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EDDY CURRENTS IN SOLID RECTANGULAR CORES

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Abstract—An expression for the eddy current loss in solid rectangular cores is obtained using linear electromagnetic field analysis. Wherefrom text book formula for eddy current loss is derived highlighting various assumptions involved. To get an insight into the current interruption phenomena, electromagnetic fields in a composite rectangular core are analyzed. It is concluded that the reduction in eddy current loss in a laminated cores is basically due to the insertion of distributed capacitors in eddy current paths. Presence of these capacitors increases the impedance of the eddy current path, reducing eddy currents and eddy current loss.

1. INTRODUCTION

Time-varying magnetic fields are established in the core of a coil carrying alternating currents. This may result in eddy currents leading to eddy current loss in the core. Expressions for eddy current loss commonly found in text books [1–3] are derived using lumped circuit approach and assumed eddy current paths. Eddy current loss per unit volume of a thin plate is given by

$$P_e = \frac{\pi^2}{6} B_m^2 f^2 T^2 \sigma \tag{1}$$

where B_m, f, T and σ indicate maximum value of flux density, supply frequency, plate thickness and conductivity respectively.

Since loss density, P_e, is proportional to the square of plate thickness T, it appears that eddy current loss can be reduced if the plate is laminated. However it has been noticed [4] that since Eq. (1) is based on a simplification of actual conditions, it is unreliable. Therefore core loss is generally estimated from curves based on laboratory tests [4].

It is an experimental fact that the eddy current loss in a core with finite cross-section, is reduced if the core is laminated [4, 5]. Fitzgerald et al. [6] observe that magnetic structures are usually built of thin sheet of laminations of the magnetic material. These laminations are insulated from each other. This greatly reduces the magnitude of eddy currents since the layers of insulation "interrupt" the current path. It is often surmised [7–9] that this "interruption" totally blocks eddy currents in one lamination from flowing into the other, thereby altering the shape and size of eddy current paths, thus reducing the eddy current loss.

Many technical papers have been published on electromagnetic transients [10–16] resulting eddy currents. In this paper, using linear electromagnetic field analysis, an expression for the eddy current loss in solid rectangular core subjected to sinusoidal excitation current is found. Wherefrom, based on simplification of the actual conditions, Eq. (1) is developed as a special case,. Also, to glean an insight into the current interruption phenomena, electromagnetic fields in a composite rectangular core are analyzed.

The work reported in the companion paper [17] takes cognizance of the current interruption in laminated cores.

2. HOMOGENEOUS RECTANGULAR CORES

Consider a long magnetic core of width W and thickness T, as shown in Fig. 1. The exciting coil carrying an alternating current

$$i = Ie^{j\omega t} \tag{2}$$

is simulated by a surface current density

$$J_o = I \cdot N \tag{3}$$

where N indicates the number of turns per unit core length.

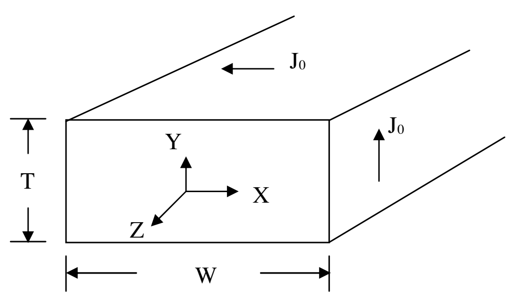

Figure 1. Solid rectangular core.

The current carrying coil will produce a magnetic field H_z. This time varying field will induce eddy currents in the conducting core. The magnetic field outside the coil is neglected. Maxwell's equations for harmonic fields are:

$$\nabla \times E = -j\omega\mu H \tag{4.1}$$

$$\nabla \times H = (\sigma + j\omega\varepsilon)E \tag{4.2}$$

$$\nabla \cdot E = 0 \tag{4.3}$$

$$\nabla \cdot H = 0 \tag{4.4}$$

Therefore, for constant permeability μ:

$$\nabla^2 H = -\gamma^2 H \tag{5}$$

where

$$\gamma = \sqrt{\omega^2\mu\varepsilon - j\omega\mu\sigma} \approx (1-j)\sqrt{\frac{\omega\mu\sigma}{2}} \tag{6}$$

