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ASYMPTOTIC ITERATION METHOD: A POWERFUL APPROACH FOR ANALYSIS OF INHOMOGENEOUS DIELECTRIC SLAB WAVEGUIDES

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Abstract—In this paper a novel approach based on Asymptotic Iteration Method (AIM) is presented to solve analytically the light propagation through one-dimensional inhomogeneous slab waveguide. Practically implemented optical slab waveguides based on traditional techniques are usually inhomogeneous and numerical methods are used to obtain guided wave characteristics. In this work, we develop analytical method for modal analysis includes Eigen modes (electric and magnetic fields distribution) and Eigen values (guided wave vector) using AIM. The developed method is applied to some especial examples.

1. INTRODUCTION

Optical integrated methods recently have been considered because of inherent potential applications in engineering fields. Also, developing easy basic cell for obtaining optical very large scale integration (VLSI) is so interesting. For this purpose slab waveguides are basic interconnection and passive elements in integrated optics. Precise analysis of dielectric slab waveguides for more understanding physical concepts is important. Especially, inhomogeneous slab waveguide which is implemented in practice using conventional fabrication methods is common device in integrated optics. For this task, in this paper we try to develop analytical method for investigation of inhomogeneous slab waveguides from modal characteristics points of view. Traditionally numerical methods were used for modal analysis of inhomogeneous waveguides. But, in these cases concepts of device operation could not explain clearly.

There are some interesting published papers for analytical analysis of slab waveguides. We try to review some of them and investigate advantages and disadvantages of these works.

Second order homogenous linear differential equations appear widely in literature and there are several techniques for obtaining exact solutions of them. Recently, a novel approach presented to solve the light propagation through one dimensional inhomogeneous slab waveguide analytically based on Nikiforov-Uvarov method [1]. This mathematical method is based on reducing the second order linear differential equations to a generalized equation of hyper-geometric type [1–7].

For clear and easy understanding of operation of these devices analytical methods are excellent and developed before in similar engineering fields or physical and mathematical research areas. One of these analytical methods is based on AIM mathematical approach which can be used for modal analysis of inhomogeneous waveguides [8–13].

The supersymmetric quantum mechanical method was applied to present some useful shape invariant index of refraction having exact solution for electromagnetic field distribution and Eigen wave vectors [14]. In this paper the authors developed new method for evaluation of the inhomogeneous slab waveguides.

Oscillator-like Hamiltonian was used to develop analytical treatment for investigation of the inhomogeneous slab waveguides with second order position dependency [15]. In this paper TM-mode electromagnetic field propagating through oscillator-like index of refraction profile was compared with Schrödinger equation of harmonic-oscillator. Then using generalized solution in quantum mechanics all optical interesting quantities determined exactly. In this method only oscillator-like index of refraction can be evaluated analytically.

PT-invariant quantum mechanical tool was used to develop analytical solution for electromagnetic fields for inhomogeneous medium having polynomial distribution versus position at most 6th order [16]. This work presents also a new class of inhomogeneous index of refraction for slab waveguide with optical characteristics analytically.

A relatively broad class of inhomogeneous index of refraction with exact solution was developed in [17] using similarity transformed harmonic oscillator algebra. The proposed method introduces a wide class of exactly solvable medium for slab waveguides.

Another analytical approach for modal analysis of inhomogeneous slab waveguide based on supersymmetric quantum mechanical method was presented in [18]. In this paper a set of shape invariant index of refraction was developed. For the obtained set of index of refraction, the guided wave vector and electric field distribution were calculated analytically.

There is close relation between general second-order ordinary differential equations and the Schrödinger equation. Based on the mentioned relation analytical solution of the Schrödinger equation for wide class of the potentials can be obtained. For this purpose the AIM is used [11–13]. Different aspects of slab waveguide including isotropic and anisotropic cases were considered in [19–25]. In these papers modal analysis were considered.

There are different alternative methods for investigation of modal analysis in optical waveguides especially inhomogeneous including layered media which discussed in [26–35].

