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ELECTROMAGNETIC RESONANCES AND FIELD DISTRIBUTIONS OF A CHIRAL FILLED SPHERICAL PERFECTLY CONDUCTING CAVITY

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Abstract—The electromagnetic resonances of a spherical cavity, with a perfectly conducting wall and filled with a homogeneous isotropic chiral medium, is studied using the spherical vector wavefunctions. The characteristic equation and the expressions for the field components, when chirality reaches its maximum value, are derived. The characteristic equation is obtained by imposing the boundary condition on the wall of the spherical cavity. The characteristic equation is solved numerically and reported for the first five modes. These modes are hybrid modes. They are classes as either hybrid electric (HE) modes or hybrid magnetic (HM) modes. The explicit expressions for the field components of the HE and HM modes are given, and the field distributions of a few modes are shown. The chirality is observed to have significant effects on the resonances and the field distributions of a chiral filled spherical perfectly conducting cavity. The results show interesting properties of the cavity, which could be applied to new applications.

1. INTRODUCTION

In recent years, many researchers have done work on special media such as chiral media and left-handed media [1, 2] in both applications and theories. The work in [3] presents a realization of a medium which exhibits both chiral and left-handed properties. This suggests that potential applications of chiral media to various practical problems such as waveguides, polarization transformers, fibers, antennas, and antenna radomes [4–10] could be realized. The problems of scattering by chiral objects have been investigated with several methods [11–17]. Analytical solutions for canonical shaped objects are recently presented by many researchers [18–24]. Several works [25–28] focus on chiral objects. The resonant frequency and Q factor of a chiral sphere [25] and a cylindrical cavity filled with a chiral medium [26] are examined. The resonant frequency [27] and Q factor [28] of a spherical cavity filled with a chiral medium are investigated. This paper is an extension of the work in [27]. The characteristic equation and the expressions for the field components for a chiral filled spherical perfectly conducting cavity shown in Figure 1 are derived when the chirality parameter reaches its maximum limit given in [29] or the absolute value of the relative chirality, defined in [25], reaches one. In this study, the spherical vector wavefunctions and the constitutive relations given in [25] are used. The characteristic equation are derived by forming the solution for the electromagnetic field inside the cavity in terms of the spherical vector wavefunctions and enforcing the boundary condition on the tangential components of the electric field on the surface of the sphere. The characteristic equation is solved numerically and reported for the first five modes. The explicit expressions for the field components of the HE

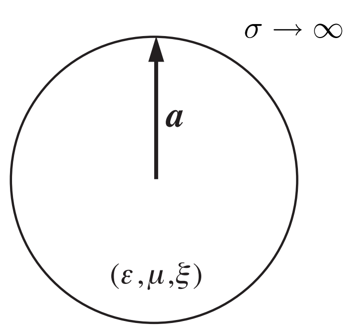


Figure 1. Spherical cavity with radius $r = a$, having a perfectly conducting wall and filled with a chiral medium.

and HM modes are given, and the field distributions of a few modes are shown. In this paper, a spherical cavity with a perfectly conducting wall is assumed.

2. THE SPHERICAL SOLUTIONS FOR THE ELECTROMAGNETIC FIELD IN A CHIRAL FILLED SPHERICAL CAVITY

The constitutive relations for a chiral medium are expressed [25] as

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} - j\xi \boldsymbol{H} \tag{1}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} + j\xi \boldsymbol{E} \tag{2}$$

where ξ is the chirality parameter and the relative chirality ξ_r is defined by

$$\xi_r = \frac{\xi}{\sqrt{\mu\varepsilon}}. \tag{3}$$

The expressions for the electromagnetic field inside a chiral filled spherical cavity ($\boldsymbol{E}_{mnr}^{chiral}, \boldsymbol{H}_{mnr}^{chiral}$) are given as [25]

$$\begin{aligned} \boldsymbol{E}_{\{e,o\}mnr}^{chiral} &= jc_{\{e,o\}mn}\Big(\boldsymbol{N}_{\{e,o\}mn}(k_+r) + \boldsymbol{M}_{\{e,o\}mn}(k_+r)\Big) \\ &\quad + d_{\{e,o\}mn}\Big(\boldsymbol{N}_{\{e,o\}mn}(k_-r) - \boldsymbol{M}_{\{e,o\}mn}(k_-r)\Big) \end{aligned} \tag{4}$$

$$\begin{aligned} \boldsymbol{H}_{\{e,o\}mnr}^{chiral} &= -\frac{1}{\eta}\Big\{c_{\{e,o\}mn}\Big(\boldsymbol{N}_{\{e,o\}mn}(k_+r) + \boldsymbol{M}_{\{e,o\}mn}(k_+r)\Big) \\ &\quad + jd_{\{e,o\}mn}\Big(\boldsymbol{N}_{\{e,o\}mn}(k_-r) - \boldsymbol{M}_{\{e,o\}mn}(k_-r)\Big)\Big\} \end{aligned} \tag{5}$$

where the spherical vector wavefunctions $\boldsymbol{M}_{\{e,o\}mn}$ and $\boldsymbol{N}_{\{e,o\}mn}$ are defined by [30, Appendix A]

