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AN IMPROVED PARTICLE SWARM OPTIMIZATION ALGORITHM FOR PATTERN SYNTHESIS OF PHASED ARRAYS

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Abstract—In this paper an improved particle swarm optimization algorithm (IPSO) for electromagnetic applications is proposed. In order to overcome the drawbacks of standard PSO, some improved mechanisms for velocity updating, the exceeding boundary control, global best perturbation and the simplified quadratic interpolation (SQI) operator are adopted. To show the effectiveness of the proposed algorithm, a selected set of numerical examples, concerned with linear as well as planar array, is presented. Simulation results show that the refined pinpointing search ability and the global search ability of the proposed algorithm are significantly improved when compared to the particle swarm optimization (PSO) and Genetic Algorithm (GA).

1. INTRODUCTION

In recent years there has been a growing interest in the design and application of phased-arrays of remote sensing radar systems [1, 2], military and commercial communication equipments [3]. However, the global synthesis of antenna arrays that generate a desired radiation pattern is a highly nonlinear optimization problem. Therefore, analytical methods are not applicable any more. Several classical methodologies for the phased-array control have been proposed aimed at defining suitable strategies for the optimal synthesis of antenna arrays [4–7]. It was shown in [8–27] that the evolutional optimization algorithms such as the genetic algorithm, ant colony optimization, particle swarm optimization and colony selection algorithm are capable of performing better and more flexible solutions than the classical optimization algorithms and the conventional analytical approaches. Nevertheless, these algorithms have been used with their own benefits and limitations in the antenna array pattern synthesis.

The standard PSO has difficulties in controlling the balance between exploration and exploitation because it tends to favor the intensification search around the "better" solutions previously found [28]. In such a context, the PSO appears to be lacking global search ability and its computational efficiency for a refined local search to pinpoint the exact optimal solution is not satisfactory [29]. Consequently, an improved particle swarm optimization is presented. Some improved mechanisms such as the design of a novel formula for velocity updating, the exceeding boundary control and global best perturbation are adopted to enhance the performance of PSO. Moreover, the simplified quadratic interpolation operator is integrated into it to improve the local search ability of PSO and to refine the pinpoint search ability. The simulation results show the effectiveness of the algorithm in synthesizing a linear array and a planar array.

2. PROBLEMS FORMULATION

The far-field radiation pattern $FF(\theta,\phi)$ is given by

$$FF(\theta,\phi) = EP(\theta) \cdot AF(\theta,\phi) \tag{1}$$

where $AF(\theta,\phi)$ is the array factor. Realistic phased arrays using aperture elements such as slots or patches typically have directive elements which give rise to an element pattern often approximated by [8]

$$EP(\theta) = \sqrt{\cos^n(\theta)} \tag{2}$$

where $n = 0$ would represent ideal isotropic array elements and $n > 0$ represents directive array elements. In this paper, n is taken to be 1.2.

For a linear array of $2N_x$ elements symmetrically placed along the y-axis, the array factor is:

$$AF(\theta,\phi) = 2\sum_{m=1}^{N_x} I_{xm}\cos\left(kx_m\sin\theta\cos\phi + \beta_m\right) \tag{3}$$

where k is the wave number and I_m, β_m and x_m are the excitation amplitude, phase and position of the mth element respectively. If $2N_y$

of such linear arrays are placed next to each other in the x direction, a rectangular array will be formed. Considering the case of symmetric excitation, the entire array factor can be written as

$$AF(\theta,\phi) = 4\sum_{m=1}^{N_x} I_{xm}\cos(kx_m\sin\theta\cos\phi)\sum_{n=1}^{N_y} I_{yn}\cos(ky_n\sin\theta\sin\phi) \tag{4}$$

It is typically desired that the magnitude $|FF(\theta,\phi)|$ of the far-field pattern remain bounded between some specified limits as

$$|FF_{\min\lim it}(\theta,\phi)| \leq |FF(\theta,\phi)| \leq |FF_{\max\lim it}(\theta,\phi)| \tag{5}$$

and a good cost measure to be minimized is defined as

$$Fitness = \sum_{m=1}^{M} \max\left(|FF(\theta_m,\phi)| - \left|FF_{\max\text{limit}}(\theta_m,\phi)\right|, 0\right)^2 \Big/ M + \sum_{m=1}^{M} \max\left(\left|FF_{\min\text{limit}}(\theta_m,\phi)\right| - |FF(\theta_m,\phi)|, 0\right)^2 \Big/ M \tag{6}$$

