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ELECTROMAGNETIC FIELD OF A HORIZONTAL ELECTRIC DIPOLE IN THE PRESENCE OF A FOUR-LAYERED REGION

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Abstract—In this paper, we study in detail the electromagnetic field excited by a horizontal electric dipole in the presence of a four-layered region, which consists of a perfect conductor, the two dielectric layers, and air above. From the derivations and analysis, it is seen that the electromagnetic field includes four wave modes: Direct wave, ideal reflected wave, trapped surface wave, and lateral wave. The wave numbers of the trapped surface wave, which are determined by the residues of the poles, are between the wave number k_0 in the air and k_2 in the lower dielectric layer. The lateral waves with the wave number being k_0 are determined by the integrations along the branch cuts. It should be pointed out that both the trapped surface wave and lateral wave can be separated into the electric-type terms and magnetic-type terms. Analysis and computations show that the trapped surface waves play major roles at large propagation distance when both the dipole point and the observation point are on or close to the air-dielectric boundary.

1. INTRODUCTION

The electromagnetic field excited by a dipole source in a layered region has been investigated widely because of its useful applications in subsurface and closed-to-the surface communication, radar and geophysical prospecting and diagnostics [1–35]. In the pioneering works by Wait [1–5], extensive investigations have been carried out for the electromagnetic field in a layered region by using asymptotic methods, contour integrations, and branch cuts. In the works by King et al. [8–13], the complete formulas have been obtained for the electromagnetic fields excited by horizontal and vertical electric dipoles in two- and three-layered regions. In a series of works by Li et al. [20–22], the dyadic Green's function technique is used to treat the electromagnetic field in a four-layered forest environment.

In the end of 20th century, the debates on the trapped surface wave excited by a dipole source in a three-layered region, which varies as $\rho^{-1/2}$ in the far-field region, was occurred between two famous professors, Wait [14] and King [15]. Subsequently, several investigators have revisited the old problem and concluded that the trapped surface wave, which is contributed by the sums of residues of the poles, can be excited efficiently by a dipole source in the three-layered region [29–35]. The new developments in [29–35], naturally, rekindle the interest in the study on the electromagnetic field of a dipole source in a four-layered region.

In the parallel paper [36], the complete formulas are derived for the electromagnetic field of a vertical electric dipole in the presence of a four-layered region. In what follows, we will attempt to treat the electromagnetic field of a horizontal electric dipole in the presence of a four-layered region.

2. ELECTROMAGNETIC FIELD EXCITED BY A HORIZONTAL ELECTRIC DIPOLE IN THE PRESENCE OF A FOUR-LAYERED REGION

2.1. Integrated Formulas for the Electromagnetic Field

The relevant geometry and Cartesian coordinate system are shown in Fig. 1, where a horizontal electric dipole in the $\hat{x}$ direction is located at

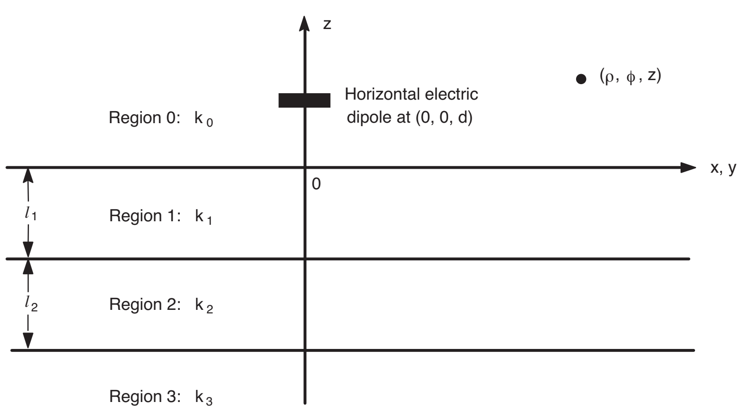

Figure 1. Geometry of a horizontal dipole in the presence of a four-layered region.

$(0, 0, d)$. Region 0 ($z \geq 0$) is the space above the two-layered dielectrics occupied by the air. Region 1 ($-l_1 \leq z \leq 0$) is the upper dielectric layer characterized by the permeability μ_0, permittivity ϵ_1, and conductivity σ_1. Region 2 ($-(l_1 + l_2) \leq z \leq -l_1$) is the lower dielectric layer characterized by the permeability μ_0, ϵ_2, and conductivity σ_2. Region 3 ($z \leq -(l_1 + l_2)$) is the rest space occupied by a perfect conductor. The wave numbers in the four-layered region are

$$k_0 = \omega\sqrt{\mu_0\epsilon_0} \tag{1}$$

$$k_j = \omega\sqrt{\mu_0\left(\epsilon_j + i\sigma_j/\omega\right)} \quad j = 1, 2 \tag{2}$$

$$k_3 \to \infty. \tag{3}$$

When the dipole point and the observation point are near the air-dielectric boundary, use is made of the time dependence $e^{-i\omega t}$, the integrated formulas of the electromagnetic field of a horizontal electric dipole in the presence of a four-layered region can be obtained readily by using (11.5.4)–(11.5.9) in the monograph by *King, Owens, and Wu* [8]. They can be written as follows:

$$\begin{aligned} E_{0\rho}(\rho, \phi, z) = {} & \frac{\omega\mu_0}{4\pi k_0^2}\cos\phi\left\{\int_0^\infty\left(k_0^2 J_0(\lambda\rho)\right.\right. \\ & \left. -\frac{\lambda^2}{2}[J_0(\lambda\rho) - J_2(\lambda\rho)]\right)\gamma_0^{-1}e^{i\gamma_0|z-d|}\lambda d\lambda \\ & +\int_0^\infty\left(\frac{\gamma_0 Q}{2}[J_0(\lambda\rho) - J_2(\lambda\rho)]\right. \\ & \left.\left. -\frac{k_0^2 P}{2\gamma_0}[J_0(\lambda\rho) + J_2(\lambda\rho)]\right)e^{i\gamma_0(z+d)}\lambda d\lambda\right\} \end{aligned} \tag{4}$$

$$\begin{aligned} E_{0\phi}(\rho, \phi, z) = {} & \frac{\omega\mu_0}{4\pi k_0^2}\sin\phi\left\{\int_0^\infty\left(k_0^2 J_0(\lambda\rho)\right.\right. \\ & \left. -\frac{\lambda^2}{2}[J_0(\lambda\rho) + J_2(\lambda\rho)]\right)\gamma_0^{-1}e^{i\gamma_0|z-d|}\lambda d\lambda \\ & +\int_0^\infty\left(\frac{\gamma_0 Q}{2}[J_0(\lambda\rho) + J_2(\lambda\rho)]\right. \\ & \left.\left. -\frac{k_0^2 P}{2\gamma_0}[J_0(\lambda\rho) - J_2(\lambda\rho)]\right)e^{i\gamma_0(z+d)}\lambda d\lambda\right\} \end{aligned} \tag{5}$$