$$\begin{aligned} \boldsymbol{M}_{\{e,o\}mn}(k_\pm r) &= \boldsymbol{a}_\theta \frac{\hat{J}_n(k_\pm r)}{k_\pm r} \frac{m P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\} \\ &\quad + \boldsymbol{a}_\phi \frac{\hat{J}_n(k_\pm r)}{k_\pm r} \sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \end{aligned} \tag{6}$$

$$\begin{aligned} \boldsymbol{N}_{\{e,o\}mn}(k_\pm r) &= \boldsymbol{a}_r n(n+1) \frac{\hat{J}_n(k_\pm r)}{(k_\pm r)^2} P_n^m(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \\ &\quad - \boldsymbol{a}_\theta \frac{\hat{J}_n'(k_\pm r)}{k_\pm r} \sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \\ &\quad + \boldsymbol{a}_\phi \frac{\hat{J}_n'(k_\pm r)}{k_\pm r} \frac{m P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\} \end{aligned} \tag{7}$$

where P_n^m is the associated Legendre polynomial of order m and degree n and $\hat{J}_n$ is the alternative spherical Bessel function of the first kind of order n given in [31]. In (6) and (7), the subscripts $\{e, o\}$ pair with either even function $\cos(m\phi)$ or odd function $\sin(m\phi)$. The subscript e pairs with the first functions in curly brackets and the subscript o pairs with the second functions in curly brackets. The wavenumbers for the equivalent media $k_\pm$ are given in terms of the free space wavenumber $k_o = \omega\sqrt{\mu_o \varepsilon_o}$ as

$$k_\pm = k_o \sqrt{\mu_r \varepsilon_r}(1 \pm \xi_r) \tag{8}$$

where $\mu = \mu_o \mu_r$ and $\varepsilon = \varepsilon_o \varepsilon_r$. These wavenumbers are nonnegative numbers when $-1 \leq \xi_r \leq 1$. In this paper, we assume that ξ_r is a positive number.

3. ELECTROMAGNETIC RESONANCE OF A CHIRAL FILLED SPHERICAL CAVITY

The electromagnetic resonances of a chiral filled spherical cavity occur when the electric field in (4) vanishes at the surface of the perfectly conducting sphere where $r = a$. That is

$$\boldsymbol{a}_r \times \boldsymbol{E}^{chiral}_{\{e,o\}mn}\Big|_{r=a} = 0 \tag{9}$$

where $\boldsymbol{a}_r$ is the unit radial vector. Using (4), (6), and (7), and the fact that the spherical vector wavefunctions are orthogonal, (9) can be expanded into two equations given by

$$jc_{\{e,o\}mn}\frac{\hat{J}_n(k_+a)}{k_+a} - d_{\{e,o\}mn}\frac{\hat{J}_n(k_-a)}{k_-a} = 0 \tag{10}$$

$$jc_{\{e,o\}mn}\frac{\hat{J}'_n(k_+a)}{k_+a} + d_{\{e,o\}mn}\frac{\hat{J}'_n(k_-a)}{k_-a} = 0. \tag{11}$$

Using $x = k_o a\sqrt{\mu_r \varepsilon_r}$ and assuming that $\xi_r \neq 1$, the nontrivial solutions for the unknown constants $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ of (10) and (11) exist only when

$$\hat{J}_n(x(1+\xi_r))\hat{J}'_n(x(1-\xi_r)) + \hat{J}_n(x(1-\xi_r))\hat{J}'_n(x(1+\xi_r)) = 0. \tag{12}$$

Equation (12) is called the characteristic equation for a chiral filled spherical cavity which is similar to the one given in [27]. Equation (12) is only valid for $\xi_r \neq 1$. When $\xi_r = 1$, $\hat{J}_n(x(1-\xi_r))$ and $\hat{J}'_n(x(1-\xi_r))$ are zero, nontrivial solutions for (12) do not exist. Therefore, the limiting value of $\hat{J}_n(z)$ and $\hat{J}'_n(z)$ as $z \to 0$ are used to derived the characteristic equation when $\xi_r = 1$. These limiting value are [31]

$$\hat{J}_n(z) = \frac{z^{n+1}}{1 \cdot 3 \cdot 5 \ldots (2n+1)} \tag{13}$$

$$\hat{J}'_n(z) = \frac{(n+1)z^n}{1 \cdot 3 \cdot 5 \ldots (2n+1)}. \tag{14}$$

When $\xi_r \to 1$, using (13) and (14), we obtain

$$\frac{\hat{J}_n(x(1-\xi_r))}{\hat{J}'_n(x(1-\xi_r))} = \frac{x(1-\xi_r)}{n+1}. \tag{15}$$

Dividing (12) with $\hat{J}'_n(x(1-\xi_r))$ and using the result from (15), (12) can be rewritten for $\xi_r \to 1$ as

$$\hat{J}_n(x(1+\xi_r)) + \frac{x(1-\xi_r)}{n+1}\hat{J}'_n(x(1+\xi_r)) = 0. \tag{16}$$