This is a two dimensional problem as fields vary along x- and y-directions only. Thus

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = -\gamma^2 H_z \tag{7}$$

The boundary conditions for the magnetic field H_z, in the core are

$$H_z = J_o, \text{ at } x = \pm W/2, \text{ over } (-T/2) < y < (T/2) \tag{8.1}$$

and

$$H_z = J_o, \text{ at } y = \pm T/2, \text{ over } (-W/2) < x < (W/2) \tag{8.2}$$

Therefore, the solution for Eq. (7) can be given as follows:

$$H_z = \sum_{p=1}^{\infty} a_m \cos\left(\frac{m\pi}{W}x\right) \cdot \frac{\cosh(\alpha_m y)}{\cosh(\alpha_m T/2)} + \sum_{q=1}^{\infty} b_n \cos\left(\frac{n\pi}{T}y\right) \cdot \frac{\cosh(\beta_n x)}{\cosh(\beta_n W/2)} \tag{9}$$

for $(-W/2) < x < (W/2)$ and $(-T/2) < y < (T/2)$

$$\text{where, } \alpha_m = \sqrt{\left(\frac{m\pi}{W}\right)^2 - \gamma^2} \tag{9.1}$$

and

$$\beta_n = \sqrt{\left(\frac{n\pi}{T}\right)^2 - \gamma^2} \tag{9.2}$$

$$m = 2p - 1$$

and

$$n = 2q - 1$$

while, the Fourier coefficients for rectangular waveforms are:

$$a_m = J_o \frac{4}{m\pi} \sin\left(\frac{m\pi}{2}\right) \tag{10.1}$$

and

$$b_n = J_o \frac{4}{n\pi} \sin\left(\frac{n\pi}{2}\right) \tag{10.2}$$

The distribution of eddy current density in the homogeneous core can be found using

$$J = \sigma E \tag{11}$$

where σ indicates the conductivity of the core material. Therefore in view of Eq. (4.2)

$$J = \delta(\nabla \times H) \tag{12}$$

where,

$$\delta = \frac{\sigma}{\sigma + j\omega\varepsilon} \tag{12.1}$$

Thus from Eqs. (9) and (12):

$$\begin{aligned} J_x = & \sum_{p=1}^{\infty} (\delta\alpha_m)\, a_m \cos\left(\frac{m\pi}{W}x\right) \cdot \frac{\sinh(\alpha_m y)}{\cosh(\alpha_m T/2)} \\ & - \sum_{q=1}^{\infty} \left(\delta\frac{n\pi}{T}\right) b_n \sin\left(\frac{n\pi}{T}y\right) \cdot \frac{\cosh(\beta_n x)}{\cosh(\beta_n W/2)} \end{aligned} \tag{13}$$

$$\begin{aligned} J_y = & \sum_{p=1}^{\infty} \left(\delta\frac{m\pi}{W}\right) a_m \sin\left(\frac{m\pi}{W}x\right) \cdot \frac{\cosh(\alpha_m y)}{\cosh(\alpha_m T/2)} \\ & - \sum_{q=1}^{\infty} (\delta\beta_n)\, b_n \cos\left(\frac{n\pi}{T}y\right) \cdot \frac{\sinh(\beta_n x)}{\cosh(\beta_n W/2)} \end{aligned} \tag{14}$$

Next, consider the complex Poynting vector and its components, as given below,

$$P = \frac{1}{2} E \times H^* \tag{15}$$

$$P_x = \frac{1}{2\sigma} J_y H_z^* \tag{15.1}$$

and

$$P_y = -\frac{1}{2\sigma} J_x H_z^* \tag{15.2}$$

Now, eddy current loss per unit core length, P_e is given as the real part of P_c, the complex power per unit core length. While

$$P_c = -2 \int_{-T/2}^{T/2} P_x|_{x=W/2}\, dy - 2 \int_{-W/2}^{W/2} P_y|_{y=T/2}\, dx \tag{16}$$

Using Eqs. (10.1), (10.2), (13) and (14), one gets in view of Eqs. (6), (9.1), (9.2) and, (12.1):

$$P_c = \frac{8}{\pi^2} J_o J_o^* j\omega\mu \left[W \sum_{p=1}^{\infty} \frac{\tanh(\alpha_m T/2)}{m^2 \alpha_m} + T \sum_{q=1}^{\infty} \frac{\tanh(\beta_n W/2)}{n^2 \beta_n} \right] \tag{17}$$

where, $m = 2p - 1$and $n = 2q - 1$.

