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OMNI-DIRECTION REFLECTION IN ONE DIMENSIONAL PHOTONIC CRYSTAL

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Abstract—It is well known that under certain conditions, one dimensional photonic crystal (1D-PC) displays total omni-directional reflection (ODR) band gaps. The enhancement of total omni-directional reflection band gap in 1D-PC is calculated theoretically. Using Transfer Matrix Method (TMM) and Bloch theorem, the reflectivity of one dimensional periodic structure for TE and TM-modes at different angles of incidence has been calculated.

1. INTRODUCTION

Over the past decades, much attention has been given to photonic crystals (PCs) as a new kind of optical materials [1–6]. These materials are based on the interaction between an optical field and materials exhibiting periodicity on the scale of wave length. The main feature of photonic crystals is that they can prohibit the propagation of electromagnetic waves within a certain frequency range called photonic band gap (PBG). The materials containing PBG have many potential applications in optoelectronics and optical communication. For the optical range within which the main application is expected, most of experimental effort has been concentrated on two dimensional and three dimensional photonic crystals. The application of two dimensional and three dimensional photonic crystals is limited. Complete two dimensional and three dimensional band gap can be realized in photonic crystals where the refractive index is periodically modulated in two- and three-dimension respectively [2, 6–10].

Besides two dimensional and three dimensional photonic crystals, the simplest periodic structure both in geometry and manipulating is a one dimensional stack of two types of layer which differs in the dielectric constants. One dimensional photonic crystal is attractive since their production is more feasible at any wavelength scale and their analytical and numerical calculations are simpler. It has been shown that one dimensional photonic crystals can totally reflect arbitrary polarized, transverse-electric (TE) and transverse-magnetic (TM), lights i.e., omnidirectional reflection, at all angles within the PBGs, can take place even in the absence of a complete band gap i.e., these crystals exhibit the property of omni-directional reflection [4, 6, 11].

Recently, several research groups worldwide have reported that a simple-to-fabricate periodic one dimensional medium can have high reflectivity over a wide range of frequencies at all incident angles, i.e., an omnidirectional reflection. The total reflection band that occurs in any direction for TE and TM polarization is called omni-directional reflector mirror. One dimensional photonic crystal designed as Bragg mirror. Mirrors are of two basic varieties, a metallic mirror or omni-directional reflector and a dielectric Bragg mirror. One dimensional photonic crystal is designed as Bragg mirror [12–20]. In particular, large parts of these one dimensional photonic crystals were designed for application in the infrared wavelength ranges. Several research groups have reported that a simpler to fabricate Bragg mirror suffices to design a low loss omni-directional reflector [21–29].

In 1998 Fink et al. [4] and Winn et al. [30] first reported that the one dimensional dielectric lattice displays total omnidirection reflection for incident light under certain conditions. They constructed a stack of nine alternating micrometer thick layers of Polystyrene and Tellurium and demonstrated omnidirectional reflection over the wavelength range from 10 to 15 micrometers. Gallas et al. [31] reported the annealing effect in the Si/SiO_2 omni reflectors. Chen et al. [32] fabricated six bilayers of SiO_2 and TiO_2 quarter wave films using the sol-gel method and found an omnidirectional PBG of about 70 nm in near IR range. Chigrin et al. [6, 18] fabricated a lattice consisting of 19 layers of Na_3AlF_6 and ZnSe and found that omnidirectional photonic band gap exists in the spectral range 604.3–638.4 nm. Lee and Yao [33] studied a wide range of realistic fabrication parameters for the formation of omnidirectional photonic band gaps (PBGs) in one-dimensional photonic crystals, theoretically as well as experimentally.

In this paper, the reflectivity of Na_3AlF_6/Ge multilayer structure has large values of total omni-directional reflection than experimentally observed. The thicknesses of the each layer are taken 50% of the total thickness of the unit cell for low and high index of refractions respectively. Such Na_3AlF_6/Ge multilayer structure has large total omni-directional reflection than experimentally observed values for same structure with 50% of total thickness of unit cell for each material.