Since the characteristic equation is continuous at $\xi_r = 1$, the solution when $\xi_r \to 1$ is the solution when $\xi_r = 1$. Letting the relative chirality in (16) equal to one, the characteristic equation for $\xi_r = 1$ is given by

$$\hat{J}_n(2x) = 0. \tag{17}$$

The above equation is also true for $\xi_r = -1$. Once the root x of the characteristic equation are obtained, the resonant frequency f_r can be computed by

$$f_r = \frac{x}{2\pi a\sqrt{\mu\varepsilon}}. \tag{18}$$

When the medium is nonchiral so that $\xi_r = 0$, (12) reduces to

$$\hat{J}_n(x)\hat{J}'_n(x) = 0. \tag{19}$$

The roots of the first term of (19) give rise to TE_{mnr} modes and the roots of the second term of (19) give rise to TM_{mnr} modes [31]. For a TE mode, the radial component of electric field E_r is zero. For a TM mode, the radial component of magnetic field H_r is zero. The modes $\{\mathrm{TM}_{mnr},\ r = 1, 2, \ldots\}$ are ordered in the order of increasing x and $\{\mathrm{TE}_{mnr},\ r = 1, 2, \ldots\}$ are similarly ordered. Equations (4) and (5) show that the electromagnetic field in a chiral filled spherical cavity is neither TE nor TM because if either E_r or H_r is identically zero, then the electromagnetic field must be identically zero. In other words, the electromagnetic field in a chiral filled spherical cavity is always a hybrid mode (HEM mode). The HEM mode which reduces to the TM_{mnr} mode when $\xi_r = 0$ is a hybrid magnetic mode called the HM_{mnr} mode. The HEM mode which reduces to the TE_{mnr} mode when $\xi_r = 0$ is a hybrid electric mode called the HE_{mnr} mode.

A resonant electromagnetic field inside a chiral filled spherical cavity ($\boldsymbol{E}^{chiral}_{mnr}, \boldsymbol{H}^{chiral}_{mnr}$) can be obtained from (4) and (5) with the coefficients $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ obtained from either (10) or (11). The relation between $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ obtained from (11) is not valid when the field is a TM mode because $\hat{J}'_n(x) = 0$ where x is a root of the characteristic equation for the TM mode. For a small value of ξ_r, it is difficult to compute the field of the HM mode because the values of $\hat{J}'_n(x(1 \pm \xi_r))$ are small. For this reason, the relation between $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ obtained from (10) is used for the expressions for the field of the HM mode and the one obtained from (11) is used for the expressions for the field of the HE mode.

The relation between $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ obtained from (10) is given as

$$d_{\{e,o\}mn} = jc_{\{e,o\}mn} \frac{\hat{J}_n\left(x^{HM}_{mnr}(1+\xi_r)\right)(1-\xi_r)}{\hat{J}_n\left(x^{HM}_{mnr}(1-\xi_r)\right)(1+\xi_r)} \tag{20}$$

where x^{HM}_{mnr} is the root of the characteristic equation for the HM_{mnr} mode. Substituting (20) into (4) and (5), using (6) and (7) and letting $c_{\{e,o\}mn} = 1$, the expressions for the resonant electromagnetid field of the HM_{mnr} modes are obtained as

$$\begin{aligned}
&\boldsymbol{a}_r \cdot \boldsymbol{E}^{chiral}_{\{e,o\}mnr} = jn(n+1) \\
&\left\{\frac{\hat{J}_n\left(r_a x^{HM}_{mnr}(1+\xi_r)\right)}{\left(r_a x^{HM}_{mnr}(1+\xi_r)\right)^2} + \frac{\hat{J}_n\left(x^{HM}_{mnr}(1+\xi_r)\right)}{\hat{J}_n\left(x^{HM}_{mnr}(1-\xi_r)\right)} \frac{\hat{J}_n\left(r_a x^{HM}_{mnr}(1-\xi_r)\right)}{\left(r_a x^{HM}_{mnr}(1-\xi_r)\right)^2} \frac{(1-\xi_r)}{(1+\xi_r)}\right\} \\
&\quad P^m_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\}
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\boldsymbol{a}_\theta \cdot \boldsymbol{E}^{chiral}_{\{e,o\}mnr} = \\
&-j\left\{\frac{\hat{J}'_n\left(r_a x^{HM}_{mnr}(1+\xi_r)\right)}{r_a x^{HM}_{mnr}(1+\xi_r)} + \frac{\hat{J}_n\left(x^{HM}_{mnr}(1+\xi_r)\right)}{\hat{J}_n\left(x^{HM}_{mnr}(1-\xi_r)\right)} \frac{\hat{J}'_n\left(r_a x^{HM}_{mnr}(1-\xi_r)\right)}{r_a x^{HM}_{mnr}(1-\xi_r)} \frac{(1-\xi_r)}{(1+\xi_r)}\right\} \\
&\quad \sin\theta P^{m\prime}_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\}
\end{aligned}$$

$$+j\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}-\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\} \tag{22}$$

$$\boldsymbol{a}_\phi\cdot\boldsymbol{E}_{\{e,o\}mnr}^{chiral}=$$

$$j\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}+\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\}$$

$$+j\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}-\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{23}$$

$$\boldsymbol{a}_r\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=-\frac{1}{\eta}n(n+1)$$

$$\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)^2}-\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)^2}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$P_n^m(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{24}$$