where $|FF(\theta_m,\phi)|$ are the far-field pattern values from (1) evaluated at M far-field angles θ_m, and $|FF_{\min\lim it}(\theta_m,\phi)|$ and $|FF_{\max\lim it}(\theta_m,\phi)|$ are the corresponding desired lower and upper bounds on the antenna pattern, respectively. In the linear array pattern synthesis, the θ_m are evenly spaced in sine space as

$$\sin(\theta_m) = 2\frac{m-1}{M-1} - 1, \qquad m = 1,\ldots,M \tag{7}$$

3. PSO ALGORITHM

3.1. Standard PSO Algorithm

The PSO algorithm is an evolutionary algorithm capable of solving difficult multidimensional optimization problems in various fields. As an evolutionary algorithm, the PSO algorithm depends on the social interaction between independent particles, during their search for the optimum solution. A population of particles is randomly generated initially. Each particle represents a potential solution and has a position represented by a position vector $\vec{x}_i$. A swarm of particles moves through the problem space, with the moving velocity of each particle represented by a velocity vector $\vec{v}_i$. Each particle keeps track of its own best position, which is associated with the best fitness it has achieved so far in a vector $\vec{p}_i$. Furthermore, the best position among all the particles obtained so far in the population is kept track of as $\vec{p}_g$. The particle's velocity update and position update are the main PSO operators, which can be expressed as:

$$\vec{v}_i(\tau+1) = w\vec{v}_i(\tau) + c_1 r_1 (\vec{p}_i(\tau) - \vec{x}_i(\tau)) + c_2 r_2 (\vec{p}_g(\tau) - \vec{x}_i(\tau)) \tag{8}$$

$$\vec{x}_i(\tau+1) = \vec{x}_i(\tau) + \vec{v}_i(\tau+1) \tag{9}$$

where c_1 and c_2 are acceleration constants and r_1 and r_2 are uniformly distributed random numbers in [0,1]. The term $\vec{v}_i$ is limited to its bounds. If the velocity violates this limit, it is set to its proper limit. w is the inertia weight factor and in general, it is set according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{T} \cdot \tau \tag{10}$$

where $w_{\max}$ and $w_{\min}$ is maximum and minimum value of the weighting factor respectively. T is the maximum number of iterations and τ is the current iteration number.

3.2. Improved Particle Swarm Optimization(IPSO)

To enhance the refined pinpointing search ability and to strike a balance between exploration and exploitation of available PSOs, the following improvements are proposed.

3.2.1. Velocity Updating

As seen from (8), since the two random parameters r_1 and r_2 are independently generated, inevitably there are cases in which they are both too large or too small. In the former, both the personal and social experiences accumulated so far are overused and the particle might be driven away from the local optimum. For the latter case, both the personal and social experiences are not fully used, so the particle might not be able to find the local optimum. Therefore, the convergence performance of the algorithm is undermined. In other words, the two random weighting parameters reflecting the experiences of his own and his companions are not completely independent. By modeling this reasoning ability into an updating formula and noting the sum of the two inter-related weighting parameters can be set to 1, one single random parameter that includes the cognitive and social experiences of the particle for updating its velocity is proposed. Therefore, the velocity is updated by using

$$\vec{v}_i(\tau+1) = w\vec{v}_i(\tau) + c_1 r_1 (\vec{p}_i(\tau) - \vec{x}_i(\tau)) + c_2(1-r_1)(\vec{p}_g(\tau) - \vec{x}_i(\tau)) \tag{11}$$

It should be noted that the communication between different particles in the proposed search procedure is set up in a hierarchical manner rather than a hierarchical manner.

3.2.2. Exceeding Boundary Control

In updating the position of particles using (9), it is very common to find the coordinates of the new particles lying outside the boundaries of the parameter space. In such cases, the popular approaches used in available PSO algorithms are either to take the boundaries as the coordinates of the new particles, or to keep the coordinates of the particle unchanged but to assign the particle with an extremely poor objective function value. However, either treatment will reduce the diversity of the particles in the searching process and reduce the global search ability of the algorithm correspondingly. Therefore, in the improved PSO algorithm, a different approach is proposed in that if a new particle moves outside the boundaries, the current velocity of the particle in question is modified using

$$\vec{v}_i(\tau)_{new} = -\frac{d}{D} \cdot \vec{v}_i(\tau) \tag{12}$$

where d is the distance between the particle and its violation boundary, and D is its variation range. It is obvious that the velocity is determined by the violation distance, the variation range and the previous velocity. Then, it can improve the diversity of the particles in the searching process and the global search ability of the algorithm to some extent.

