be obtained in such a slab.

1. INTRODUCTION

In recent years, left-handed metamaterial (LHM), which possesses simultaneously negative permittivity and permeability, has attracted much attention because of its exotic electromagnetic properties and the potential applications [1–4, 20–30]. In 2003, the behavior of wave propagation in media of which not all the principal elements of the permeability and permittivity tensors have the same sign is investigated [5]. The k-surface of this kind "indefinite media" can be hyperbola or ellipse for lossless cases. A consequent question is what would happen when the k-surface changes from a hyperbola to an ellipse. In this paper we investigate the transition behavior of the k-surface in a non-magnetic anisotropic indefinite slab. It is found that, when one principle element of the permittivity tensor satisfying Drude model changes from negative to positive at the plasma frequency, the k-surface does not show a sudden change from hyperbola to ellipse if a small loss is taken into account. In fact, after introducing a small loss, the shape of the k-surface can be a combination of a hyperbola and an ellipse, and a selective high directional transmission can be obtained in such a slab.

2. THE TRANSITION THE k-SURFACE

Consider a nonmagnetic uniaxial media with a scalar permeability $\mu = 1$ and a permittivity tensor

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{1}$$

which is given in the principal coordinates (x-y-z system). The optical axis is along z-direction with a dielectric constant $\varepsilon_\parallel$, as shown in Fig. 1. For a conventional uniaxial medium, $\varepsilon_\perp$ and $\varepsilon_\parallel$ are both positive. In this paper, we assume $\varepsilon_\parallel$ obey the frequency dependence of Drude model

$$\varepsilon_\parallel = 1 - \omega_p^2/[\omega(\omega + i\gamma_e)], \tag{2}$$

where ω_p is the plasmas frequency (we will normalize $\omega_p = 1$ in following calculations) and γ_e stands for the material loss. The dielectric constants in the directions perpendicular to the optical axis are both simply assumed to be $\varepsilon_\perp = 1.6{+}0.01i$. In the GHz frequencies, the idea to achieve this frequency dependence of $\varepsilon_\parallel$ and $\varepsilon_\perp$ can be realized by a composite of periodically arranged metallic thin wires aligned along the optical axis [11–13].

Consider such an anisotropic slab with thickness d as shown in Fig. 1, where the angle between the optical axis of the slab and the z-axis is θ (note that the optical axis is in the x-z plane).

The dispersion relation for a TM wave with the magnetic field polarized along the y axis can be expressed as

$$\begin{aligned} &k_x^2\left(\varepsilon_\perp \cos^2\theta + \varepsilon_\parallel \sin^2\theta\right) + k_{2z}^2\left(\varepsilon_\parallel \cos^2\theta + \varepsilon_\perp \sin^2\theta\right) \\ &+2\sin\theta\cos\theta k_x k_{2z}\left(\varepsilon_\parallel - \varepsilon_\perp\right) - \varepsilon_\parallel\varepsilon_\perp(\omega/c)^2 = 0, \end{aligned} \tag{3}$$

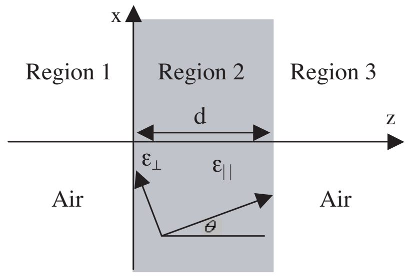
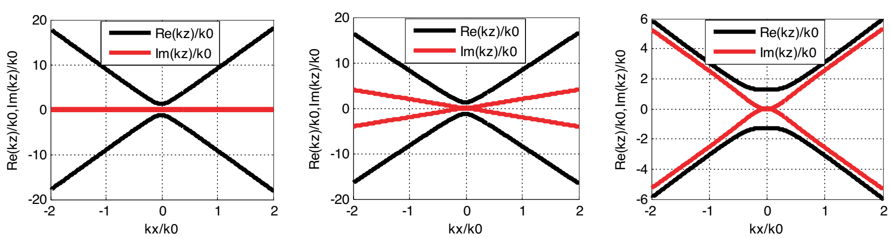


Figure 1. An anisotropic slab with $\varepsilon_\parallel = 1 - \omega_p^2/[\omega(\omega + i\gamma_e)]$ and $\varepsilon_\perp = 1.6$.

where c is the speed of light in air. For an arbitrarily given k_x determined by the incidence, generally within the slab there should be two solutions for k_{2z} with opposite directions, say k_2^i, k_2^r. However, when $\varepsilon_\parallel \cos^2\theta + \varepsilon_\perp \sin^2\theta = 0$, there will be only one solution of k_{2z} for any k_x. This is just the case discussed in [ ], however in this paper, we will show the solution in an more intuitive way.