$$\boldsymbol{a}_\theta\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=$$

$$\frac{1}{\eta}\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}-\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\}$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}+\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\} \tag{25}$$

$$\boldsymbol{a}_\phi\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}-\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\}$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1+\xi_r)\right)}{r_a x_{mnr}^{HM}(1+\xi_r)}+\frac{\hat{J}_n\left(x_{mnr}^{HM}(1+\xi_r)\right)}{\hat{J}_n\left(x_{mnr}^{HM}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HM}(1-\xi_r)\right)}{r_a x_{mnr}^{HM}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$
$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \qquad (26)$$

where $r_a = r/a$. When the medium is nonchiral so that $\xi_r = 0$, $\boldsymbol{a}_r \cdot \boldsymbol{H}_{\{e,o\}mnr}^{chiral}$ in (24) is zero and the above equations reduce to the resonant electromagnetic field of the TM_{mnr} modes.

The relation between $c_{\{e,o\}mn}$ and $d_{\{e,o\}mn}$ obtained from (11) is given as

$$d_{\{e,o\}mn} = -jc_{\{e,o\}mn}\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)(1-\xi_r)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)(1+\xi_r)} \qquad (27)$$

where x_{mnr}^{HE} is the root of the characteristic equation for the HE_{mnr} mode. Substituting (27) into (4) and (5), using (6) and (7) and letting $c_{\{e,o\}mn} = 1$, the expressions for the resonant electromagnetic field of the HE_{mnr} modes are obtained as

$$\boldsymbol{a}_r \cdot \boldsymbol{E}_{\{e,o\}mnr}^{chiral} = jn(n+1)$$
$$\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)^2}-\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)^2}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$
$$P_n^m(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \qquad (28)$$

$$\boldsymbol{a}_\theta \cdot \boldsymbol{E}_{\{e,o\}mnr}^{chiral} =$$
$$-j\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}-\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$
$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\}$$
$$+j\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}+\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$
$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\} \qquad (29)$$

$$\boldsymbol{a}_\phi \cdot \boldsymbol{E}_{\{e,o\}mnr}^{chiral} =$$
$$j\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}-\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\}$$

$$+j\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}+\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{30}$$

$$\boldsymbol{a}_r\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=-\frac{1}{\eta}n(n+1)$$

$$\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)^2}+\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)^2}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$P_n^m(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{31}$$

$$\boldsymbol{a}_\theta\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=$$

$$\frac{1}{\eta}\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}+\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\}$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}-\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\} \tag{32}$$

$$\boldsymbol{a}_\phi\cdot\boldsymbol{H}_{\{e,o\}mnr}^{chiral}=$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}+\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n'\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$m\frac{P_n^m(\cos\theta)}{\sin\theta}\{-\sin(m\phi),\cos(m\phi)\}$$

$$-\frac{1}{\eta}\left\{\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1+\xi_r)\right)}{r_a x_{mnr}^{HE}(1+\xi_r)}-\frac{\hat{J}_n'\left(x_{mnr}^{HE}(1+\xi_r)\right)}{\hat{J}_n'\left(x_{mnr}^{HE}(1-\xi_r)\right)}\frac{\hat{J}_n\left(r_a x_{mnr}^{HE}(1-\xi_r)\right)}{r_a x_{mnr}^{HE}(1-\xi_r)}\frac{(1-\xi_r)}{(1+\xi_r)}\right\}$$

$$\sin\theta P_n^{m\prime}(\cos\theta)\{\cos(m\phi),\sin(m\phi)\} \tag{33}$$

When the medium is nonchiral so that $\xi_r = 0$, $\boldsymbol{a}_r\cdot\boldsymbol{E}_{\{e,o\}mnr}^{chiral}$ in (28) is zero and the above equations reduce to the resonant electromagnetic field of the TE_{mnr} modes. Equations (24) and (31) show that the radial component of the magnetic field does not vanish on the conducting surface at $r_a = 1$ when $\xi_r \neq 0$. Using (2), (21), and (24), one can easily show that the radial component of the magnetic flux density for the HM_{mnr} mode is always vanish on the conducting surface. The same result can be obtained for the HE_{mnr} mode.

The field components of the HM and HE modes, given in the above equations, become invalid when $\xi_r = 1$. The solutions for the resonant electromagnetic field when $\xi_r \to 1$ can be derived from either (21)–(26) or (28)–(33) by using the limiting values of $\hat{J}_n(z)$ and $\hat{J}'_n(z)$ as $z \to 0$. Since the solutions for the resonant electromagnetic field are continuous at $\xi_r = 1$, the solutions when $\xi_r \to 1$ are the solutions when $\xi_r = 1$. Letting the relative chirality in the resulting equations equal to one, the expressions for the resonant electromagnetic for $\xi_r = 1$ are obtained as

$$\boldsymbol{a}_r \cdot \boldsymbol{E}^{chiral}_{\{e,o\}mnr} = jn \left\{ (n+1) \frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{\left(2r_a x^{HM,HE}_{mnr}\right)^2} - (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \right\} P^m_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \tag{34}$$