Also, electromagnetic wave equation for inhomogeneous linear media can be converted to general second order ordinary differential equation in one-dimensional cases. So, analytical evaluation of the Maxwell equations can be done in this domain. For this purpose one-dimensional case of linear inhomogeneous media is considered in this work. Then using AIM approach, relation between the index of refraction and parameters of the second-order ordinary differential equation in the case of exactly solvable problems are extracted. Finally, the characteristic of the inhomogeneous media are obtained.

Organization of the paper is as follows:
Mathematical background is presented in Section 2. In this section the Maxwell's equations is arranged in the form of wave equation for comparing with standard second order homogeneous linear differential equation. Next, using AIM exact solution of the wave equation is obtained. Results and discussion of the presented method is presented in Section 3. Finally, the paper ends with a short conclusion.

2. MATHEMATICAL BACKGROUND

In this section mathematical background is presented for management of the light propagation through inhomogeneous slab waveguide. We consider this subject in the next two sub-sections. First, we manipulate the Maxwell's equations to review the wave equation in inhomogeneous slab waveguides. Secondly, AIM is used to obtain analytical solution of the general homogeneous second order differential equations.

2.1. Light Propagation through Inhomogeneous Media

We first consider Maxwell's equations with a plan wave incident upon an inhomogeneous slab waveguides as shown in Fig. 1.

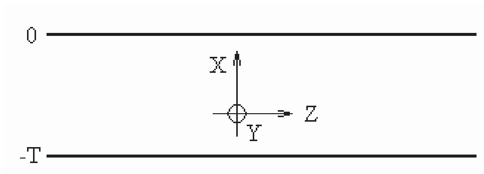

Figure 1. Schematic of slab waveguide.

For simplicity, we put the x-z plan in the incidence plan, such that $\partial/\partial y = 0$. In addition, optical characteristics, permittivity $\varepsilon(x)$ and permeability $\mu(x)$, are supposed as functions of slab's height x. Using the Maxwell's equation and considering the slab waveguide geometry, we obtain a second order differential equation for TE-Mode of electromagnetic field in inhomogeneous waveguides. Actually, we have two independent TE and TM waves in this two dimensional space. In principle, since these two modes are similar to each other; so, we just consider TM mode. Now, let us start by introducing the plan wave as especial solution of the wave equation

$$\mathrm{H} = \mathrm{H}(x)e^{i(\omega t - kz)},$$

and the following forms for the Maxwell equations

$$\nabla \times \mathrm{E} = -i\omega\mu(x)\mathrm{H}, \quad \nabla \times \mathrm{H} = i\omega\varepsilon(x)\mathrm{E},$$

in which, $\mathrm{H}(x)$ is y-component of magnetic field. Using mathematical combination of the Maxwell's equations, one gets

$$\nabla \times \left(\nabla \times \frac{\mathrm{H}}{\varepsilon(x)}\right) = \omega^2\mu(x)\mathrm{H}, \tag{1}$$

where, we have assumed that there are no any charge density ($\nabla \cdot D = 0$) and magnetic monopole ($\nabla \cdot B = 0$). Eq. (1) leads to the following differential equation

$$\mathrm{H}''(x) - \frac{\varepsilon'(x)}{\varepsilon(x)}\mathrm{H}'(x) + \left[\omega^2\mu(x)\varepsilon(x) - k^2\right]\mathrm{H}(x) = 0. \tag{2}$$

In this equation, the prime indicates differentiation with respect to x. This is a second order differential equation, which can be exactly solved for some functions $\varepsilon(x)$ and $\mu(x)$. In the next section, we will introduce an appropriate mathematical method for this purpose.

2.2. Asymptotic Iteration Method

AIM is proposed for obtaining the exact solution of a second order differential equation in the following form

$$Y''(x) - \lambda_0(x)Y'(x) - s_0(x)Y(x) = 0, \tag{3}$$

where $\lambda_0(x) \neq 0$ and $s_0(x)$ are coefficients of the differential equation and are well defined functions as well as sufficiently differentiable. In order to obtain a general solution to the equation the following procedure is supposed.