Therefore the loss density P_e, in the rectangular core is given by:

$$P_e = \frac{8}{\pi^2} J_o J_o^* \omega\mu \left[\begin{array}{l} W \sum\limits_{p=1}^{\infty} \dfrac{\alpha_{mi} \sinh(\alpha_{mr} T) - \alpha_{mr} \sin(\alpha_{mi} T)}{m^2 \left(\alpha_{mr}^2 + \alpha_{mi}^2\right) \left[\cosh(\alpha_{mr} T) + \cos(\alpha_{mi} T)\right]} \\ + T \sum\limits_{q=1}^{\infty} \dfrac{\beta_{ni} \sinh(\beta_{nr} W) - \beta_{nr} \sin(\beta_{ni} W)}{n^2 \left(\beta_{nr}^2 + \beta_{ni}^2\right) \left[\cosh(\beta_{nr} W) + \cos(\beta_{ni} W)\right]} \end{array} \right] \tag{18}$$

where,

$$\alpha_{mr} = \mathrm{Re}[\alpha_m]$$
$$= \frac{1}{\sqrt{2}} \left[\sqrt{\left\{ \left(\frac{m\pi}{W}\right)^2 - \omega^2\mu\varepsilon \right\}^2 + \omega^2\mu^2\sigma^2} + \left\{ \left(\frac{m\pi}{W}\right)^2 - \omega^2\mu\varepsilon \right\} \right]^{\frac{1}{2}} \tag{18.1}$$

$$\alpha_{mi} = \mathrm{Im}[\alpha_m]$$
$$= \frac{1}{\sqrt{2}} \left[\sqrt{\left\{ \left(\frac{m\pi}{W}\right)^2 - \omega^2\mu\varepsilon \right\}^2 + \omega^2\mu^2\sigma^2} - \left\{ \left(\frac{m\pi}{W}\right)^2 - \omega^2\mu\varepsilon \right\} \right]^{\frac{1}{2}} \tag{18.2}$$

$$\beta_{nr} = \mathrm{Re}[\beta_n]$$
$$= \frac{1}{\sqrt{2}} \left[\sqrt{\left\{ \left(\frac{n\pi}{T}\right)^2 - \omega^2\mu\varepsilon \right\}^2 + \omega^2\mu^2\sigma^2} + \left\{ \left(\frac{n\pi}{T}\right)^2 - \omega^2\mu\varepsilon \right\} \right]^{\frac{1}{2}} \tag{18.3}$$

$$\beta_{ni} = \mathrm{Im}\left[\beta_n\right] = \frac{1}{\sqrt{2}}\left[\sqrt{\left\{\left(\frac{n\pi}{T}\right)^2 - \omega^2\mu\varepsilon\right\}^2 + \omega^2\mu^2\sigma^2} - \left\{\left(\frac{n\pi}{T}\right)^2 - \omega^2\mu\varepsilon\right\}\right]^{\frac{1}{2}} \quad (18.4)$$

The expression for eddy current loss per unit core length given in Eq. (18) is quite involved. This can, however, be simplified by noting that the hyperbolic functions are usually much larger than sinusoidal functions. Thus for large values of $(\alpha_{mr} \cdot T)$ and $(\beta_{nr} \cdot W)$, on setting tan hyperbolic functions to unity:

$$P_e \approx \frac{8}{\pi^2} J_o J_o^* j\omega\mu \left[W \sum_{p=1}^{\infty} \frac{\alpha_{mi}}{m^2 (\alpha_{mr}^2 + \alpha_{mi}^2)} + T \sum_{q=1}^{\infty} \frac{\beta_{ni}}{n^2 (\beta_{nr}^2 + \beta_{ni}^2)} \right] \quad (19)$$

where, $m = 2p - 1$ and $n = 2q - 1$.