$$\begin{aligned} E_{0z}(\rho, \phi, z) = {} & \frac{i\omega\mu_0}{4\pi k_0^2}\cos\phi \\ & \int_0^\infty(\pm e^{i\gamma_0|z-d|} + Qe^{i\gamma_0(z+d)})J_1(\lambda\rho)\lambda^2 d \quad \begin{matrix} d < z \\ 0 \leq z \leq d \end{matrix} \end{aligned} \tag{6}$$

$$
\begin{aligned}
B_{0\rho}(\rho,\phi,z) \;&=\; -\frac{\mu_0}{4\pi}\sin\phi\left\{\pm\int_0^\infty J_0(\lambda\rho)e^{i\gamma_0|z-d|}\lambda d\lambda\right.\\
&\quad+\int_0^\infty\left(\frac{Q}{2}[J_0(\lambda\rho)+J_2(\lambda\rho)]\right.\\
&\quad\left.\left.-\frac{P}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)]\right)e^{i\gamma_0(z+d)}\lambda d\lambda\right\}\quad \begin{matrix} d<z \\ 0\le z\le d\end{matrix} \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
B_{0\phi}(\rho,\phi,z) \;&=\; -\frac{\mu_0}{4\pi}\cos\phi\left\{\pm\int_0^\infty J_0(\lambda\rho)e^{i\gamma_0|z-d|}\lambda d\lambda\right.\\
&\quad+\int_0^\infty\left(\frac{Q}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)]\right.\\
&\quad\left.\left.-\frac{P}{2}[J_0(\lambda\rho)+J_2(\lambda\rho)]\right)e^{i\gamma_0(z+d)}\lambda d\lambda\right\}\quad \begin{matrix} d<z \\ 0\le z\le d\end{matrix} \qquad (8)
\end{aligned}
$$

$$
B_{0z}(\rho,\phi,z) = \frac{i\mu_0}{4\pi}\sin\phi\int_0^\infty[e^{i\gamma_0|z-d|}-Pe^{i\gamma_0(z+d)}]\gamma_0^{-1}J_1(\lambda\rho)\lambda^2 d\lambda \qquad (9)
$$

where

$$
Q = -\frac{k_1^2\gamma_0 + ik_0^2\gamma_1\dfrac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2}}{k_1^2\gamma_0 - ik_0^2\gamma_1\dfrac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2}} \qquad (10)
$$

$$
P = -\frac{\gamma_0 - i\gamma_1\dfrac{\gamma_2-\gamma_1\tan\gamma_1 l_1\tan\gamma_2 l_2}{\gamma_1\tan\gamma_2 l_2+\gamma_2\tan\gamma_1 l_1}}{\gamma_0 + i\gamma_1\dfrac{\gamma_2-\gamma_1\tan\gamma_1 l_1\tan\gamma_2 l_2}{\gamma_1\tan\gamma_2 l_2+\gamma_2\tan\gamma_1 l_1}} \qquad (11)
$$

$$
\gamma_j = \sqrt{k_j^2-\lambda^2} \qquad j=0,1,2 \qquad (12)
$$

$$
k_j = \omega\sqrt{\mu_o\epsilon_j} \qquad j=0,1,2. \qquad (13)
$$

It is convenient to rewrite (4)–(9) in the following forms.

$$
E_{0\rho}(\rho,\phi,z)=\frac{\omega\mu_0}{4\pi k_0^2}\cos\phi[F_{\rho 0}(\rho,z-d)-F_{\rho 0}(\rho,z+d)+F_{\rho 1}(\rho,z+d)] \qquad (14)
$$

$$
E_{0\phi}(\rho,\phi,z)=\frac{\omega\mu_0}{4\pi k_0^2}\sin\phi[F_{\phi 0}(\rho,z-d)-F_{\phi 0}(\rho,z-d)+F_{\phi 1}(\rho,z+d)] \qquad (15)
$$

$$
E_{0z}(\rho,\phi,z)=\frac{i\omega\mu_0}{4\pi k_0^2}\cos\phi[F_{z0}(\rho,z-d)-F_{z0}(\rho,z-d)+F_{z1}(\rho,z+d)] \qquad (16)
$$

$$
B_{0\rho}(\rho,\phi,z)=-\frac{\mu_0}{4\pi}\sin\phi[G_{\rho 0}(\rho,z-d)-G_{\rho 0}(\rho,z+d)+G_{\rho 1}(\rho,z+d)] \qquad (17)
$$

$$
B_{0\phi}(\rho,\phi,z)=-\frac{\mu_0}{4\pi}\cos\phi\,[G_{\phi 0}(\rho,z-d)-G_{\phi 0}(\rho,z+d)+G_{\phi 1}(\rho,z+d)] \qquad (18)
$$

$$B_{0z}(\rho,\phi,z)=\frac{i\mu_0}{4\pi}\sin\phi\left[G_{z0}(\rho,z-d)-G_{z0}(\rho,z+d)+G_{z1}(\rho,z+d)\right] \quad (19)$$

where

$$F_{\rho 0}(\rho,z-d) = \int_0^\infty \left\{\frac{\gamma_0}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho)+J_2(\lambda\rho)]\right\} e^{i\gamma_0|z-d|}\lambda d\lambda \quad (20)$$

$$F_{\rho 0}(\rho,z+d) = \int_0^\infty \left\{\frac{\gamma_0}{2}[J_0(\lambda\rho)-J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho)+J_2(\lambda\rho)]\right\} e^{i\gamma_0(z+d)}\lambda d\lambda \quad (21)$$

$$F_{\phi 0}(\rho,z-d) = \int_0^\infty \left(\frac{\gamma_0}{2}[J_0(\lambda\rho)+J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho)-J_2(\lambda\rho)]\right) e^{i\gamma_0|z-d|}\lambda d\lambda \quad (22)$$

$$F_{\phi 0}(\rho,z+d) = \int_0^\infty \left(\frac{\gamma_0}{2}[J_0(\lambda\rho)+J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho)-J_2(\lambda\rho)]\right) e^{i\gamma_0(z+d)}\lambda d\lambda \quad (23)$$

$$F_{z0}(\rho,z-d) = \pm\int_0^\infty J_1(\lambda\rho)e^{i\gamma_0|z-d|}\lambda^2 d\lambda \quad \begin{matrix} d<z \\ 0\le z\le d\end{matrix} \quad (24)$$

$$F_{z0}(\rho,z+d) = \int_0^\infty J_1(\lambda\rho)e^{i\gamma_0(z+d)}\lambda^2 d\lambda \quad (25)$$

$$G_{\rho 0}(\rho,z-d) = \pm\int_0^\infty J_0(\lambda\rho)e^{i\gamma_0|z-d|}\lambda d\lambda \quad \begin{matrix} d<z \\ 0\le z\le d\end{matrix} \quad (26)$$

$$G_{\rho 0}(\rho,z+d) = \int_0^\infty J_0(\lambda\rho)e^{i\gamma_0(z+d)}\lambda d\lambda \quad (27)$$

$$G_{\phi 0}(\rho,z-d) = \pm\int_0^\infty J_0(\lambda\rho)e^{i\gamma_0|z-d|}\lambda d\lambda = G_{\rho 0}(\rho,z-d) \quad \begin{matrix} d<z \\ 0\le z\le d\end{matrix} \quad (28)$$

$$G_{\phi 0}(\rho,z+d) = \int_0^\infty J_0(\lambda\rho)e^{i\gamma_0(z+d)}\lambda d\lambda = G_{\rho 0}(\rho,z+d) \quad (29)$$

$$G_{z0}(\rho,z-d) = \int_0^\infty J_1(\lambda\rho)\gamma_0^{-1}e^{i\gamma_0|z-d|}\lambda^2 d\lambda \quad (30)$$

$$G_{z0}(\rho,z+d) = \int_0^\infty J_1(\lambda\rho)\gamma_0^{-1}e^{i\gamma_0(z+d)}\lambda^2 d\lambda. \quad (31)$$