3.2.3. Global Best Perturbation

The particle swarm optimization (PSO) algorithm has been shown to converge rapidly during the initial stages of a global search, but around global optimum, the search will become very slow. For that reason, the global best perturbation operator is defined as follows: the global best can be expressed as: $\vec{p}_g = (p_1 \dots \bar{p}_k \dots p_n)$

where

$$\bar{p}_k \in \left\{ \max\left(p_k - \mu\frac{p_k^{\max} - p_k^{\min}}{2}, p_k^{\min}\right), \min\left(p_k + \mu\frac{p_k^{\max} - p_k^{\min}}{2}, p_k^{\max}\right) \right\} \tag{13}$$

$p_k^{\max}$, $p_k^{\min}$ are upper and lower bounds of p_k respectively, μ is the perturbation parameter, which decreases with the increase of iterations.

$$\mu(\tau) = 1 - r^{[1-(\tau/T)]^b} \tag{14}$$

where r is uniform random number in [0,1], T is the maximum number of iterations, τ is the current iteration number, and b is the shape parameter. From (13), it can be seen that at the initial stage of evolution, for small value of r, $\mu(\tau) \approx 1$, the variation range is large in this case. However, in the later evolution, when τ approaches T, $\mu(\tau) \approx 0$, the variation range becomes small and the particles search in the local domain. By using formulas (13) and (14), the stagnant globalbest may be activated again so that it has a higher probability to find the global best.

3.2.4. A Simplified Quadratic Interpolation Operator

As a powerful local search operator, the three-point quadratic interpolation method is integrated into PSO in order to improve its local search ability and avoid its premature convergence.

Denote three particles by $x^a = [x_1^a, \dots, x_n^a]^T$, $x^b = [x_1^b, \dots, x_n^b]^T$, $x^c = [x_1^c, \dots, x_n^c]^T$, and calculate their fitness values $f_a = fit\,(x^a)$, $f_b = fit\left(x^b\right)$, $f_c = fit\,(x^c)$. Suppose that $f_a > f_b$ and $f_c > f_b$, then the approximate minimal point $\bar{x} = [\bar{x}_1, \dots, \bar{x}_n]^T$ derived from the three-point quadratic interpolation is expressed as follows:

$$\bar{x}_i = \frac{1}{2}\left\{ \frac{\left[\left(x_i^b\right)^2 - (x_i^c)^2\right] f_a + \left[(x_i^c)^2 - (x_i^a)^2\right] f_b + \left[(x_i^a)^2 - \left(x_i^b\right)^2\right] f_c}{\left(x_i^b - x_i^c\right) f_a + \left(x_i^c - x_i^a\right) f_b + \left(x_i^a - x_i^b\right) f_c} \right\}, \quad i = 1, \dots, n \tag{15}$$

The sequential steps of the proposed algorithm are given below.

Step 1: Randomly initialize the population of P individuals within the variable constraint range.

Step 2: Calculate the fitness of the population from the fitness function.

Step 3: Compare each individual's evaluation value with its own best $\vec{p}_i(\tau)$ evolved so far. The best evaluation value among the $\vec{p}_i(\tau)$ is denoted as $\vec{p}_g(\tau)$.

Step 4: Modify the velocity of each individual according to Eqs. (11) and (9).

Step 5: Check the velocity components, if the position violates the boundaries, then modify it according to Eqs. (12).

Step 6: Local search

Step 6.1: Compare the fitness values of all the individuals, order ascendingly and choose the three best individuals, denoted by x^b, x^a, and x^c, respectively, such that $f_b = fit\left(x^b\right) \leq f_a = fit\left(x^a\right) \leq f_c = fit\left(x^c\right)$.

Step 6.2: For some $i \in \{1, 2, \ldots, n\}$, if $\left(x_i^b - x_i^c\right) f_a + \left(x_i^c - x_i^a\right) f_b + \left(x_i^a - x_i^b\right) f_c < \varepsilon \left(\varepsilon = 10^{-5}\right)$, then let $\bar{x} = x^b$, and $fit\left(\bar{x}\right) = x^b$, go to Step 6.4, otherwise, go to Step 6.3.