$$\begin{aligned}\boldsymbol{a}_\theta \cdot \boldsymbol{E}^{chiral}_{\{e,o\}mnr} = & -j \left\{ \frac{\hat{J}'_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} - (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \right\} \\ & \sin\theta P^{m\prime}_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \\ & + j \frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} m \frac{P^m_n(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\}\end{aligned} \tag{35}$$

$$\begin{aligned}\boldsymbol{a}_\phi \cdot \boldsymbol{E}^{chiral}_{\{e,o\}mnr} = & j \left\{ \frac{\hat{J}'_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} - (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \right\} \\ & m \frac{P^m_n(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\} \\ & + j \frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \sin\theta P^{m\prime}_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\}\end{aligned} \tag{36}$$

$$\boldsymbol{a}_r \cdot \boldsymbol{H}^{chiral}_{\{e,o\}mnr} = -\frac{1}{\eta} n \left\{ (n+1) \frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{\left(2r_a x^{HM,HE}_{mnr}\right)^2} + (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \right\} P^m_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \tag{37}$$

$$\boldsymbol{a}_\theta \cdot \boldsymbol{H}^{chiral}_{\{e,o\}mnr} = \frac{1}{\eta}\left\{\frac{\hat{J}'_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} + (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}}\right\}$$
$$\sin\theta P^{m\prime}_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\}$$
$$-\frac{1}{\eta}\frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} m \frac{P^m_n(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\} \tag{38}$$

$$\boldsymbol{a}_\phi \cdot \boldsymbol{H}^{chiral}_{\{e,o\}mnr} = -\frac{1}{\eta}\left\{\frac{\hat{J}'_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} + (r_a)^n \frac{\hat{J}'_n\left(2x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}}\right\}$$
$$m\frac{P^m_n(\cos\theta)}{\sin\theta}\{-\sin(m\phi), \cos(m\phi)\}$$
$$-\frac{1}{\eta}\frac{\hat{J}_n\left(2r_a x^{HM,HE}_{mnr}\right)}{2r_a x^{HM,HE}_{mnr}} \sin\theta P^{m\prime}_n(\cos\theta)\{\cos(m\phi), \sin(m\phi)\} \tag{39}$$

4. NUMERICAL RESULTS

The roots of the characteristic equation are solved numerically. Equation (17) is used for the characteristic equation when $\xi_r = 1$. For the dielectric filled spherical cavity, TE and TM solutions are computed using (19). Each solution is in excellent agreement with that published in [31] and is used as an initial value to obtain a solution for each hybrid mode. We assume that each solution is a continuous function of ξ_r. A previously computed solution for a value of ξ_r is used as an initial value to obtain a solution for a larger ξ_r. The increment of ξ_r must be small enough such that the solution does not jump to a different mode. Figure 2 shows x for the first five modes as a function of the relative chirality ξ_r. It can be observed that the HE_{m11} and the HM_{m31} modes are degenerate modes whose $x = 4.397$ when $\xi_r = 0.436$. The resonant frequencies of these two modes can be adjusted with the parameter ξ_r. This shows a useful property for a design of a spherical cavity filter [32].

Figures 3 and 6 show the magnetic field distributions on the spherical conducting surface of the HE_{o111} mode and the HM_{o131} mode, respectively. Figures 3(b) and 6(b) are the resonant magnetic fields of the degenerate modes with $\xi_r = 0.436$. It is observed that the fields of these two modes are not orthogonal and can easily couple. Figures 4 and 7 show the electric field distributions on the yz-plane of the HE_{o111}

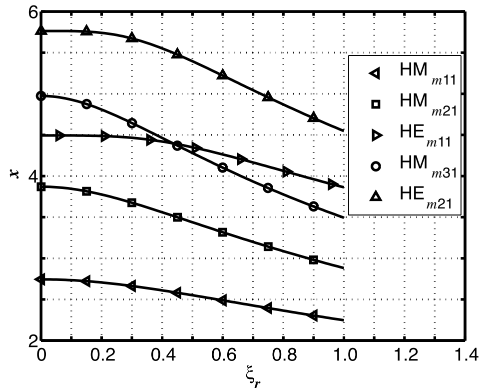


Figure 2. Root x of the characteristic equation for the first five modes.