A further simplification is possible if both (π/W) and (π/T) are large compared to $\sqrt{\omega\mu\sigma}$. Therefore for small values of σ, in view of Eqs. (18.1)–(18.4), Eq. (19) reduces to:

$$P_e \approx \frac{4}{\pi^5} S J_o J_o^* \omega^2 \mu^2 \sigma \left[W^4 + T^4 \right] \quad (20)$$

where,

$$S = \sum_{p=1}^{\infty} \frac{1}{m^5} \quad (20.1)$$

where, $m = 2p - 1$.

The value of S found from tables [18, 19] is

$$S = 1.00452 \approx 1 \quad (20.2)$$

Alternatively for large values of σ, only the first terms in the two fast converging infinite series involved in Eq. (19) could be retained. Then, from Eqs. (9.1) and (9.2):

$$\alpha_1 \approx \beta_1 \approx j\gamma \quad (21)$$

Thus, using Eq. (6)

$$\alpha_1 \approx \beta_1 \approx (1 + j) \sqrt{\frac{\omega\mu\sigma}{2}} \quad (22)$$

Therefore from Eq. (19), for large values of σ

$$P_e \approx \frac{4\sqrt{2}}{\pi^2} J_o J_o^* \sqrt{\frac{\omega\mu}{\sigma}}\,[W+T] \tag{23}$$

A large conducting plate of thickness T, can be considered as a special case of the rectangular core with its width, W, tending to infinity. While, the eddy current loss per unit plate volume, p_e, is given by

$$p_e = (P_e/WT)|_{w\to\infty} \tag{24}$$

Therefore, in view of Eqs. (9.1), (6), (18) and (24), one gets:

$$p_e \approx J_o J_o^* \sqrt{\frac{\omega\mu}{2\sigma}} \left[\frac{1}{T} \frac{\sinh\left(\sqrt{\frac{\omega\mu\sigma}{2}}\cdot T\right) - \sin\left(\sqrt{\frac{\omega\mu\sigma}{2}}\cdot T\right)}{\cosh\left(\sqrt{\frac{\omega\mu\sigma}{2}}T\right) + \cos\left(\sqrt{\frac{\omega\mu\sigma}{2}}T\right)} \right] \tag{25}$$

Therefore, for thick plates

$$p_e \approx J_o J_o^* \sqrt{\frac{\omega\mu}{2\sigma}} \left[\frac{1}{T}\right] \tag{25.1}$$

while for thin plates,

$$p_e \approx J_o J_o^* \frac{\sigma}{24} \omega^2 \mu^2 T^2 \tag{25.2}$$

This leads to Eq. (1), on substituting $2\pi f$ for ω and B_m^2 for $\mu^2 J_o J_o^*$.

3. COMPOSITE MAGNETIC CORES

Let the homogeneous core in Fig. 1 be replaced by a composite core, made up of two different materials and placed symmetrically inside the current carrying coil, as shown in Fig. 2.

Three core-regions can be identified, viz, region-1 (or central region) for $(-T_1/2) < y < (T_1/2)$; region-2 (or top region) for $(T_1/2) < y < (T/2)$ and region-3 (or bottom region) for $(-T/2) < y < (-T_1/2)$. Each region extends over $(-W/2) < x < (W/2)$. In view of symmetry it will be sufficient to consider, say, the first two regions. We shall use suffix-1, to indicate region 1 and suffix-2, to indicate region-2.

Noting that H_{1z} is an even function of x and y; further, it satisfies Eqs. (7) and (8.1). Let

$$H_{1z} = \sum_{p=1}^{\infty} c'_m \cos\left(\frac{m\pi}{W}x\right) \cdot \frac{\cosh(\alpha_{1m} y)}{\cosh(\alpha_{1m} T/2)}$$

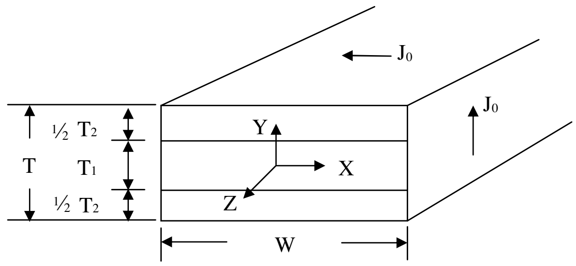

Figure 2. Composite magnetic core.