In order not to lose generality, we assume k_2^i, k_2^r are complex for a given k_x, and we plot respectively the real and imaginary parts of k_{2z} in one figure by calculating Eq. (3) with $\omega = 0.99\omega_p$, $\theta = 0°$ and different γ_e, as shown in Fig. 2. We see that the increasing of the loss leads to the increasing of the imaginary part of k_{2z}, except in the normal incidence case.



(a) $\gamma_e = 0$ (b) $\gamma_e = 0.01$ (c) $\gamma_e = 0.1$

Figure 2. The k-surface of the dispersion relation of the indefinite media with $\omega = 0.99\omega_p$, $\theta = 0°$ and (a) $\gamma_e = 0$, (b) $\gamma_e = 0.01$, (c) $\gamma_e = 0.1$, respectively.

In Fig. 3, we investigate the k-surface of the dispersion relation when the sign of the real part of $\varepsilon_\parallel$ changes from negative to positive. For a lossless case, the negative or positive $\varepsilon_\parallel$ corresponds to a hyperbolic and an elliptic k-surface, respectively [5]. However, as shown in Fig. 3, after introducing a small loss, for example $\gamma_e = 0.01$, no sudden change of the k-surface occurs as $\varepsilon_\parallel$ changes its sign, instead, the real part of the k-surfaces became combinations of hyperbolas and ellipses, which brings drastically electromagnetic behaviors near the plasma frequency.

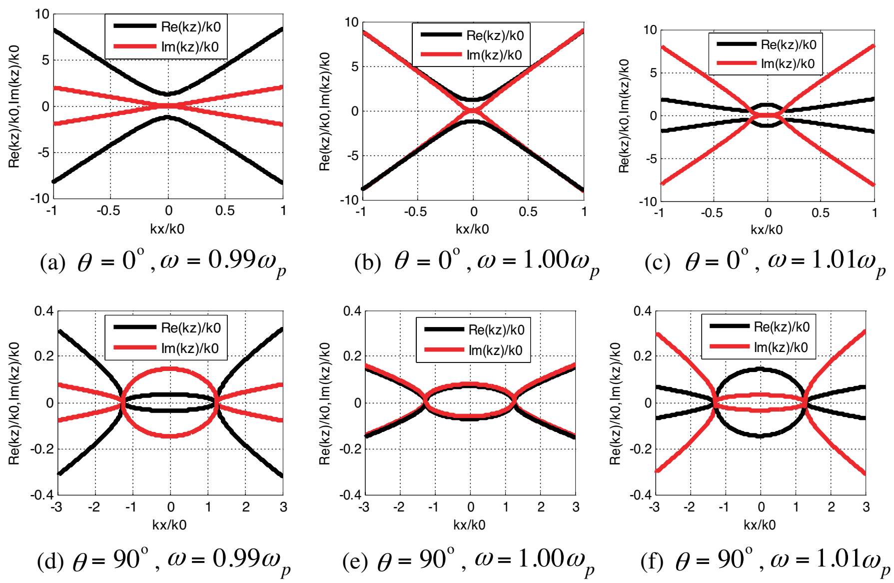


Figure 3. The transition of the k-surface from hyperbola to ellipse with $\gamma_e = 0.01$. (a) $\theta = 0^\circ$, $\omega = 0.99\omega_p$, (b) $\theta = 0^\circ$, $\omega = 1.00\omega_p$, (c) $\theta = 0^\circ$, $\omega = 1.01\omega_p$, (d) $\theta = 90^\circ$, $\omega = 0.99\omega_p$, (e) $\theta = 90^\circ$, $\omega = 1.00\omega_p$, (f) $\theta = 90^\circ$, $\omega = 1.01\omega_p$.