It is noted that the first and second terms in (14)–(19), which had been evaluated by King, Owens, and Wu [8], stand for the direct wave and ideal reflected wave, respectively. In the next step, it is necessary to evaluate the rest integrals. The third terms in (14)–(19) can be separated into electric-type (TM) and magnetic-type (TE) terms. They are

$$F_{\rho 1}(\rho, z+d) = F_{\rho 2}(\rho, z+d) + F_{\rho 3}(\rho, z+d) \tag{32}$$
$$F_{\phi 1}(\rho, z+d) = F_{\phi 2}(\rho, z+d) + F_{\phi 3}(\rho, z+d) \tag{33}$$
$$G_{\rho 1}(\rho, z+d) = G_{\rho 2}(\rho, z+d) + G_{\rho 3}(\rho, z+d) \tag{34}$$
$$G_{\phi 1}(\rho, z+d) = G_{\phi 2}(\rho, z+d) + G_{\phi 3}(\rho, z+d) \tag{35}$$

where

$$F_{\rho 2}(\rho, z+d) = \int_0^\infty \frac{\gamma_0(Q+1)}{2}[J_0(\lambda\rho) - J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{36}$$
$$F_{\rho 3}(\rho, z+d) = -\int_0^\infty \frac{k_0^2(P-1)}{2\gamma_0}[J_0(\lambda\rho) + J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{37}$$
$$F_{\phi 2}(\rho, z+d) = \int_0^\infty \frac{\gamma_0(Q+1)}{2}[J_0(\lambda\rho) + J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{38}$$
$$F_{\phi 3}(\rho, z+d) = -\int_0^\infty \frac{k_0^2(P-1)}{2\gamma_0}[J_0(\lambda\rho) - J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{39}$$
$$G_{\rho 2}(\rho, z+d) = \int_0^\infty \frac{Q+1}{2}[J_0(\lambda\rho) + J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{40}$$
$$G_{\rho 3}(\rho, z+d) = -\int_0^\infty \frac{P-1}{2}[J_0(\lambda\rho) - J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{41}$$
$$G_{\phi 2}(\rho, z+d) = \int_0^\infty \frac{Q+1}{2}[J_0(\lambda\rho) - J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda \tag{42}$$
$$G_{\phi 3}(\rho, z+d) = -\int_0^\infty \frac{P-1}{2}[J_0(\lambda\rho) + J_2(\lambda\rho)]e^{i\gamma_0(\rho, z+d)}\lambda d\lambda \tag{43}$$
$$F_{z1}(\rho, z+d) = \int_0^\infty (Q+1)J_1(\lambda\rho)e^{i\gamma_0(z+d)}\lambda^2 d\lambda \tag{44}$$
$$G_{z1}(\rho, z+d) = -\int_0^\infty (P-1)J_1(\lambda\rho)\gamma_0^{-1}e^{i\gamma_0(z+d)}\lambda^2 d\lambda. \tag{45}$$

In this paper, the integrals $F_{z1}, F_{\rho 2}, F_{\phi 2}, G_{\rho 2}$, and $G_{\phi 2}$ involving the factor $(Q+1)$ are defined as the electric-type (TM) terms, and the integrals $G_{z1}, F_{\rho 3}, F_{\phi 3}, G_{\rho 3}$, and $G_{\phi 3}$ involving the factor $(P-1)$ are defined as the magnetic-type (TE) terms.

2.2. Evaluations for the Electric-type (TM) Terms

Considering γ_0, γ_1, and γ_2 are even functions of λ, and using the relations between Bessel function and Hankel function

$$J_n(\lambda\rho) = \frac{1}{2}\left[H_n^{(1)}(\lambda\rho) + H_n^{(2)}(\lambda\rho)\right] \tag{46}$$

$$H_n^{(1)}(-\lambda\rho) = H_n^{(2)}(\lambda\rho)(-1)^{n+1} \tag{47}$$

$$J_0(\lambda\rho) + J_2(\lambda\rho) = \frac{2}{\lambda\rho}J_1(\lambda\rho) \tag{48}$$

Substitution (46)–(48) into (36) yields to

$$\begin{aligned} F_{\rho 2}(\rho, z+d) \ = \ & -\frac{i}{2}\int_{-\infty}^{\infty} \frac{k_0^2\gamma_0\gamma_1(\gamma_1 k_2^2 \tan\gamma_1 l_1 + \gamma_2 k_1^2 \tan\gamma_2 l_2)}{q(\lambda)} \\ & \times \left[H_0^{(1)}(\lambda\rho) - H_2^{(1)}(\lambda\rho)\right] e^{i\gamma_0(z+d)}\lambda d\lambda \end{aligned} \tag{49}$$

where

$$\begin{aligned} q(\lambda) \ = \ & k_1^2\gamma_0(\gamma_1 k_2^2 - \gamma_2 k_1^2 \tan\gamma_1 l_1 \tan\gamma_2 l_2) \\ & -ik_0^2\gamma_1(\gamma_1 k_2^2 \tan\gamma_1 l_1 + \gamma_2 k_1^2 \tan\gamma_2 l_2). \end{aligned} \tag{50}$$

For convenience, it is necessary to decompose $F_{\rho 2}$ into two terms $F_{\rho 2}^{(1)}$ and $F_{\rho 2}^{(2)}$. They are expressed as follows:

$$\begin{aligned} F_{\rho 2}^{(1)} \ = \ & -\frac{i}{2}k_0^2 k_1^2 \\ & \int_{-\infty}^{\infty} \frac{\gamma_0\gamma_1\gamma_2 \tan\gamma_2 l_2 \left[H_0^{(1)}(\lambda\rho) - H_2^{(1)}(\lambda\rho)\right]}{q(\lambda)} e^{i\gamma_0(z+d)}\lambda d\lambda \end{aligned} \tag{51}$$

$$\begin{aligned} F_{\rho 2}^{(2)} \ = \ & -\frac{i}{2}k_0^2 k_2^2 \\ & \int_{-\infty}^{\infty} \frac{\gamma_0\gamma_1^2 \tan\gamma_1 l_1 \left[H_0^{(1)}(\lambda\rho) - H_2^{(1)}(\lambda\rho)\right]}{q(\lambda)} e^{i\gamma_0(z+d)}\lambda d\lambda. \end{aligned} \tag{52}$$