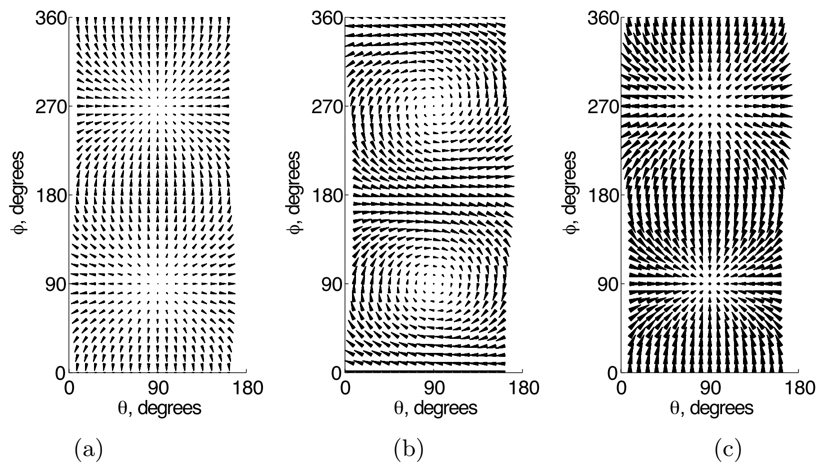
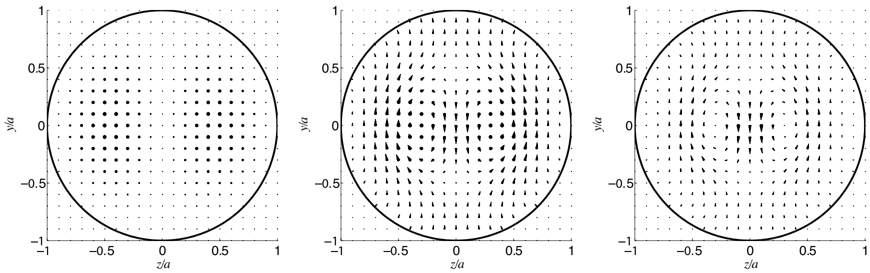


Figure 3. Magnetic field distributions on the conducting surface of HE_{o111} mode (a) $\xi_r = 0$ with $x = 4.493$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.863$.

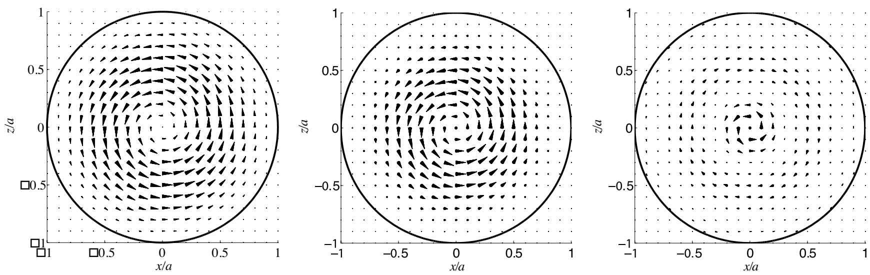
mode and the HM_{o131} mode, respectively. Figures 5 and 8 show the electric field distributions on the xz-plane of the HE_{o111} mode and the HM_{o131} mode, respectively. Figures 3–8 show that the chirality has significant effects on both electric and magnetic field distributions.

Figures 3(c) and 9(a) show that the resonant magnetic field on the spherical conducting surface of the HE_{o111} mode when $\xi_r = 1$ is similar to the field of the TE_{o112} mode. Figures 6(c) and 9(b) show



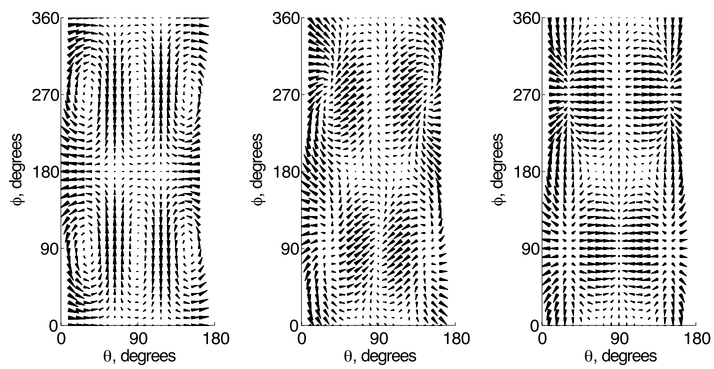
(a) (b) (c)

Figure 4. Electric field distributions on the yz-plane of HE_{o111} mode (a) $\xi_r = 0$ with $x = 4.493$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.863$.



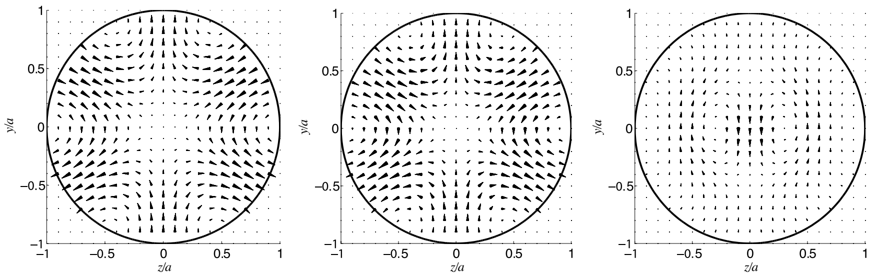
(a) (b) (c)

Figure 5. Electric field distributions on the xz-plane of HE_{o111} mode (a) $\xi_r = 0$ with $x = 4.493$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.863$.



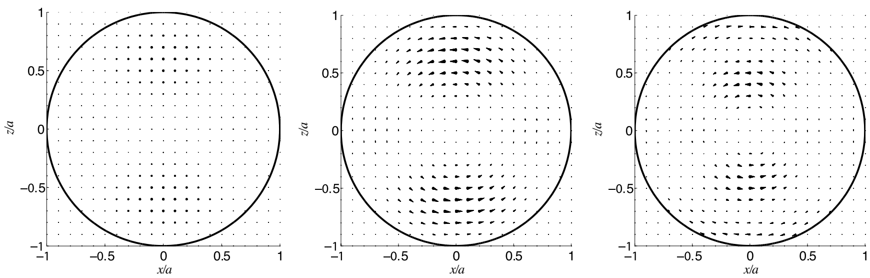
(a) (b) (c)

Figure 6. Magnetic field distributions on the conducting surface of HM_{o131} mode (a) $\xi_r = 0$ with $x = 4.973$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.494$.