$$+\sum_{q=1}^{\infty} c''_n \cos\left(\frac{n\pi}{T_1}y\right) \cdot \frac{\cosh\left(\beta_{1n}x\right)}{\cosh\left(\beta_{1n}W/2\right)} \tag{26}$$

where, in view of Eq. (7)

$$\alpha_{1m} = \sqrt{\left(\frac{m\pi}{W}\right)^2 - \gamma_1^2} \tag{26.1}$$

$$\beta_{1n} = \sqrt{\left(\frac{n\pi}{T_1}\right)^2 - \gamma_1^2} \tag{26.2}$$

$$\gamma_1 = \sqrt{(-j\omega\mu_1)(\sigma_1 + j\omega\varepsilon_1)} \tag{26.3}$$

and in view of Eq. (8.1)

$$c''_n = J_o \frac{4}{n\pi} \cdot \sin\left(\frac{n\pi}{2}\right) \tag{26.4}$$

while c'_m indicate a set of arbitrary constants and $m = 2p - 1$ and $n = 2q - 1$.

Now, H_{2z} is an even function of x only. It satisfies Eqs. (7) and (8.1). Further,

$$H_{2z} = H_{1z}, \text{ at } y = T_1/2 \text{ over } (-W/2) < x < (W/2) \tag{27.1}$$

and

$$H_{2z} = J_o, \text{ at } y = T/2 \text{ over } (-W/2) < x < (W/2) \tag{27.2}$$

Therefore,

$$H_{2z} = \sum_{p=1}^{\infty} \cos\left(\frac{m\pi}{W}x\right)\left[d_m \frac{\sinh \alpha_{2m}(y - T_1/2)}{\sinh(\alpha_{2m}T_2/2)} - c'_m \frac{\sinh \alpha_{2m}(y - T/2)}{\sinh(\alpha_{2m}T_2/2)}\right]$$

$$+\sum_{q=1}^{\infty} c''_n \cos\frac{n2\pi}{T_2}\left(y-\frac{T_1}{2}-\frac{T_2}{4}\right)\cdot\frac{\cosh(\beta_{2n}x)}{\cosh(\beta_{2n}W/2)} \tag{28}$$

where, in view of Eq. (7),

$$\alpha_{2m} = \sqrt{\left(\frac{m\pi}{W}\right)^2-\gamma_2^2} \tag{28.1}$$

$$\beta_{2n} = \sqrt{\left(\frac{n2\pi}{T_2}\right)^2-\gamma_2^2} \tag{28.2}$$

$$\gamma_2 = \sqrt{(-j\omega\mu_2)(\sigma_2+j\omega\varepsilon_2)} \tag{28.3}$$

and in view of Eq. (27.2)

$$d_m = J_o\frac{4}{m\pi}\sin\left(\frac{m\pi}{2}\right) \tag{28.4}$$

and $m = 2p-1,\ n = 2q-1$.

To find the arbitrary constant c'_m, consider the distribution of electric field in the two regions. In view of Eqs. (12), (12.1), (26) and (28), the distribution of eddy current density in region-1 is obtained as:

$$\begin{aligned} J_{1x} =& \sum_{p=1}^{\infty}(\delta_1\alpha_{1m})\,c'_m\cos\left(\frac{m\pi x}{W}\right)\frac{\sinh(\alpha_{1m}y)}{\cosh\left(\alpha_{1m}\frac{T_1}{2}\right)} \\ &-\sum_{q=1}^{\infty}\left(\delta_1\frac{n\pi}{T_1}\right)c''_n\sin\left(\frac{n\pi y}{T_1}\right)\frac{\cosh(\beta_{1n}x)}{\cosh\left(\beta_{1n}\frac{W}{2}\right)} \end{aligned} \tag{29.1}$$