As we mentioned, when $\varepsilon_\parallel \cos^2\theta + \varepsilon_\perp \sin^2\theta = 0$, there is only one solution for k_{2z} with a given k_x in a lossless case [14], as shown in Fig. 4(a). However, after introducing a small loss of the material, we can always find two solutions for a given k_x, as shown in Figs. 4(b) and (c). It appears that assuming lossless for a negative permittivity yields in some case ambiguity which can be solved after introducing a small material loss.

3. HIGH DIRECTIVE TRANSMISSION PROPERTY OF THE INDEFINITE SLAB

As is shown in Figs. 3(a)–(c), the imaginary part of k_{2z} is very small when k_x is in the vicinity of zero, while the imaginary part of k_{2z} is

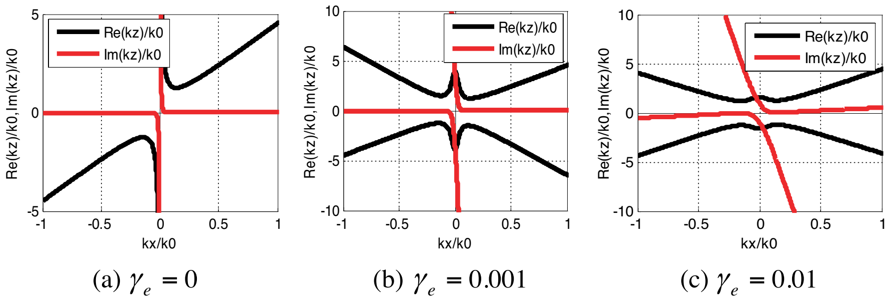


Figure 4. The k-surface for $\varepsilon_\parallel \cos^2\theta + \varepsilon_\perp \sin^2\theta = 0$ with $\theta = 6.425°$, $\omega = 0.99\omega_p$ and (a) $\gamma_e = 0$, (b) $\gamma_e = 0.001$, (c) $\gamma_e = 0.01$.

a comparably large value if k_x is away from the vicinity of zero. This property of k-surface gives birth to the high transmission directivity. Only the wave with $k_x = 0$ can propagate through this slab.

To examine the property of high directive transmission, a more general case is considered. Let a monochromatic H-polarized wave $\overline{H} = \hat{y}e^{i(k_z z + k_x x)}$ (with the time dependence $e^{-i\omega t}$) be obliquely incident upon an anisotropic slab with thickness d as shown in Fig. 1. The permittivity tensor of the anisotropic medium is described as

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp \cos^2\theta + \varepsilon_\parallel \sin^2\theta & 0 & -\sin\theta\cos\theta(\varepsilon_\perp - \varepsilon_\parallel) \\ 0 & \varepsilon_\perp & 0 \\ -\sin\theta\cos\theta(\varepsilon_\perp - \varepsilon_\parallel) & 0 & \varepsilon_\perp \sin^2\theta + \varepsilon_\parallel \cos^2\theta \end{pmatrix} \tag{4}$$

The analytical treatment is omitted here for the sake of brevity. By using the boundary condition, the reflection and transmission coefficients are found to be

$$T = \frac{2k_{1z}\left(k_2^i - k_2^r\right)m}{e^{i\left(k_{1z} - k_2^r - k_2^i\right)d}\left(e^{ik_2^r d}p - e^{ik_2^i d}q\right)} \tag{5}$$

where

$$p = \left(k_{1z} + k_2^i m - n\right)\left(k_{1z} - k_2^r m + n\right), \quad p = \left(k_{1z} + k_2^r m - n\right)\left(k_{1z} - k_2^i m + n\right),$$

$$m = \frac{\cos^2\theta}{\varepsilon_\perp} + \frac{\sin^2\theta}{\varepsilon_\parallel}, \quad n = k_x \sin\theta \cdot \cos\theta \cdot \left(\frac{1}{\varepsilon_\perp} - \frac{1}{\varepsilon_\parallel}\right).$$

In Figs. 5(a) and (b), the magnitude of transmission coefficient T was plotted as a function of k_x (normalized by $k_0 = \omega/c$) and ω

(normalize by ω_p) with different material loss γ_e. From these figures, we clearly see that the oblique incident wave can not propagate through the slab if ω is close to ω_p, since the magnitude of transmission coefficient T is close to zero when $k_x \neq 0$. In addition, the smaller the material loss, the higher of the directivity of the transmission. A full wave simulation is carried out to verify the high transmission directivity. As shown in Fig. 5(c), only the wave with $k_x = 0$ can propagate through the slab.