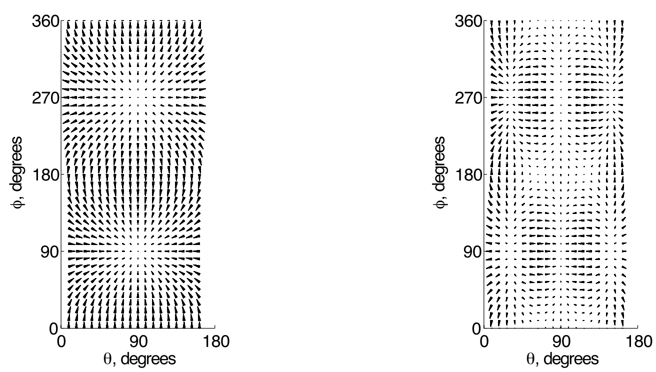
(a) (b) (c)

Figure 7. Electric field distributions on the yz-plane of HM_{o131} mode (a) $\xi_r = 0$ with $x = 4.973$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.494$.



(a) (b) (c)

Figure 8. Electric field distributions on the xz-plane of HM_{o131} mode (a) $\xi_r = 0$ with $x = 4.973$ (b) $\xi_r = 0.436$ with $x = 4.397$ (c) $\xi_r = 1$ with $x = 3.494$.



(a) (b)

Figure 9. Magnetic field distributions on the conducting surface for (a) TE_{o112} mode with $x = 7.725$ (b) TE_{o131} mode with $x = 6.988$.

that the resonant magnetic field on the spherical conducting surface of the HM_{o131} mode when $\xi_r = 1$ is similar to the field of the TE_{o131} mode. This shows that the resonant magnetic field for $\xi_r = 1$ on the spherical conducting surface of either the HE_{o111} mode or the HM_{o131} mode is similar to the field for TE mode whose resonant frequency is twice higher.

5. CONCLUSION

In this paper the characteristic equation for a chiral filled spherical cavity is derived by using the spherical vector wavefunctions. The detailed derivation for the characteristic equation is given when the relative chirality reaches its maximum value. The expressions for the field components of the HE and HM modes given in this paper are valid for any values of the chirality. The roots of the characteristic equation are numerically solved and shown for the first five modes. It shows that the HE_{m11} and the HM_{m31} modes are degenerate at $\xi_r = 0.436$. The field distributions of the HE_{o111} and HM_{o131} modes are shown and compared with the field distributions of the TE modes. It shows that the HE_{m11} and the HM_{m31} modes with $\xi_r = 0.436$ could easily couple and the resonant magnetic field of each mode for $\xi_r = 1$ on the spherical conducting surface is similar to the field of the associated TE mode with a twice higher resonant frequency. These properties of a chiral filled spherical cavity could give rise to new applications. It can be observed that the chirality has significant effects on both electric and magnetic field distributions.

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REFERENCES

1. Grzegorczyk, T. M. and J. A. Kong, "Review of left-handed metamaterials: Evolution from theoretical and numerical studies to potential applications," *J. Electromagn. Waves Appl.*, Vol. 20, No. 14, 2053–2064, 2006.
2. Chen, H., B.-I. Wu, and J. A. Kong, "Review of electromagnetic theory in left-handed materials," *J. Electromagn. Waves Appl.*, Vol. 20, No. 15, 2137–2151, 2006.