and

$$\begin{aligned} J_{1y} =& \sum_{p=1}^{\infty}\left(\delta_1\frac{m\pi}{W}\right)c'_m\sin\left(\frac{m\pi x}{W}\right)\frac{\cosh(\alpha_{1m}y)}{\cosh\left(\alpha_{1m}\frac{T_1}{2}\right)} \\ &-\sum_{q=1}^{\infty}(\delta_1\beta_{1n})c''_n\cos\left(\frac{n\pi y}{T_1}\right)\frac{\sinh(\beta_{1n}x)}{\cosh\left(\beta_{1n}\frac{W}{2}\right)} \end{aligned} \tag{29.2}$$

where, $m = 2p-1,\ n = 2q-1$ and

$$\delta_1 = \frac{\sigma_1}{(\sigma_1+j\omega\varepsilon_1)} \tag{29.3}$$

and in region-2, as:

$$J_{2x} = \sum_{p=1}^{\infty} (\delta_2 \alpha_{2m}) \cos\left(\frac{m\pi x}{W}\right) \times \left[d_m \frac{\cosh \alpha_{2m} \left(y - \frac{T_1}{2}\right)}{\sinh\left(\alpha_{2m} \frac{T_2}{2}\right)} - c'_m \frac{\cosh \alpha_{2m} \left(y - \frac{T}{2}\right)}{\sinh\left(\alpha_{2m} \frac{T_2}{2}\right)} \right] - \sum_{q=1}^{\infty} \left(\delta_2 \frac{n2\pi}{T_2}\right) c''_n \sin \frac{n2\pi}{T_2} \left(y - \frac{T_1}{2} - \frac{T_2}{4}\right) \frac{\cosh(\beta_{2n} x)}{\cosh\left(\beta_{2n} \frac{W}{2}\right)} \quad (30.1)$$

and

$$J_{2y} = \sum_{p=1}^{\infty} \left(\delta_2 \frac{m\pi}{W}\right) \sin\left(\frac{m\pi x}{W}\right) \times \left[d_m \frac{\sinh \alpha_{2m} \left(y - \frac{T_1}{2}\right)}{\sinh\left(\alpha_{2m} \frac{T_2}{2}\right)} - c'_m \frac{\sinh \alpha_{2m} \left(y - \frac{T}{2}\right)}{\sinh\left(\alpha_{2m} \frac{T_2}{2}\right)} \right] - \sum_{q=1}^{\infty} (\delta_2 \beta_{2n}) c''_n \cos \frac{n2\pi}{T_2} \left(y - \frac{T_1}{2} - \frac{T_2}{4}\right) \frac{\sinh(\beta_{2n} x)}{\cosh\left(\beta_{2n} \frac{W}{2}\right)} \quad (30.2)$$

where, $m = 2p - 1$, $n = 2q - 1$ and

$$\delta_2 = \frac{\sigma_2}{(\sigma_2 + j\omega\varepsilon_2)} \quad (30.3)$$

Now, since,

$$\frac{J_{1x}}{\sigma_1} = \frac{J_{2x}}{\sigma_2}, \text{ at } y = T_1/2, \text{ over } (-W/2) < x < (W/2) \quad (31)$$

Eqs. (29.1) and (30.1) give,

$$\sum_{p=1}^{\infty} \left(\frac{\delta_1}{\sigma_1} \alpha_{1m}\right) \tanh(\alpha_{1m} T_1/2)\, c'_m \cos\left(\frac{m\pi}{W} x\right) - \sum_{q=1}^{\infty} \left(\frac{\delta_1}{\sigma_1} \frac{n\pi}{T_1}\right) \sin(n\pi/2)\, c''_n \frac{\cosh(\beta_{1n} x)}{\cosh(\beta_{1n} W/2)}$$

$$= \sum_{p=1}^{\infty} \left(\frac{\delta_2}{\sigma_2}\alpha_{2m}\right)\left[d_m \mathrm{cosech}\,(\alpha_{2m}T_2/2) - c'_m \coth(\alpha_{2m}T_2/2)\right] \cos\left(\frac{m\pi}{W}x\right)$$
$$+ \sum_{q=1}^{\infty} \left(\frac{\delta_2}{\sigma_2}\frac{n2\pi}{T_2}\right) \sin(n\pi/2)\, c''_n \frac{\cosh(\beta_{2n}x)}{\cosh(\beta_{2n}W/2)}$$
$$\text{over } (-W/2) < x < (W/2) \qquad (32)$$

and where, $m = 2p - 1$, $n = 2q - 1$.