In order to evaluate the above two integrals, we shift the contour around the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The next main tasks are to determine the poles and to evaluate the integrations along the branch cuts Γ_0, Γ_1, and Γ_2. The poles of the integrands satisfy the following equation.

$$\begin{aligned} q(\lambda) \ = \ & k_1^2\gamma_0(\gamma_1 k_2^2 - \gamma_2 k_1^2 \tan\gamma_1 l_1 \tan\gamma_2 l_2) \\ & -ik_0^2\gamma_1(\gamma_1 k_2^2 \tan\gamma_1 l_1 + \gamma_2 k_1^2 \tan\gamma_2 l_2) = 0. \end{aligned} \tag{53}$$

Comparing with the case of a vertical dipole as addressed by Xu et al. [36], it is seen that the pole Equation (53) is same with that of the vertical-dipole case. Thus, $F_{\rho 2}^{(1)}$ can be written in the form

$$F_{\rho 2}^{(1)} = 2\pi i\left(-\frac{i}{2}k_0^2k_1^2\right)$$
$$\sum_j \frac{\gamma_{0E}^*\gamma_{1E}^*\gamma_{2E}^*\tan\gamma_{2E}^* l_2\Big[H_0^{(1)}(\lambda_{jE}^*\rho)-H_2^{(1)}(\lambda_{jE}^*\rho)\Big]}{q'(\lambda_{jE}^*)}e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^*$$
$$-\frac{i}{2}k_0^2k_1^2\int_{\Gamma_0+\Gamma_1+\Gamma_2}\frac{\gamma_0\gamma_1\gamma_2\tan\gamma_2 l_2\Big[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)\Big]}{q(\lambda)}e^{i\gamma_0(z+d)}\lambda d\lambda \quad (54)$$

where λ_{jE}^* are poles of electric-type (TM) wave,

$$\begin{aligned} q'(\lambda) = & -k_1^2\frac{\lambda}{\gamma_0}\left(\gamma_1k_2^2-\gamma_2k_1^2\tan\gamma_1l_1\tan\gamma_2l_2\right)\\ & +ik_0^2\frac{\lambda}{\gamma_1}\left(\gamma_1k_2^2\tan\gamma_1l_1+\gamma_2k_1^2\tan\gamma_2l_2\right)\\ & +k_1^2\gamma_0\left[-\frac{\lambda}{\gamma_1}k_2^2+\frac{\lambda}{\gamma_2}k_1^2\tan\gamma_1l_1\tan\gamma_2l_2\right.\\ & \left.+k_1^2\gamma_2\lambda\left(\frac{l_1}{\gamma_1}\sec^2\gamma_1l_1\tan\gamma_2l_2+\frac{l_2}{\gamma_2}\tan\gamma_1l_1\sec^2\gamma_2l_2\right)\right]\\ & +ik_0^2\gamma_1\lambda\cdot\left(\frac{k_2^2\tan\gamma_1l_1}{\gamma_1}+\frac{k_1^2\tan\gamma_2l_2}{\gamma_2}\right.\\ & \left.+k_2^2l_1\sec^2\gamma_1l_1+k_1^2l_2\sec^2\gamma_2l_2\right) \end{aligned} \quad (55)$$

$$\gamma_{nE}^* = \gamma_n^*(\lambda_{jE}^*) \quad n = 0,1,2 \quad (56)$$

Next, we will evaluate the integrals along the branch cuts Γ_0, Γ_1, and Γ_2. Similar to the three-layered case addressed by Tang and Hong [31], it is easily verified that the integrations along the branch cuts Γ_1 and Γ_2 are zero for the integrals in (54). Subjecting to the far-field condition of $k_0\rho \gg 1$ and $z+d \ll \rho$, it is seen that the dominant contribution of the integration along Γ_0 comes from the vicinity of k_0. Let $\lambda = k_0(1_i\tau^2)$, at the vicinity of k_0, the values $H_n^{(1)}(\lambda\rho)$, γ_0, γ_1, and γ_2 are approximated as

$$H_n^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_0\rho}}e^{i(k_0\rho-\frac{1}{4}\pi-n\frac{\pi}{2})}\cdot e^{-k_0\rho\tau^2} \quad n = 0,1,2 \quad (57)$$

$$\gamma_0 = \sqrt{k_0^2-\lambda^2} \approx k_0 e^{i\frac{3}{4}\pi}\sqrt{2}\tau, \tag{58}$$

$$\gamma_1 = \sqrt{k_1^2-\lambda^2} \approx \sqrt{k_1^2-k_0^2} = \gamma_{10}, \tag{59}$$

$$\gamma_2 = \sqrt{k_2^2-\lambda^2} \approx \sqrt{k_2^2-k_0^2} = \gamma_{20}, \tag{60}$$

Then, with the change of variable $t=\tau+\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}$, the integral along branch cut Γ_0 in (54) can be written in following form.

$$I_1 = \frac{2\sqrt{2}k_0^4 e^{-i\frac{\pi}{4}}\sqrt{\frac{1}{\pi k_0\rho}}\gamma_{10}\gamma_{20}\tan\gamma_{20}l_2 \exp\left[ik_0\rho+i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}\right)^2\right]}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2} \times\int_{-\infty}^{\infty}\frac{e^{-k_0\rho t^2}\left(t-\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2}{t-e^{i\frac{\pi}{4}}\Delta_1'}dt \tag{61}$$

where

$$\Delta_1' = \frac{z+d}{\sqrt{2}\rho} - i\frac{k_0\gamma_{10}\left(\gamma_{10}k_2^2\tan\gamma_{10}l_1+\gamma_{20}k_1^2\tan\gamma_{20l_2}\right)}{\sqrt{2}k_1^2\left(\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2\right)}. \tag{62}$$

The integral in (62) can be evaluated directly. It is

$$\int_{-\infty}^{\infty}\frac{e^{-k_0\rho t^2}\left(t-\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2}{t-e^{i\frac{\pi}{4}}\Delta_1'}dt = e^{i\frac{\pi}{4}}\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_1'-\sqrt{2}\frac{z+d}{\rho}\right)-\pi e^{-ip_1^*}\left(\Delta_1'-\frac{1}{\sqrt{2}}\frac{z+d}{\rho}\right)^2\mathrm{erfc}(\sqrt{-ip_1^*}) \tag{63}$$

where $p_1^* = k_0\rho\Delta_1'^2$, and the phase of $\sqrt{ik_0\rho\Delta_1'^2}$ in (64) requires to be

$$\left|\mathrm{Arg}\sqrt{-i\triangle_1'^2 k_0\rho}\right| \le \frac{\pi}{4}. \tag{64}$$

The error function and Fresnel integral are defined by

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}dt, \tag{65}$$

$$F(p^*) = \frac{1}{2}(1+i)-\int_0^{p^*}\frac{e^{it}}{\sqrt{2\pi t}}dt. \tag{66}$$

Taking into account the relation between the error function and Fresnel integral

$$\operatorname{erfc}(\sqrt{-ip^*}) = \sqrt{2}e^{-i\frac{\pi}{4}}F(p^*). \tag{67}$$