2. THEORY

The Helmholtz equation for considered periodic structure (photonic crystal);

$$\vec{\nabla}^2\vec{E} + \frac{\omega^2}{c^2}\varepsilon_r\mu_r\vec{E} = 0 \tag{1}$$

where $\varepsilon(x)\mu(x) = n^2(x)$, with $n(x)$ is the refractive index. We deled with electromagnetic wave propagated in the x-direction and polarized linearly. We take the y-axis is the direction of the polarization. The electric field of the propagated wave is denoted by the complex function $E(x, y)$ for convenience

$$E(x, y) = E(x)e^{i\beta y} \tag{2}$$

where $E(x) = A_j e^{ik_{jx}x} + B_j e^{-ik_{jx}x}$ for $n_j = \sqrt{\varepsilon_j}$, $\beta = \frac{\omega}{c}n_j\sin\theta$, $k_{jx} = \frac{\omega}{c}n_j\cos\theta_j$, θ_j is the angle of incidence, A_j, B_j are coefficients of reflected and incidence wave respectively with $j = 1, 2$. The wave vectors of first and second medium are k_{1x} and k_{2x} respectively. The magnetic field vector $\vec{H}$ can be obtained along y-axis;

$$\vec{H} = \frac{i}{\omega\mu}\vec{\nabla}\times\vec{E} \tag{3}$$

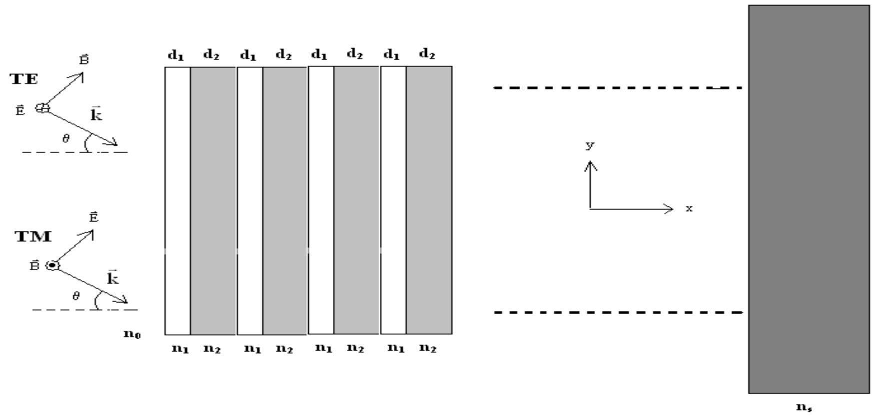

Figure 1. Schematic of the multilayer system with the incident wave vector **k**, the electromagnetic mode convention E and B are the electric and magnetic fields, respectively.

A periodic arrangement of a multilayer film with refractive indices n_1 and n_2 and thickness d_1 and d_2 respectively is shown in Figure 1. This structure is surrounded by the refractive indices n_0 and n_s, called incidence and substrate medium. The solution for Equation (1) will be superposition of plane waves traveling to the right and to the left. Say, for layer with index n_1, the right going and left going plane waves have amplitudes A_1 and B_1 respectively and for layer with index n_2 right going and left going planes waves have amplitudes C_1 and D_1 respectively. Hence for layer with index n_1 the solution of Equation (2) is

$$E(x) = A_1 e^{ik_{1x}x} + B_1 e^{-ik_{1x}x} \tag{4}$$

and

$$E(x) = C_1 e^{ik_{2x}(x-d_1)} + D_1 e^{-ik_{2x}(x-d_1)} \tag{5}$$

for the layer with index n_2. The parameter k_{1x} and k_{2x} will be called the wave number, and the definition is given by $k_{1x} = \frac{\omega}{c} n_1 \cos\theta_1$ and $k_{2x} = \frac{\omega}{c} n_2 \cos\theta_2$. At the interface between layers ($x = d_1$), the Equations (2) and (3) are continuous. This gives a relation between plane waves amplitude;

$$\begin{pmatrix} C_1 \\ D_1 \end{pmatrix} = M_{12} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{6}$$

with

$$M_{12} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{k_{1x}}{k_{2x}}\right)e^{ik_{1x}d_1} & \frac{1}{2}\left(1-\frac{k_{1x}}{k_{2x}}\right)e^{-ik_{1x}d_1} \\ \frac{1}{2}\left(1-\frac{k_{1x}}{k_{2x}}\right)e^{ik_{1x}d_1} & \frac{1}{2}\left(1+\frac{k_{1x}}{k_{2x}}\right)e^{-ik_{1x}d_1} \end{pmatrix} \tag{7}$$

and also at $x = d$, the interface between layer with index n_2 and n_1, continuity of the Equations (2) and (3) which gives;

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M_{21} \begin{pmatrix} C_1 \\ D_1 \end{pmatrix} \tag{8}$$

where the matrix M_{21} is the same as (7) but with interchanging the indices. Using matrix Equations (7) and (8) and eliminating C_1 and D_1, we found the matrix for binary structure;