By differentiating Eq. (3) with respect to x one finds

$$Y'''(x) - \lambda_1(x)Y'(x) - s_1(x)Y(x) = 0,$$

where $\lambda_1(x) = \lambda_0' + \lambda_0^2 + s_0$ and $s_1(x) = s_0' + s_0\lambda_0$ are new coefficients related to the coefficients in the original differential equation.

The second derivative of Eq. (3) leads to

$$Y''''(x) - \lambda_2(x)Y'(x) - s_2(x)Y(x) = 0,$$

where $\lambda_2(x) = \lambda_1' + \lambda_1\lambda_0 + s_1$ and $s_2(x) = s_1' + s_0\lambda_1$ are new coefficients appeared in new version of the basic differential equation.

Finally, by taking mth derivative of Eq. (3) one obtains the following equation:

$$Y^{(m)}(x) - \lambda_{m-2}(x)Y'(x) - s_{m-2}(x)Y(x) = 0, \tag{4}$$

where

$$\begin{aligned} \lambda_m(x) &= \lambda_{m-1}' + \lambda_{m-1}\lambda_0 + s_{m-1}, \\ s_m(x) &= s_{m-1}' + s_0\lambda_{m-1} \\ m &= 1, 2, \ldots . \end{aligned} \tag{5}$$

Clearly, the following relation can be obtained from Eq. (4),

$$\frac{Y^{(m+2)}(x)}{Y^{(m+1)}(x)} = \frac{\lambda_m \left[Y'(x) + \dfrac{s_m}{\lambda_m} Y(x) \right]}{\lambda_{m-1} \left[Y'(x) + \dfrac{s_{m-1}}{\lambda_{m-1}} Y(x) \right]}, \tag{6}$$

and by introducing the asymptotic aspect of the iteration method for sufficiently large value of m

$$\frac{s_m}{\lambda_m} = \frac{s_{m-1}}{\lambda_{m-1}} \equiv \alpha. \qquad (7)$$

Consequently Eq. (6) reduces to following simple form

$$\frac{Y^{(m+2)}(x)}{Y^{(m+1)}(x)} = \frac{\lambda_m}{\lambda_{m-1}}. \qquad (8)$$

Clearly, by integrating (8), we find

$$Y^{(m+1)}(x) = Ce^{\int \frac{\lambda_m(x)}{\lambda_{m-1}(x)} dx}, \qquad (9)$$

where C is the integration constant. Using relation (5) and the definition of (7) one can rewrite (9) as follows

$$Y^{(m+1)}(x) = C\lambda_{m-1}(x)e^{\int [\alpha(x)+\lambda_0(x)]dx} \qquad (10)$$

Substituting (10) into (4) leads to the first order differential equation

$$Y'(x) + \alpha Y(x) - Ce^{\int [\alpha+\lambda_0(x)]dx} = 0.$$

The general solution of the last deferential equation is

$$Y(x) = e^{-\int \alpha(x)dx}[C' + C \int e^{\int [\lambda_0(x)+2\alpha(x)]dx} dx], \qquad (11)$$

where C' is a new integration constant.

Finally, the energy eigenvalues are obtained from the roots of the following condition

$$\lambda_m(x)s_{m-1}(x) - \lambda_{m-1}(x)s_m(x) = 0 \quad m = 1, 2, 3, \ldots . \qquad (12)$$

2.3. Some Examples

Now, we use AIM to obtain the exact solutions (Eigenvalues and Eigenfunctions) of Eq. (2) for some given permittivity and permeability functions. Let us start with

$$\varepsilon(x) = \varepsilon_0 e^{x^2}, \quad \mu(x) = \mu_0 e^{-x^2}.$$

Substituting in Eq. (2) leads to

$$\mathrm{H}''(x) - 2x\mathrm{H}'(x) + \left(\frac{\omega^2}{c^2} - k^2\right)\mathrm{H}(x) = 0,$$

where $\varepsilon_0\mu_0 = 1/c^2$.