Then, we obtain

$$\begin{aligned}F_{\rho 2}^{(1)} &= \pi k_0^2 k_1^2 \\ &\sum_j \frac{\gamma_{0E}^*\gamma_{1E}^*\gamma_{2E}^* \tan\gamma_{2E}^* l_2 \left[H_0^{(1)}(\lambda_{jE}^*\rho) - H_2^{(1)}(\lambda_{jE}^*\rho)\right]}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^* \\ &+ \frac{2\sqrt{2}k_0^4\gamma_{10}\gamma_{20}\tan\gamma_{20}l_2}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2} \\ &\left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_1' - \sqrt{2}\frac{z+d}{\rho}\right) + i\sqrt{2}\pi\left(\Delta_1' - \frac{1}{\sqrt{2}}\frac{z+d}{\rho}\right)^2 e^{-ip_1^*}F(p_1^*)\right]\end{aligned} \tag{68}$$

where $r_2 = \sqrt{\rho^2 + (z+d)^2} \approx \rho\left[1 + \frac{1}{2}\left(\frac{z+d}{\rho}\right)\right]$. Similarly, the integral (52) can be evaluated readily.

$$\begin{aligned}F_{\rho 2}^{(2)} &= \pi k_0^2 k_2^2 \\ &\sum_j \frac{\gamma_{0E}^*\gamma_1^2(\lambda_{jE}^*)\tan\gamma_{1E}^* l_1 \left[H_0^{(1)}(\lambda_{jE}^*\rho) - H_2^{(1)}(\lambda_{jE}^*\rho)\right]}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^* \\ &+ \frac{2\sqrt{2}k_0^4\frac{k_2^2}{k_1^2}\gamma_{10}^2\tan\gamma_{10}l_1}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2} \\ &\left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_1' - \sqrt{2}\frac{z+d}{\rho}\right) + i\sqrt{2}\pi\left(\Delta_1' - \frac{1}{\sqrt{2}}\frac{z+d}{\rho}\right)^2 e^{-ip_1^*}F(p_1^*)\right]\end{aligned} \tag{69}$$

Then, the final expression of $F_{\rho 2}$ can be written in the following form.

$$\begin{aligned}F_{\rho 2}(\rho, z+d) = \pi k_0^2 \sum_j &\frac{\gamma_{0E}^*\gamma_{1E}^*\left[k_2^2\gamma_{1E}^*\tan\gamma_{1E}^* l_1 + k_1^2\gamma_{2E}^*\tan\gamma_{2E}^* l_2\right]}{q'(\lambda_{jE}^*)} \\ &\cdot \left[H_0^{(1)}(\lambda_{jE}^*\rho) - H_2^{(1)}(\lambda_{jE}^*\rho)\right] \cdot e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^* \\ &+ \frac{2\sqrt{2}k_0^4\gamma_{10}\left(\frac{k_2^2}{k_1^2}\gamma_{10}\tan\gamma_{10}l_1 + \gamma_{20}\tan\gamma_{20}l_2\right)}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}\end{aligned}$$

$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_1'-\sqrt{2}\frac{z+d}{\rho}\right)\right.$$
$$\left.+i\sqrt{2}\pi\left(\Delta_1'-\frac{1}{\sqrt{2}}\frac{z+d}{\rho}\right)^2 e^{-ip_1^*}F(p_1^*)\right]. \tag{70}$$

Evidently, with the the similar procedures, the rest terms of the electric-type (TM) field can also be evaluated. We write

$$F_{\phi 2}(\rho,z+d)=\pi k_0^2\sum_j\frac{\gamma_{0E}^*\gamma_{1E}^*\left[k_2^2\gamma_{1E}^*\tan\gamma_{1E}^*l_1+k_1^2\gamma_{2E}^*\tan\gamma_{2E}^*l_2\right]}{q'(\lambda_{jE}^*)}$$
$$\cdot\left[H_0^{(1)}(\lambda_{jE}^*\rho)+H_2^{(1)}(\lambda_{jE}^*\rho)\right]\cdot e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^*$$
$$-\frac{i2\sqrt{2}k_0^3\gamma_{10}\left(\frac{k_2^2}{k_1^2}\gamma_{10}\tan\gamma_{10}l_1+\gamma_{20}\tan\gamma_{20}l_2\right)}{\rho\left(\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2\right)}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}$$
$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_1'-\sqrt{2}\frac{z+d}{\rho}\right)\right.$$
$$\left.+i\sqrt{2}\pi\left(\Delta_1'-\frac{1}{\sqrt{2}}\frac{z+d}{\rho}\right)^2 e^{-ip_1^*}F(p_1^*)\right] \tag{71}$$

$$G_{\phi 2}(\rho,z+d) \;=\; -\pi k_0^2\sum_j\frac{\gamma_{1E}^*\left[k_2^2\gamma_{1E}^*\tan\gamma_{1E}^*l_1+k_1^2\gamma_{2E}^*\tan\gamma_{2E}^*l_2\right]}{q'(\lambda_{jE}^*)}$$
$$\cdot\left[H_0^{(1)}(\lambda_{jE}^*\rho)-H_2^{(1)}(\lambda_{jE}^*\rho)\right]\cdot e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^*$$
$$-\frac{2k_0^3\gamma_{10}\left(\frac{k_2^2}{k_1^2}\gamma_{10}\tan\gamma_{10}l_1+\gamma_{20}\tan\gamma_{20}l_2\right)}{\gamma_{10}k_2^2-\gamma_{20}k_2^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}\sqrt{\frac{1}{\pi k_0\rho}}e^{-k_0r_2}$$
$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+i\sqrt{2}\pi\left(\Delta_1'-\frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_1^*}F(p_1^*)\right] \tag{72}$$

$$G_{\rho 2}(\rho,z+d) \;=\; -\pi k_0^2\sum_j\frac{\gamma_{1E}^*\left[k_2^2\gamma_{1E}^*\tan\gamma_{1E}^*l_1+k_1^2\gamma_{2E}^*\tan\gamma_{2E}^*l_2\right]}{q'(\lambda_{jE}^*)}$$
$$\cdot\left[H_0^{(1)}(\lambda_{jE}^*\rho)+H_2^{(1)}(\lambda_{jE}^*\rho)\right]\cdot e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^*$$
$$+\frac{i2k_0^2\gamma_{10}\left(\frac{k_2^2}{k_1^2}\gamma_{10}\tan\gamma_{10}l_1+\gamma_{20}\tan\gamma_{20}l_2\right)}{\rho\left(\gamma_{10}k_2^2-\gamma_{20}k_2^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2\right)}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}$$