Considering the Fourier series expansion:

$$\frac{\cosh(\beta_n x)}{\cosh(\beta_n W/2)} = \sum_{p=1}^{\infty} \left[\frac{4}{W} \frac{\frac{m\pi}{W}\sin\left(\frac{m\pi}{2}\right)}{\left(\frac{m\pi}{W}\right)^2 + \beta_n^2}\right] \cos\left(\frac{m\pi}{W}x\right) \qquad (33)$$

over $(-W/2) < x < (W/2)$ and where, $m = 2p - 1$.

We get from Eq. (32)

$$\left[\left(\frac{\delta_1}{\sigma_1}\alpha_{1m}\right)\tanh(\alpha_{1m}T_1/2) + \left(\frac{\delta_2}{\sigma_2}\alpha_{2m}\right)\coth(\alpha_{2m}T_2/2)\right] c'_m$$
$$= \left(\frac{\delta_2}{\sigma_2}\alpha_{2m}\right)\mathrm{cosec}\,(\alpha_{2m}T_2/2)\, d_m + \frac{4}{W}\frac{m\pi}{W}\sin\left(\frac{m\pi}{2}\right)$$
$$\left[\left(\frac{\delta_1}{\sigma_1}\frac{1}{T_1}\right)\sum_{q=1}^{\infty}\frac{n\pi\sin\left(\frac{n\pi}{2}\right)}{\beta_{1n}^2 + \left(\frac{m\pi}{W}\right)^2} c''_n + \left(\frac{\delta_2}{\sigma_2}\frac{2}{T_2}\right)\sum_{q=1}^{\infty}\frac{n\pi\sin\left(\frac{n\pi}{2}\right)}{\beta_{2n}^2 + \left(\frac{m\pi}{W}\right)^2} c''_n\right] \qquad (34)$$

where, $n = 2q - 1$.

Now, in view of Eqs. (26.1), (26.2) and (26.4)

$$\sum_{q=1}^{\infty}\frac{n\pi\sin\left(\frac{n\pi}{2}\right)}{\beta_{1n}^2 + \left(\frac{m\pi}{W}\right)^2} c''_n = \sum_{n-odd}^{\infty}\frac{4J_o\,(T_1/\pi)^2}{n^2 + \left(\frac{T_1}{\pi}\alpha_{1m}\right)^2} \qquad (34.1)$$

where, $n = 2q - 1$ and in view of Eqs. (28.1), (28.2) and (26.4)

$$\sum_{q=1}^{\infty}\frac{n\pi\sin\left(\frac{n\pi}{2}\right)}{\beta_{2n}^2 + \left(\frac{m\pi}{W}\right)^2} c''_n = \sum_{q=1}^{\infty}\frac{4J_o\,(T_2/2\pi)^2}{n^2 + \left(\frac{T_2}{2\pi}\alpha_{2m}\right)^2} \qquad (34.2)$$

where, $n = 2q - 1$.

These infinite series can be summed up [11, 12]. Thus, using the identity:

$$\sum_{q=1}^{\infty} \frac{1}{n^2 + \theta^2} \equiv \frac{\pi}{4\theta} \tanh\left(\frac{\pi}{2}\theta\right) \tag{35}$$

where, $n = 2q-1$ and Eqs. (34) and (28.4), the expression for c'_m found as:

$$c'_m = \frac{\left[\begin{array}{l} \frac{4}{W} J_o \sin\left(\frac{m\pi}{2}\right) \frac{\delta_1}{\sigma_1} \left\{\frac{m\pi/W}{\alpha_{1m}} \tanh(\alpha_{1m} T_1/2)\right\} \\ + \frac{4}{W} J_o \sin\left(\frac{m\pi}{2}\right) \frac{\delta_2}{\sigma_2} \left\{\frac{m\pi/W}{\alpha_{2m}} \tanh(\alpha_{2m} T_2/4)\right. \\ \left. + \frac{\alpha_{2m}}{m\pi/W} \operatorname{cosech}(\alpha_{2m} T_2/2)\right\} \end{array}\right]}{\left[\left(\frac{\delta_1}{\sigma_1}\alpha_{1m}\right) \tanh(\alpha_{1m} T_1/2) + \left(\frac{\delta_2}{\sigma_2}\alpha_{2m}\right) \coth(\alpha_{2m} T_2/2)\right]} \tag{36}$$