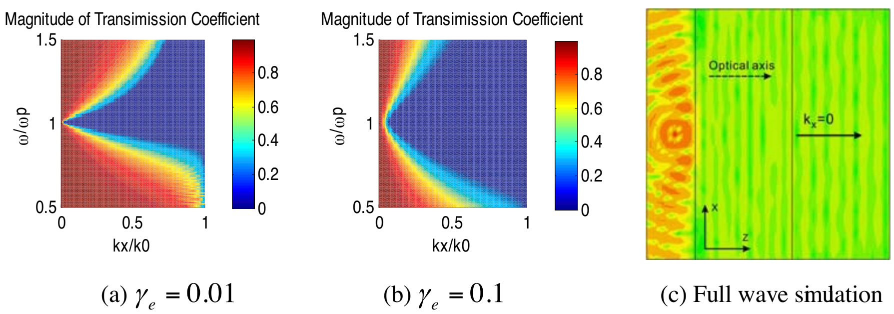
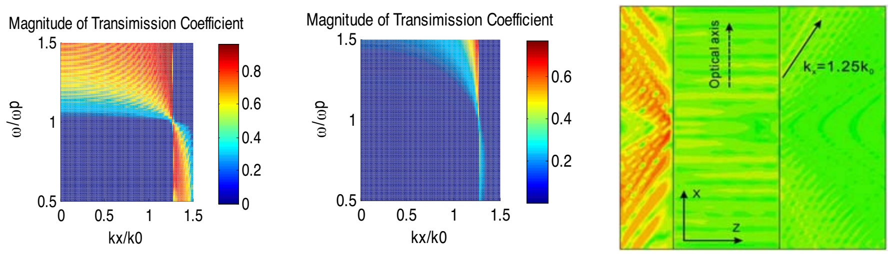


Figure 5. The transmission coefficient as a function of the transverse wave vector k_x, frequency ω and with (a) $\gamma_e = 0.01$, (b) $\gamma_e = 0.1$. The thickness of the slab is 10λ (wavelength of the source) and $\theta = 0°$ in this calculation. (c) The full wave verification of the high transmission directivity property with $\omega = \omega_p$ and $\gamma_e = 0.01$. A magnetic dipole is placed 1λ away from the left hand side of the slab.

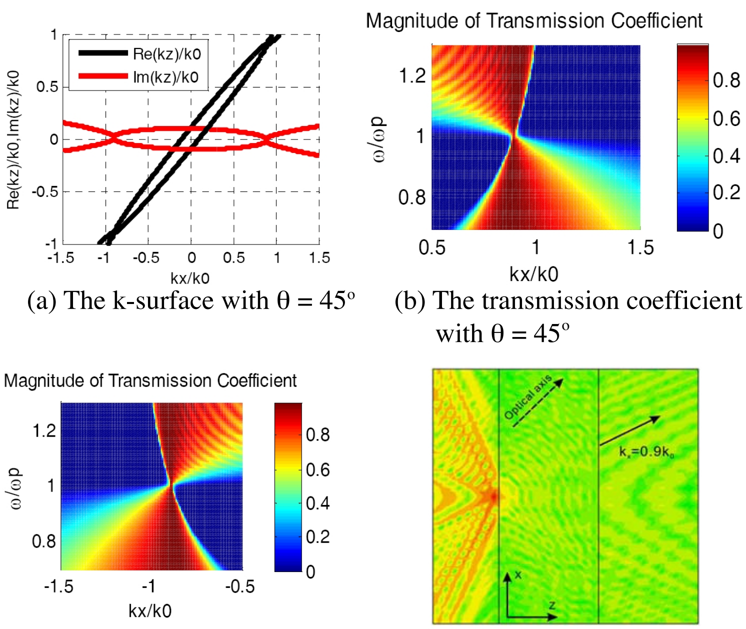
As shown in Figs. 3(d)–(f), the imaginary part of k_{2z} is very small when $k_x = 1.25k_0$, while it is a comparably large value if $k_x \neq 1.25k_0$. So we also expect that the high directivity transmission property can be observed when $k_x = 1.25k_0$. In order to study the transmission property In this case, the air in the region1 and region3 is replaced by a dielectric with and $\mu = 1$, since the wave is an evanescent type in the air with $k_x = 1.25k_0$. The transmission coefficient T as a function of the transverse wave vector k_x and frequency ω is shown in Fig. 6, and only the wave whose transverse wave vector k_x is close to $1.25k_0$ can propagate through the slab. From Fig. 6(b), as the loss of the material increases, the magnitude of the transmission coefficient decreases, but the high directivity still exists. A full wave simulation is also carried out for this case as shown in Fig. 6(c).