$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+i\sqrt{2}\pi\left(\Delta_1'-\frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_1^*}F(p_1^*)\right] \tag{73}$$

$$\begin{aligned}
&F_{z1}(\rho,z+d)= \\
&\quad 2\pi k_0^2\sum_j\frac{\gamma_{1E}^*\left[k_2^2\gamma_{1E}^*\tan\gamma_{1E}^*l_1+k_1^2\gamma_{2E}^*\tan\gamma_{2E}^*l_2\right]H_1^{(1)}(\lambda_{jE}^*\rho)}{q'(\lambda_{jE}^*)} \\
&\quad\cdot e^{i\gamma_{0E}^*(z+d)}\lambda_{jE}^{*2}+\frac{i2k_0^4\gamma_{10}\left(\frac{k_2^2}{k_1^2}\gamma_{10}\tan\gamma_{10}l_1+\gamma_{20}\tan\gamma_{20}l_2\right)}{\gamma_{10}k_2^2-\gamma_{20}k_2^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2} \\
&\quad\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+i\sqrt{2}\pi\left(\Delta_1'-\frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_1^*}F(p_1^*)\right].
\end{aligned} \tag{74}$$

2.3. Evaluations for the Terms of Magnetic-type (TE) Field

Substitutions (46)–(48) into (39) yields to

$$\begin{aligned}
&F_{\phi3}(\rho,z+d)= \\
&\frac{k_0^2}{2}\int_{-\infty}^{\infty}\frac{(\gamma_1\tan\gamma_2l_2+\gamma_2\tan\gamma_1l_1)\left[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)\right]}{p(\lambda)}e^{i\gamma_0(z+d)}\lambda d\lambda
\end{aligned} \tag{75}$$

where

$$p(\lambda)=\gamma_0\gamma_1\tan\gamma_2l_2+\gamma_0\gamma_2\tan\gamma_1l_1+i\gamma_1\gamma_2-i\gamma_1^2\tan\gamma_1l_1\tan\gamma_2l_2. \tag{76}$$

It is convenient to decompose $F_{\phi3}$ into $F_{\phi3}^{(1)}$ and $F_{\phi3}^{(2)}$. They are expressed as follows:

$$F_{\phi3}^{(1)}=\frac{k_0^2}{2}\int_{-\infty}^{\infty}\frac{\gamma_1\tan\gamma_2l_2\left[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)\right]}{p(\lambda)}e^{i\gamma_0(z+d)}\lambda d\lambda \tag{77}$$

$$F_{\phi3}^{(2)}=\frac{k_0^2}{2}\int_{-\infty}^{\infty}\frac{\gamma_2\tan\gamma_1l_1\left[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)\right]}{p(\lambda)}e^{i\gamma_0(z+d)}\lambda d\lambda. \tag{78}$$

In order to evaluate $F_{\phi3}^{(1)}$, it is necessary to examine the pole equation of the magnetic-type terms.

$$p(\lambda)=\gamma_0\gamma_1\tan\gamma_2l_2+\gamma_0\gamma_2\tan\gamma_1l_1+i\gamma_1\gamma_2-i\gamma_1^2\tan\gamma_1l_1\tan\gamma_2l_2=0. \tag{79}$$

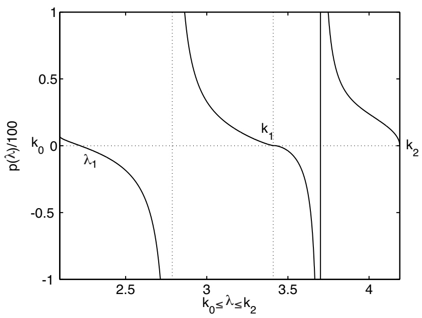

Figure 2. Roots of $p(\lambda)$ for $f = 100\,\mathrm{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4.0$, $l_1 = l_2 = 0.8\,\mathrm{m}$.

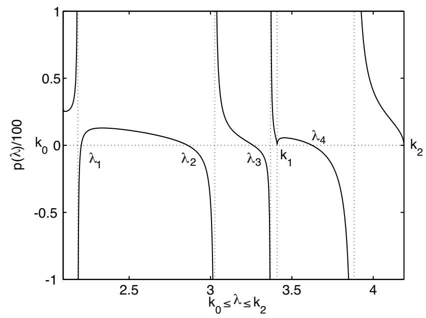

Figure 3. Roots of $p(\lambda)$ for $f = 100\,\mathrm{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4.0$, $l_1 = 3.0\,\mathrm{m}$, and $l_2 = 1.0\,\mathrm{m}$.

Clearly, the poles may exist in the interval $k_0 \leq \lambda \leq k_2$, and k_1 is a removable pole. As illustrated in Figs. 2 and 3, the poles can be determined by using Newton method. Then, we have

$$F_{\phi 3}^{(1)} = 2\pi i \cdot \frac{k_0^2}{2} \sum_j \frac{\gamma_{1B}^* \tan \gamma_{2B}^* l_2 \left[H_0^{(1)}(\lambda_{jB}^* \rho) - H_2^{(1)}(\lambda_{jB}^* \rho)\right]}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^*(z+d)} \lambda_{jB}^* + \frac{k_0^2}{2} \int_{\Gamma_0+\Gamma_1+\Gamma_2} \frac{\gamma_1 \tan \gamma_2 l_2 \left[H_0^{(1)}(\lambda\rho) - H_2^{(1)}(\lambda\rho)\right]}{p(\lambda)} e^{i\gamma_0(z+d)} \lambda d\lambda \quad (80)$$

where λ_{jB}^* are the poles of magnetic-type (TE) field.

$$\begin{aligned} p'(\lambda) = & -\lambda \left(\frac{\gamma_1}{\gamma_0} \tan \gamma_2 l_2 + \frac{\gamma_0}{\gamma_1} \tan \gamma_2 l_2 + \frac{\gamma_0 \gamma_1 l_2}{\gamma_2} \sec^2 \gamma_2 l_2 + \frac{\gamma_2}{\gamma_0} \tan \gamma_1 l_1 \right. \\ & + \frac{\gamma_0}{\gamma_2} \tan \gamma_1 l_1 + \frac{\gamma_0 \gamma_2 l_1}{\gamma_1} \sec^2 \gamma_1 l_1 + i\frac{\gamma_2}{\gamma_1} + i\frac{\gamma_1}{\gamma_2} - i2 \tan \gamma_1 l_1 \tan \gamma_2 l_2 \\ & \left. - i\gamma_1 l_1 \sec^2 \gamma_1 l_1 \tan \gamma_2 l_2 - i\frac{\gamma_1^2 l_2}{\gamma_2} \tan \gamma_1 l_1 \sec^2 \gamma_2 l_2 \right) \end{aligned} \quad (81)$$

$$\gamma_{nB}^* = \gamma_n^*(\lambda_{jB}^*) \quad n = 0, 1, 2. \quad (82)$$

Similar to the case of the electric-type (TM) field, it is seen that the integrations along the branch cuts Γ_1 and Γ_2 are zero in (81). As $k_0\rho \gg 1$ and $z + d \ll \rho$, the dominant contribution of the integration along the branch cut Γ_0 comes from the vicinity of k_0. Let $\lambda = k_0(1_i\tau^2)$, and using the notation $t = \tau + \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}$, the integral along Γ_0 in (81) can be evaluated readily.