By comparing the last equation with Eq. (3) one can write $\lambda_0(x)$ and $s_0(x)$ functions, and calculate $\lambda_m(x)$ and $s_m(x)$ using recurrence relation (6) for several values of m as follows

$$\begin{aligned}
\lambda_0(x) &= 2x \\
s_0(x) &= \left(\frac{\omega^2}{c^2} - k^2\right) \\
\lambda_1(x) &= 4x^2 + 2 - \left(\frac{\omega^2}{c^2} - k^2\right) \\
s_1(x) &= -2x\left(\frac{\omega^2}{c^2} - k^2\right) \\
\lambda_2(x) &= 4x\left[2x^2 + 3 - \left(\frac{\omega^2}{c^2} - k^2\right)\right] \\
s_2(x) &= -\left(\frac{\omega^2}{c^2} - k^2\right)\left[4x^2 + 4 - \left(\frac{\omega^2}{c^2} - k^2\right)\right] \\
&\ldots \text{etc.}
\end{aligned} \tag{13}$$

By combing these results it is straightforward to show that

$$\lambda_m(x)s_{m-1}(x) - \lambda_{m-1}(x)s_m(x) = \prod_{n=0}^{m+1}\left(\frac{\omega^2}{c^2} - k^2 - 2n\right) \quad m = 0,\, 1,\, 2,\, \ldots$$

Clearly the quantization condition (12) implies that $\frac{\omega^2}{c^2} - k^2$ should be a non-negative and even integer

$$\frac{\omega^2}{c^2} - k^2 = 2n \quad n = 0,\, 1,\, 2,\, \ldots .$$

According to this condition, energy Eigenvalues are obtained as follows

$$k = \sqrt{\frac{\omega^2}{c^2} - 2n}, \quad n = 0,\, 1,\, 2,\, \ldots .$$

These results can be used to calculate the ratio $s_n(x)/\lambda_n(x)$ and consequently for obtaining general solution via (11) as follows

$$\begin{aligned}
n = 0, &\quad \frac{s_0}{\lambda_0} = \frac{s_1}{\lambda_1} = \ldots = 0 &&\Rightarrow \quad H_0(x) = 1 \\
n = 1, &\quad \frac{s_1}{\lambda_1} = \frac{s_2}{\lambda_2} = \ldots = -\frac{1}{x} &&\Rightarrow \quad H_1(x) = 2x \\
n = 2, &\quad \frac{s_2}{\lambda_2} = \frac{s_3}{\lambda_3} = \ldots = -\frac{4x}{2x^2 - 1} &&\Rightarrow \quad H_2(x) = 4x^2 - 2 \\
&\qquad \ldots \text{ etc.}
\end{aligned}$$

It can be easily shown that the general form of the solution is in the following form

$$H_n(x) = (2x)^n - \frac{n(n-1)}{1!}(2x)^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2!}(2x)^{n-4} + \dots ,$$

which is known as Hermite polynomials.

Table 1. Some of exactly solvable permittivity $\varepsilon(x)$, permeability $\mu(x)$, their eigenvalues and functions.