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M_{21}M_{12} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{9a}$$

or

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M_{i,j} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{9b}$$

where $M_{i,j} = M_{21}M_{12}$. The matrix element of $M_{i,j}$ is given by; $M_{1,1} = e^{ik_{1x}d_1}\left[\cos(k_{2x}d_2) + \frac{1}{2}i\left(\gamma + \frac{1}{\gamma}\right)\sin(k_{2x}d_2)\right]$, $M_{1,2} = e^{-i\,k_{1x}d_1}\left[+\frac{1}{2}i\left(\frac{1}{\gamma} - \gamma\right)\sin(k_{2x}d_2)\right]$, $M_{2,1} = \bar{M}_{1,2}$, $M_{2,2} = \bar{M}_{1,1}$, with $\gamma = \frac{k_{1x}}{k_{2x}}$ for TE-mode and $\gamma = \frac{k_{1x}\times n_2^2}{k_{2x}\times n_1^2}$ for TM-mode.

For the period of the lattices, $\vec{E}(x, K) = U_K(x)e^{iK(\omega)x}$ with $U_K(x)$ is a complex valued periodic function of the lattices $U_K(x) = U_K(x + d)$. The thickness of the unit cell is given as $d(= d1 + d2)$. The parameter $K(\omega)$ is called the Bloch wave number or Dispersion Relation. There is an explicit expression for $K(\omega)$ as follows;

$$K(\omega) = \frac{1}{d}\cos^{-1}\left(\frac{1}{2}Tr\left[M_{i,j}\right]\right) \tag{10}$$

with $M_{i,j}$ given in Equation (9). So Equation (10) becomes;

$$\begin{aligned} K(\omega) \;=\; & \frac{1}{d}\cos^{-1}\left[\cos\left(k_{1x}d_1\right)\times\cos\left(k_{2x}d_2\right)\right. \\ & \left.-\frac{1}{2}\left(\gamma+\frac{1}{\gamma}\right)\times\sin\left(k_{1x}d_1\right)\times\sin\left(k_{2x}d_2\right)\right] \end{aligned} \tag{11}$$

Considering the structure of N period of layers with alternately refractive indices n_1 and n_2 and thickness d_1 and d_2, the coefficients of propagating states in right and left side of the multilayer structures are calculated by multiplying transfer matrices of each cell.

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = (M_{i,j})_1 \times (M_{i,j})_2 \times (M_{i,j})_3 \ldots (M_{i,j})_N \begin{pmatrix} A_N \\ B_N \end{pmatrix} \tag{12}$$

The reflectance coefficient is given by,

$$r_N = \left(\frac{B_0}{A_0} \right)_{B_N=0} \tag{13}$$

The reflectance is obtained by taking absolute value of the reflectance coefficient,

$$R = (|r_N|)^2 \tag{14}$$

3. RESULTS AND DISCUSSION

For the sake of numerical calculation we take Na_3AlF_6/Ge dielectric material with low and high index contrast. The refractive index for Na_3AlF_6 is $n_1 = 1.34$ and for Ge is $n_2 = 4.2$. Taking thickness of the each layer same $a = b$ and $d = a + b = 115\,\text{nm}$ in the visible region. Applying transfer matrix method [34, 35] a graph is plotted between reflectance and wavelength. Figure 2 and Figure 3 shows the reflectivity of calculated spectra at different angle 0°, 20°, 40°, 60° and 85° for TE and TM mode respectively and the ODR is found at 0° to 86° shown in Figure 4.

Table 1. For Na_3AlF_6/Ge $d = 115\,\text{nm}$ and $a = b$).

Angle (deg)	**TE (nm)**	**Band width**	**TM (nm)**	**Band width**
0	540–894	354	540–894	354
20	535–891	356	539–871	332
40	522–883	361	535–810	271
60	508–874	366	533–733	200
85	500–868	368	536–670	134

ODR range ... ... 540–670 nm and ODR width ... ... 130 nm

It is clear from the Table that the transmission range for TE mode is 540–894 nm at normal incidence; at the angle of 20° the range is

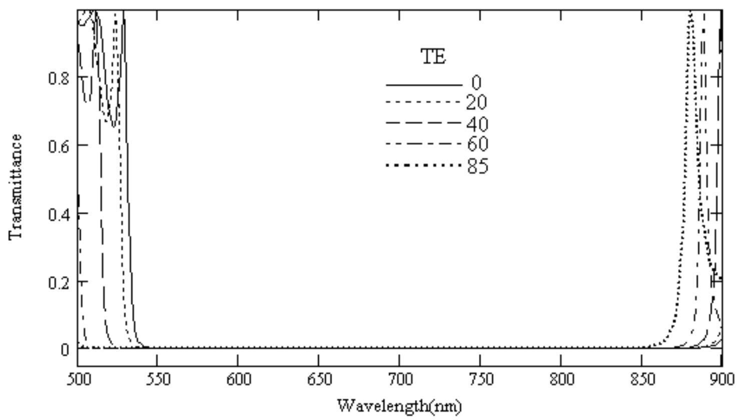

Figure 2. Transmittance curve Vs wavelength (nm) of Na_3AlF_6/Ge multilayer structure for TE mode at different angle.