$$\begin{aligned} I_2 &= -\frac{i2k_0^3\sqrt{\frac{1}{\pi k_0\rho}}\gamma_{10}\tan\gamma_{20}l_2\exp\left[ik_0\rho + i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}\right)^2\right]}{\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1} \\ &\quad \int_{-\infty}^{\infty}\frac{e^{-k_0\rho t^2}\left(t - \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)}{t - e^{i\frac{\pi}{4}}\Delta_2'}dt \\ &= -\frac{i2k_0^3\sqrt{\frac{1}{\pi k_0\rho}}\gamma_{10}\tan\gamma_{20}l_2\exp\left[ik_0\rho + i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}\right)^2\right]}{\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1} \\ &\quad \cdot\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(-ip_2^*)\right] \end{aligned} \tag{83}$$

where

$$\Delta_2' = \frac{z+d}{\sqrt{2}\rho} + i\frac{\gamma_{10}\gamma_{20} - \gamma_{10}^2\tan\gamma_{10}l_1\tan\gamma_{20}l_2}{\sqrt{2}k_0(\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1)}. \tag{84}$$

Then, we get

$$\begin{aligned} F_{\phi 3}^{(1)} &= i\pi k_0^2\sum_j\frac{\gamma_{1B}^*\tan\gamma_{2B}^*l_2\left[H_0^{(1)}(\lambda_{jB}^*\rho) - H_2^{(1)}(\lambda_{jB}^*\rho)\right]}{p'(\lambda_{jB}^*)}e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^* \\ &\quad -\frac{i2k_0^3\gamma_{10}\tan\gamma_{20}l_2}{\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2} \\ &\quad \cdot\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(-ip_2^*)\right]. \end{aligned} \tag{85}$$

Similarly, we have

$$\begin{aligned} F_{\phi 3}^{(2)} &= i\pi k_0^2\sum_j\frac{\gamma_{2B}^*\tan\gamma_{1B}^*l_1\left[H_0^{(1)}(\lambda_{jB}^*\rho) - H_2^{(1)}(\lambda_{jB}^*\rho)\right]}{p'(\lambda_{jB}^*)}e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^* \\ &\quad -\frac{i2k_0^3\gamma_{20}\tan\gamma_{10}l_1}{\gamma_{10}\tan\gamma_{20}l_2 + \gamma_{20}\tan\gamma_{10}l_1}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2} \\ &\quad \cdot\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(-ip_2^*)\right]. \end{aligned} \tag{86}$$

The final expression of $F_{\phi 3}$ can be written as follows:

$$F_{\phi 3}(\rho, z+d) = i\pi k_0^2 \sum_j \frac{\gamma_{1B}^* \tan\gamma_{2B}^* l_2 + \gamma_{2B}^* \tan\gamma_{1B}^* l_1}{p'(\lambda_{jB}^*)}$$
$$\cdot \left[H_0^{(1)}(\lambda_{jB}^*\rho) - H_2^{(1)}(\lambda_{jB}^*\rho)\right] \cdot e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^* - i2k_0^3\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}$$
$$\cdot \left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(-ip_2^*)\right]. \tag{87}$$

Following the similar procedures, it is obtained readily.

$$F_{\rho 3}(\rho, z+d) = i\pi k_0^2 \sum_j \frac{\gamma_{1B}^* \tan\gamma_{2B}^* l_2 + \gamma_{2B}^* \tan\gamma_{1B}^* l_1}{p'(\lambda_{jB}^*)}$$
$$\cdot \left[H_0^{(1)}(\lambda_{jB}^*\rho) + H_2^{(1)}(\lambda_{jB}^*\rho)\right] e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^*$$
$$-\frac{2k_0^2}{\rho}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(-ip_2^*)\right] \tag{88}$$

$$G_{\rho 3}(\rho, z+d) = i\pi \sum_j \frac{\gamma_{0B}^*\left[\gamma_{1B}^* \tan\gamma_{2B}^* l_2 + \gamma_{2B}^* \tan\gamma_1(\lambda_{jB}^* l_1)\right]}{p'(\lambda_{jB}^*)}$$
$$\cdot \left[H_0^{(1)}(\lambda_{jB}^*\rho) - H_2^{(1)}(\lambda_{jB}^*\rho)\right] \cdot e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^* + i2\sqrt{2}k_0^2\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}$$
$$\cdot \left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_2' - \sqrt{2}\frac{z+d}{\rho}\right) + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)^2 e^{-ip_2^*}F(p_2^*)\right] \tag{89}$$

$$G_{\phi 3}(\rho, z+d) = i\pi \sum_j \frac{\gamma_{0B}^*\left[\gamma_{1B}^* \tan\gamma_{2B}^* l_2 + \gamma_{2B}^* \tan\gamma_1(\lambda_{jB}^* l_1)\right]}{p'(\lambda_{jB}^*)}$$
$$\cdot \left[H_0^{(1)}(\lambda_{jB}^*\rho) + H_2^{(1)}(\lambda_{jB}^*\rho)\right] \cdot e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^* + \frac{2\sqrt{2}k_0}{\rho}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}$$
$$\cdot \left[\sqrt{\frac{\pi}{k_0\rho}}\left(\Delta_2' - \sqrt{2}\frac{z+d}{\rho}\right) + i\sqrt{2}\pi\left(\Delta_2' - \frac{z+d}{\sqrt{2}\rho}\right)^2 e^{-ip_2^*}F(p_2^*)\right] \tag{90}$$

$$G_{z1}(\rho, z+d) =$$
$$2\pi i \sum_j \frac{\gamma_{1B}^* \tan\gamma_{2B}^* l_2 + \gamma_{2B}^* \tan\gamma_1(\lambda_{jB}^*) l_1 H_1^{(1)}(\lambda_{jB}^*\rho)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^*(z+d)}\lambda_{jB}^{*2}$$

$$-2k_0^2\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}\left[\sqrt{\frac{\pi}{k_0\rho}}+i\sqrt{2}\pi\left(\Delta_2'-\frac{z+d}{\sqrt{2}\rho}\right)e^{-ip_2^*}F(p_2^*)\right]. \quad (91)$$

2.4. Final Formulas of the Six Field Components

With the above results for the trapped surface wave and lateral wave, and those for the direct wave and ideal reflected wave addressed by *King, Owens, and Wu* [8], the final expressions for the six components can be obtained readily.