To give one more example, we also study the case of $\theta = 45°$. In Fig. 7(a), the imaginary part of the $k_{2z}$is very small when $k_x = 0.9k_0$, while it is a comparably large value if $k_x \neq 0.9k_0$. So the high directivity transmission property is observed in Fig. 7(b) with $k_x =$



(a) $\gamma_e = 0.01$ (b) $\gamma_e = 0.1$ (c) Full wave simulation

Figure 6. The transmission coefficient as a function of the transverse wave vector k_x and frequency ω. The thickness of the slab is 10λ (wavelength of the source) and $\theta = 90°$ in this calculation. (a) $\gamma_e = 0.01$, (b) $\gamma_e = 0.1$, (c) The full wave simulation with and $\mu = 1$ as the background material.



(a) The k-surface with θ = 45° (b) The transmission coefficient with θ = 45°

(c) The transmission coefficient with θ = 135° (d) The full wave simulation

Figure 7. (a) The k-surface with $\omega = 1.00\omega_p$ with $\theta = 45°$ and $\gamma_e = 0.01$. (b) The transmission coefficient with $\theta = 45°$. (c) The transmission coefficient with $\theta = 135°$. (d) The full wave simulation with and $\mu = 1$ as the background material.

$0.9k_0$. The magnitude of the Transmission coefficient with $\theta = 135^\circ$ is shown in Fig. 7(c), and a full wave simulation is also carried out for this case as shown in Fig. 7(d).

4. CONCLUSION

We investigate the k-surface of an anisotropic indefinite slab whose principal elements of the permittivity tensor can have different signs. It is found that, if the material loss is taken into account, the k-surface does not show a sudden change from hyperbola to ellipsis when the principle element of the permittivity tensor changes from a negative value to a positive one. In fact, after introducing a small material loss, the shape of the k-surface can be a combination of a hyperbola and an ellipse, so the conventional method to categorize the k-surface into hyperbolic and elliptic types seems not appropriate for the indefinite media with material loss. The transmission property of the anisotropic slab is also discussed during this transition of the principle element of the permittivity tensor. It is shown that the high directive transmission property of the anisotropic slab can be observed. The back physics of the high directivity transmission property, as shown in Fig. 3 and Fig. 7, is due to the imaginary part of the k_{2z}, that is the imaginary part of k_{2z} is a small value for some specific k_x, while is a comparably large value for other k_xs. In addition, by adjusting the orientation of the optical axis, we can get different high directivity for different k_xs. Since the electrically anisotropic material discussed here can be easily fabricated in GHz frequencies, our result may solve the ambiguity resulted from the sudden change of k-surface in the lossless case and find some practical application in designing of new microwave devices such as high directive antennas and etc.

The electrically anisotropic material discussed here can be easily fabricated in GHz frequencies by a composite of periodically arranged metallic thin wires aligned along the optical axis. In optical region, the anisotropic indefinite media had been experimentally fabricated based on a composite of altering layers of dielectric and metal [19], and it had become a research-focus recently because of its application in far-field imaging [17–19]. Our result here may solve the ambiguity of indefinite media resulted from the sudden change of k-surface in the lossless case and find some practical application in designing of new devices such as high directive antennas and etc.

ACKNOWLEDGMENT

This work is supported by the NSFC (Nos. 60531020, 60671003), ZJNSF (No. R105253), the NCET-07-0750, and the Ph.D. Programs Foundation of MEC (No. 20070335120).

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