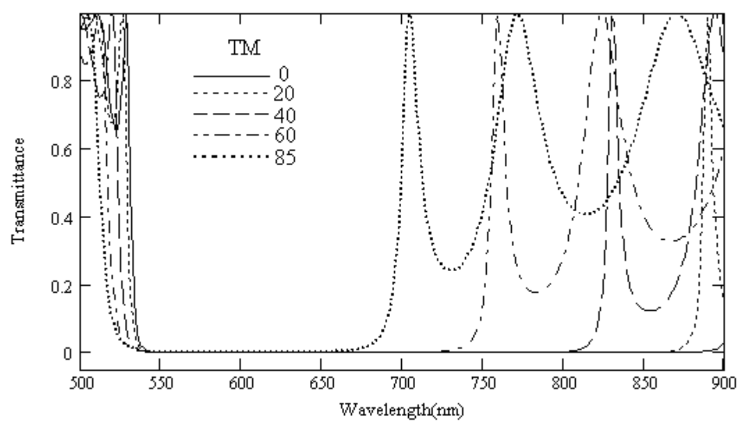

Figure 3. Transmittance curve Vs wavelength (nm) of Na_3AlF_6/Ge multilayer structure for TM mode at different angle.

535–891 nm; at the angle of 40° the range is 522–883 nm; at the angle of 60° the range is 508–874 nm; at the angle of 85° the range is 500–868 nm. The transmission range for TM mode is 540–894 nm at normal incidence; at the angle of 20° the range is 539–871 nm; at the angle of 40° the range is 535–810 nm; at the angle of 60° the range is 533–733 nm; at the angle of 85° the range is 536–670 nm.Hence the total omnidirection range for this multilayer structure is lies between 670–540 nm and the width of the omnidirection range is very large 130 nm.

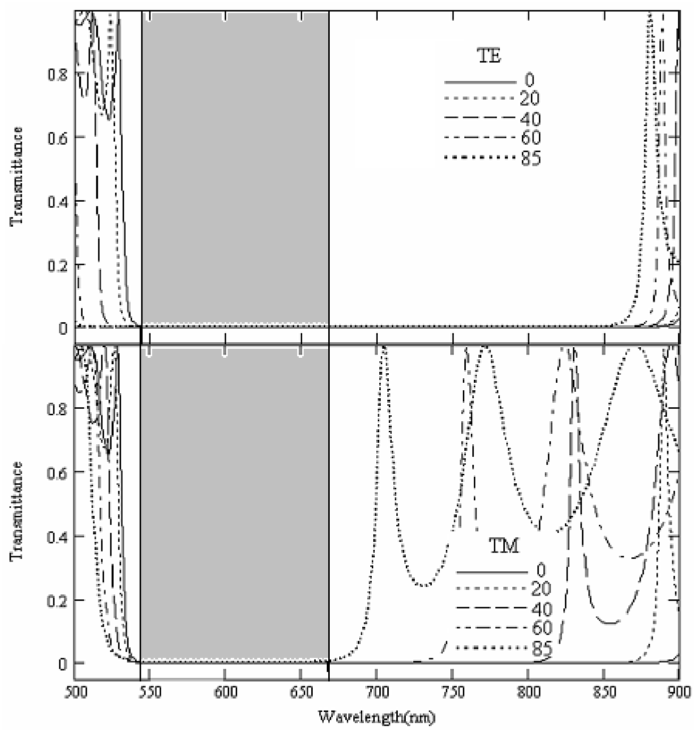

Figure 4. Omnidirectional gap of Na_3AlF_6/Ge multilayer structure for TE and TM mode at different angle.

4. CONCLUSION

In conclusion, we have shown theoretically that a one-dimensional dielectric structure can exhibit total omnidirectional reflection of incident light. Such a structure can be used as a very useful optical device in the optical industry. These findings should stimulate new experiments on controllable spontaneous emission of atoms, molecules and solid state micro-structures in the optical range [18].

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