$$E_{0\rho}(\rho,\phi,z)=$$
$$\frac{\omega\mu_0}{4\pi k_0^2}\cos\phi\left\{-\left[\frac{2k_0}{r_1^2}+\frac{2i}{r_1^3}+\left(\frac{z-d}{r_1}\right)^2\left(\frac{ik_0^2}{r_1}-\frac{3k_0}{r_1^2}-\frac{3i}{r_1^3}\right)\right]e^{ik_0r_1}\right.$$
$$\left.+\left[\frac{2k_0}{r_2^2}+\frac{2i}{r_2^3}+\left(\frac{z+d}{r_2}\right)^2\left(\frac{ik_0^2}{r_2}-\frac{3k_0}{r_2^2}-\frac{3i}{r_2^3}\right)\right]e^{ik_0r_2}+F_{\rho2}+F_{\rho3}\right\} \quad (92)$$

$$E_{0\phi}(\rho,\phi,z) = \frac{\omega\mu_0}{4\pi k_0^2}\sin\phi\left[-\left(\frac{ik_0^2}{r_1}-\frac{k_0}{r_1^2}-\frac{i}{r_1^3}\right)e^{ik_0r_1}\right.$$
$$\left.+\left(\frac{ik_0^2}{r_2}-\frac{k_0}{r_2^2}-\frac{i}{r_2^3}\right)e^{ik_0r_2}+F_{\phi2}+F_{\phi3}\right] \quad (93)$$

$$E_{0z}(\rho,\phi,z) = \frac{i\omega\mu_0}{4\pi k_0^2}\cos\phi\left[-\left(\frac{\rho}{r_1}\right)\left(\frac{z-d}{r_1}\right)\left(\frac{k_0^2}{r_1}+\frac{3ik_0}{r_1^2}-\frac{3}{r_1^3}\right)e^{ik_0r_1}\right.$$
$$\left.+\left(\frac{\rho}{r_2}\right)\left(\frac{z+d}{r_2}\right)\left(\frac{k_0^2}{r_2}+\frac{3ik_0}{r_2^2}-\frac{3}{r_2^3}\right)e^{ik_0r_2}+F_{z1}\right] \quad (94)$$

$$B_{0\rho}(\rho,\phi,z) = -\frac{\mu_0}{4\pi}\sin\phi\left[-\left(\frac{z-d}{r_1}\right)\left(\frac{ik_0}{r_1}-\frac{1}{r_1^2}\right)e^{ik_0r_1}\right.$$
$$\left.+\left(\frac{z+d}{r_2}\right)\left(\frac{ik_0}{r_2}-\frac{1}{r_2^2}\right)e^{ik_0r_2}+G_{\rho2}+G_{\rho3}\right] \quad (95)$$

$$B_{0\phi}(\rho,\phi,z) = -\frac{\mu_0}{4\pi}\cos\phi\left[-\left(\frac{z-d}{r_1}\right)\left(\frac{ik_0}{r_1}-\frac{1}{r_1^2}\right)e^{ik_0r_1}\right.$$
$$\left.+\left(\frac{z+d}{r_2}\right)\left(\frac{ik_0}{r_2}-\frac{1}{r_2^2}\right)e^{ik_0r_2}+G_{\phi2}+G_{\phi3}\right] \quad (96)$$

$$B_{0z}(\rho,\phi,z) = \frac{i\mu_0}{4\pi}\sin\phi\left[-\left(\frac{\rho}{r_1}\right)\left(\frac{k_0}{r_1}+\frac{i}{r_1^2}\right)e^{ik_0r_1}\right.$$
$$\left.+\left(\frac{\rho}{r_2}\right)\left(\frac{ik_0}{r_2}-\frac{1}{r_2^2}\right)e^{ik_0r_2}+G_{z1}\right]. \quad (97)$$

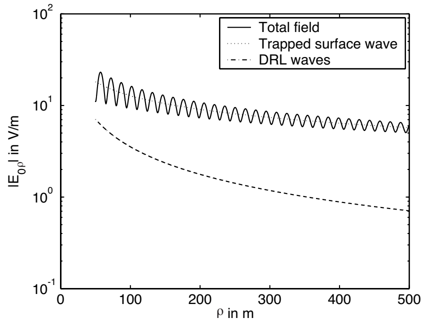

Figure 4. The electric field $|E_\rho|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1l_1 = k_2l_2 = 0.7$, and $z = d = 0\,\text{m}$.

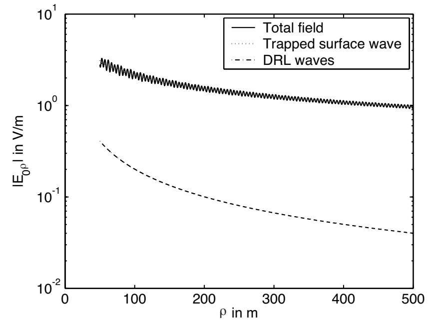

Figure 5. The electric field $|E_\rho|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1l_1 = k_2l_2 = 1.5$, and $z = d = 0\,\text{m}$.

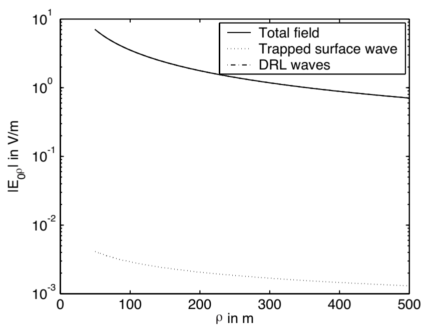

Figure 6. The electric field $|E_\rho|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1l_1 = k_2l_2 = 0.7$, and $z = d = 3\,\text{m}$.

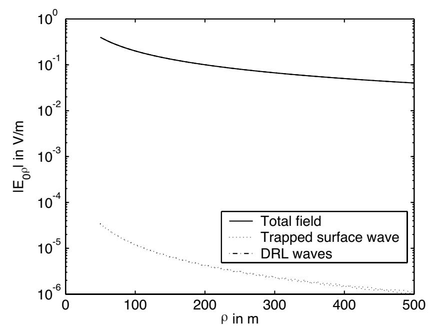

Figure 7. The electric field $|E_\rho|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1l_1 = k_2l_2 = 1.5$, and $z = d = 3\,\text{m}$.

If Region 1 is made the air by setting $k_1 = k_0$ or both Regions 1 and 2 are made the same dielectric by setting $k_1 = k_2$, the above results reduce to the corresponding results for the three-layered case as addressed by Tang and Hong [31].

3. CONCLUSIONS AND COMPUTATIONS

From the above derivations and analysis, the complete formulas have been obtained for the electromagnetic field of a horizontal electric dipole in the presence of a four-layered region. It is seen that both the trapped surface wave and the lateral wave can be separated into the electric-type (TM) and magnetic-type (TE) terms. The trapped surface wave with its wave number being between k_0 and k_2, is determined by the sum of the residues of the poles. The lateral wave with its wave number k_0 is determined by the integration of the branch cut.

For the radical electric field component $|E_{0\rho}(\rho, 0, z)|$ at $z = d = 0$ m, with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65, \epsilon_{12r} = 4.0$, the total field, the trapped surface wave, and the DRL waves are computed for $k_1 l_1 = k_2 l_2 = 0.7$ and $k_1 l_1 = k_2 l_2 = 1.5$, respectively. It is noted that the DRL waves include the direct wave, the ideal reflected wave, and the lateral wave. Similar to those shown in Figs. 4 and 5, the corresponding results at $z = d = 3\,\text{m}$ are shown in Figs. 6 and 7, respectively. When both the dipole point and the observation point are on the air-dielectric boundary, the total field is determined by the trapped surface wave. Once the dipole point or the observation point is away to the boundary, the trapped surface wave attenuates rapidly and the total field is determined primarily by the lateral wave. In practical applications, we can change the thicknesses of the above and lower dielectric layers to fulfil the required results.

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