4. DISCUSSION

Expressions for eddy current loss in the solid rectangular core are given by eqns. (18), (19), (20) and (23). From each of these equations it can be concluded that for a given core area and coil current, the loss density is minimum for a core with square cross section. Incidentally, for each metre of coil length, both the amount of copper in the coil as well as the copper loss will also be minimum in the case of a square cross section.

For the rectangular core, the eddy current density, J_y, is given by eqn. (14). This component of eddy current density vanishes as the core width, W, tends to infinity. Therefore, in large plates, eddy currents flow parallel to the plate surfaces. Thus if the plate is laminated, eddy currents are not interrupted. However, because of non-zero thickness of interlaminar insulation, laminating large plates alters the distribution of eddy currents in the plate volume.

Consider the composite core shown in Fig. 2. It may be seen from Eqs. (29.2) and (30.2), that the normal component of both, the displacement- and the conduction-current densities are continuous at the boundary between the two adjacent regions, provided that the relaxation times for these regions are identical.

In general, these current densities are discontinuous at the interface of two adjacent regions. This indicates the existence of a distribution of time-varying surface charge density on the interface.

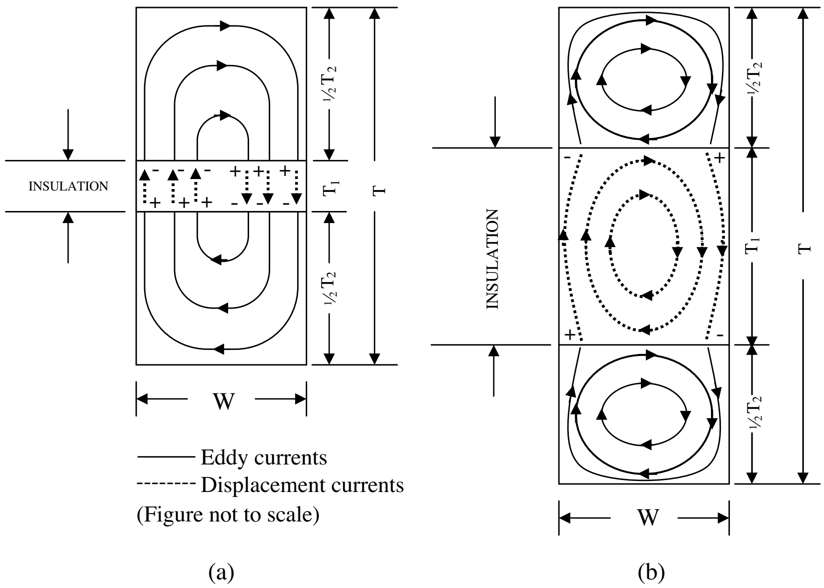

Figure 3. Cross-sectional view of composite cores with perfect insulation. (a) Small insulation thickness, (b) Large insulation thickness.

In view of eqns. (29.1), (29.2) and (29.3), there will be no eddy currents in region-1 when the conductivity of this region, s1, is zero. Eddy currents in region-2 and -3, flowing in open paths deposit charges on the two interfaces between conducting and non-conducting regions, vide Fig. 3. The region-1 provides a distributed capacitance in the eddy current paths. For a non-zero conductivity of region-1, σ_1, eddy currents flow through leaky capacitance. It is therefore, concluded that even with perfect interlaminar insulation, eddy currents in a lamination are not totally restrained from flowing into another.

5. CONCLUSION

The presence of capacitance in the eddy current path increases the impedance of the path. Thus reducing eddy currents and eddy current loss for a given core flux.

An application of the theory discussed here, is presented in the companion paper [[17]. Lastly, it may be pointed out that the treatment given in this paper can be readily adapted for cores made of left-handed materials with simultaneously negative permittivity and permeability [20–22].

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