$a > 0,\ b > 0$

N	$\varepsilon(x)$	$\mu(x)$	$\mathrm{H}(x)$	k
1	$\varepsilon_0 e^{x^2}$	$\mu_0 e^{-x^2}$	$\mathrm{H}_n(x)$	$\sqrt{\left(\frac{\omega}{c}\right)^2 - 2n}$
2	ε_0	$\mu_0 x^2$	$e^{-\omega^2 x^2/2c}\mathrm{H}_n(\sqrt{\omega/c}x)$	$(2n+1)(\frac{\omega}{c})$
3	ω^{-2} $\times x^2$	$ax - b$	$x^{(\sqrt{4b+1}-1)/2}e^{-kx}$ $\times L_n^{\sqrt{4b+1}}(2kx)$	$\frac{a}{2n+1+\sqrt{4b+1}}$
4	ω^{-2} $\times\sqrt{x}$	$\frac{1}{4}\times(\frac{a}{\sqrt{x}}$ $-\frac{b}{x\sqrt{x}})$	$x^{\sqrt{b+1/4}-1/2}e^{-kx^2}$ $\times L_n^{\sqrt{b+1/4}}(2kx^2)$	$\frac{a}{2(4n+2b+3)}$
5	ω^{-2} $\times x^{3/2}$	$a\sqrt{x} - \frac{b}{\sqrt{x}}$	$x^{(\sqrt{16b+1}-1)/4}e^{-kx}$ $\times L_n^{\sqrt{4b+1/4}}(2kx)$	$\frac{a}{2n+1+\sqrt{4b+1/4}}$
6	ε_0	$\frac{a}{\varepsilon_0}e^{-\alpha x}$ $-\frac{b}{\varepsilon_0}e^{-2\alpha x}$	$e^{(-kx+\frac{\omega\sqrt{b}}{\alpha}e^{-\alpha x})}$ $\times L_n^{2k/\alpha}(-\frac{2\omega\sqrt{b}}{\alpha}e^{-\alpha x})$	$-\frac{\alpha}{2}\times\left[\frac{\omega a}{\alpha\sqrt{b}}\right.$ $\left.+(2n+1)\right]$
7	ε_0	$-\frac{a}{x^2}+\frac{b}{x}$	$x^{\ell+1}e^{-x/n}$ $\times L_{n-\ell-1}^{2\ell+1}(\omega^2\varepsilon_0 bx/n)$ $\omega^2\varepsilon_0 a = \ell(\ell+1)$	$\frac{\omega^2\varepsilon_0 b}{2n+1+\sqrt{1+4\omega^2\varepsilon_0 a}}$
8	$\frac{k^2/\omega^2}{1-x^2}$	$1 - x^2$ $-\frac{1}{1-x^2}+\frac{a}{k^2}$	$(1-x^2)^{k/2}P_n^{(k,k)}(x)$	$\sqrt{a+1/4}$ $-(n+1/2)$
9	ε_0	$-\mu_0$ $\times\frac{e^{-\alpha x}}{(1-e^{-\alpha x})^2}$	$e^{-kx}(1-e^{-\alpha x})^{\ell/2}$ $\times P_n^{(2k/\alpha,\ell-1)}(1-2e^{-\alpha x})$ $\ell = 1+\sqrt{1+4(\omega/c\alpha)^2}$	$\frac{\alpha}{4}\times[\ell-1$ $-(2n+1)]$
10	ε_0	$\mu_0\times\cosh^{-2}$ $\omega\sqrt{\varepsilon_0}x$	$(1-tgh^2(\omega\sqrt{\varepsilon_0}x))^{b/2}$ $\times P_n^{(b,b)}(tgh(\omega\sqrt{\varepsilon_0}x))$ $b = k/\omega\sqrt{\varepsilon_0}$	$\frac{1}{2}\omega\sqrt{\varepsilon_0}$ $\times[\sqrt{1+4\mu_0}$ $-(2n+1)]$

3. RESULTS AND DISCUSSION

One can investigate the exact solutions of Eq. (2), as well as corresponding eigenvalues for some permittivity and permeability functions using AIM approach. The results for some calculated examples are given in Table 1.

It was shown that the AIM approach can be used to develop analytical solution for a broad class of inhomogeneous slab waveguide. The calculated results for some examples were illustrated in Table 1. In this table inhomogeneous medium and modal characteristics were demonstrated.

4. CONCLUSION

In this paper, analytical approach for modal analysis of the inhomogeneous slab waveguide has been developed based on the AIM. According to developed analytical method guided wave vector and magnetic field distribution for some interesting examples were given in Table 1. In general, the introduced technique can be extended to wide class of the index of refractions.

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