3. Cheng, X., H. Chen, L. Ran, B.-I. Wu, T. M. Grzegorczyk, and J. A. Kong, "A bianisotropic left-handed metamaterials compose of s-ring resonator," *PIERS Online*, Vol. 3, No. 5, 593–598, 2007
4. Hussain, A., M. Faryad, and Q. A. Naqvi, "Fractional curl operator and fractional chiro-waveguide," *J. Electromagn. Waves Appl.*, Vol. 21, No. 8, 1119–1129, 2007.
5. Panin, S. B., P. D. Smith, and A. Y. Poyedinchuk, "Elliptical to linear polarization transformation by a grating on a chiral medium," *J. Electromagn. Waves Appl.*, Vol. 21, No. 13, 1885–1899, 2007.
6. Nair, A. and P. K. Choudhury, "On the analysis of field patterns in chirofibers," *J. Electromagn. Waves Appl.*, Vol. 21 No. 15, 2277–2286, 2007.
7. Engheta, N. and M. W. Kowarz, "Antenna radiation in the presence of a chiral sphere," *J. Appl. Phys.*, Vol. 67, No. 2, 639–647, 1990.
8. Li, L.-W., M.-S. Leong, P.-N. Jiao, and W.-X. Zhang, "Analysis of a passive circular loop antenna radiating in the presence of a layered chiral sphere using method of moments," *J. Electromagn. Waves Appl.*, Vol. 16 No. 11, 1593–1611, 2002.
9. Pelet, P. and N. Engheta, "The theory of chirowaveguides," *IEEE Trans. Antennas Propagat.*, Vol. 38, No. 1, 90–98, January 1990.
10. Tretyakov, S. A. and A. A. Sochava, "Proposed composite material for nonreflecting shields and antenna radomes," *Electronic Letters*, Vol. 29, No. 12, 1048–1049, June 1993.
11. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, "Scattering and absorption characteristics of lossy dielectric, chiral, nonspherical objects," *Appl. Opt.*, Vol. 24, No. 23, 4146–4154, December 1985.
12. Kluskens, M. S. and E. H. Newman, "Scattering by a multilayer chiral cylinder," *IEEE Trans. Antennas Propagat.*, Vol. 39, No. 1, 91–96, January 1991.
13. Al-Kanhal, M. A. and E. Arvas, "Electromagnetic scattering from a chiral cylinder of arbitrary cross section," *IEEE Trans. Antennas Propagat.*, Vol. 44, No. 7, 1041–1048, July 1996.
14. Worasawate, D., J. R. Mautz, and E. Arvas, "Electromagnetic scattering from an arbitrarily shaped three-dimensional homogeneous chiral body," *IEEE Trans. Antennas Propagat.*, Vol. 51, No. 5, 1077–1084, May 2003.
15. Khatir, B. N., M. Al-Kanhal, and A. Sebak, "Electromagnetic wave scattering by elliptic chiral cylinder," *J. Electromagn. Waves*

Appl., Vol. 20 No. 10, 1377–1390, 2006.

16. Kuzu, L., V. Demir, A. Z. Elsherbeni, and E. Arvas, "Electromagnetic scattering from arbitrarily shaped chiral objects using the finite difference frequency domain method," *Progress In Electromagnetics Research*, PIER 67, 1–24, 2007.
17. Mei, C., M. Hasanovic, J. K. Lee, and E. Arvas, "Electromagnetic scattering from an arbitrarily shaped three-dimensional inhomogeneous bianisotropic body," *PIERS Online*, Vol. 3, No. 5, 680–684, 2007.
18. Zhao, J. X., "Numerical and analytical formulizations of the extended Mie theory for solving the sphere scattering problem," *J. Electromagn. Waves Appl.*, Vol. 20, No. 7, 967–983, 2006.
19. Ruppin, R., "Scattering of electromagnetic radiation by a perfect electromagnetic conductor sphere," *J. Electromagn. Waves Appl.*, Vol. 20, No. 12, 1569–1576, 2006.
20. Chen, X., "Time-reversal operator for a small sphere in electromagnetic fields," *J. Electromagn. Waves Appl.*, Vol. 21, No. 9, 1219–1230, 2007.
21. Li, Y.-L., J.-Y. Huang, and M.-J. Wang, "Investigation of electromagnetic interaction between a spherical target and a conducting plane," *J. Electromagn. Waves Appl.*, Vol. 21, No. 12, 1703–1715, 2007.
22. Valagiannopoulos, C. A., "Electromagnetic scattering from two eccentric metamaterial cylinders with frequency-dependent permittivities differing slightly each other," *Progress In Electromagnetics Research B*, Vol. 3, 23–34, 2008.
23. Kukharchik, P. D., V. M. Serdyuk, and J. A. Titovitsky, "Diffraction of hybrid modes in a cylindrical cavity resonator by a transverse circular slot with a plane anisotropic dielectric layer," *Progress In Electromagnetics Research B*, Vol. 3, 73–94, 2008.
24. Li, Y.-L., J.-Y. Huang, M.-J. Wang, and J. Zhang, "Scattering field for the ellipsoidal targets irradiated by an electromagnetic wave with arbitrary polarizing and propagating direction," *Progress In Electromagnetics Research Letters*, Vol. 1, 221–235, 2008.
25. Worasawate, D., J. R. Mautz, and E. Arvas, "Electromagnetic resonances and Q factor of a chiral sphere," *IEEE Trans. Antennas Propagat.*, Vol. 52, No.1, 213–219, January 2004.
26. Rao, T. C. K., "Resonant frequency and Q-factor of a cylindrical cavity containing a chiral medium," *Int. J. Electronics*, Vol. 73, No. 1, 183–191, 1992.

27. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, "Eigenmodes of a chiral sphere with a perfectly conducting coating," *J. Phys. D: Appl. Phys.*, Vol. 22, 825–828, 1989.
28. Hui, H. T. and E. K. N. Yung, "The quality factor of a spherical cavity filled with a chiral medium," *J. Electromagn. Waves Appl.*, Vol. 15 No. 1, 41–52, 2001.
29. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Wave in Chiral and Bi-Isotropic Media*, Artech House, Boston, 1994.
30. Worasawate, D., "Electromagnetic scattering from an arbitrarily shaped three-dimensional chiral body," Ph.D. Dissertation, Syracuse University, 2002.
31. Harrington, R. F., *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961.
32. Lai, S.-L. and W.-G. Lin, "A five mode single spherical cavity microwave filter," *IEEE Microwave Theory and Techniques Society International Microwave Symposium Digest 1992*, Vol. 2, 909–912, June 1–5, 1992.