Step 6.3: Calculate $\bar{x}$ by using Eqs. (15), and then calculate the fitness value $fit\left(\bar{x}\right)$.

Step 6.4: If $fit\left(\bar{x}\right) < f_b$, then replace the worst solution in current population with $\bar{x}$, otherwise, replace the worst solution in current population with x^b.

Step 7: Perturb the global best individual according to Eqs. (13) and (14).

Step 8: Repeat steps 2–7 until a stopping criterion, such as a sufficiently good solution being discovered or a maximum number of generations being completed, is satisfied. The best scoring individual in the population is taken as the final answer.

4. NUMERICAL RESULTS

The capabilities of the proposed hybrid algorithm will be assessed by presenting the results of a linear array and a planar array. In order to demonstrate the superiority of IPSO, in each simulation, the performance of IPSO is compared with standard PSO and GA. The parameters used in PSO and GA are selected the same as those used in IPSO, which ensure a fair comparison in computation efficiency and solution quality.

4.1. Synthesis of a Linear Array

We consider a 100 elements half-wavelength-spaced linear array, The excitation amplitude distribution is symmetric with respect to the center of the array. Because of symmetry, only 50 amplitudes are to be optimized.

For design specifications, the following parameters have been selected for the proposed algorithm: population size = 80; generation = 2000; inertia weight factor w is set by Eqs. (10), where $w_{\max} = 0.9$ and $w_{\min} = 0.4$; acceleration constant $c_1 = 2.0$ and $c_2 = 2.0$. Fig. 1 shows the normalized absolute power pattern in dB and the common amplitude distribution obtained by IPSO. Fig. 2 shows the normalized absolute power pattern in dB obtained by PSO and GA. It is clear that the pattern obtained by PSO and GA cannot satisfy the specifications. It can be found from Table 1 that IPSO procedure possesses the dominant speed and precision in the optimization compared with other algorithms.

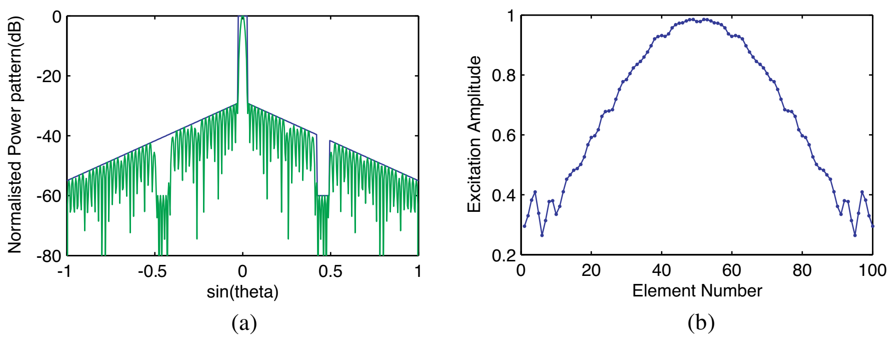

Figure 1. (a) Radiation pattern for the linear antenna array using IPSO, (b) excitation amplitude distributions for the linear antenna array by amplitude-only synthesis.

Table 1. Performance comparisons for different methods of linear array design.

Method	Best fitness value	Cost function evaluations
GA	0.0339	160000
PSO	0.0447	160000
IPSO	0.0109	20480

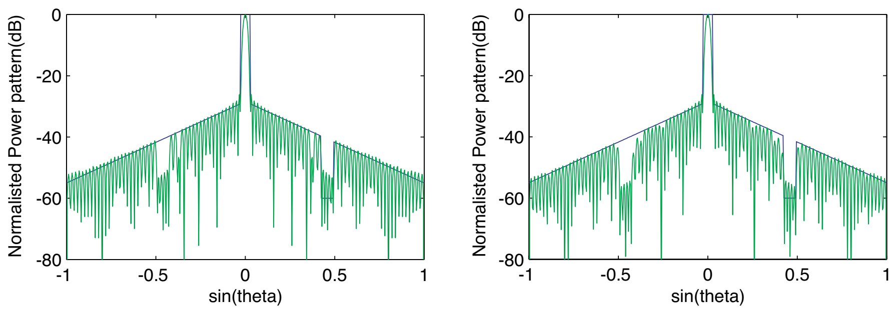

Figure 2. (a) Radiation pattern for the linear antenna array using PSO, (b) radiation pattern for the linear antenna array using GA.

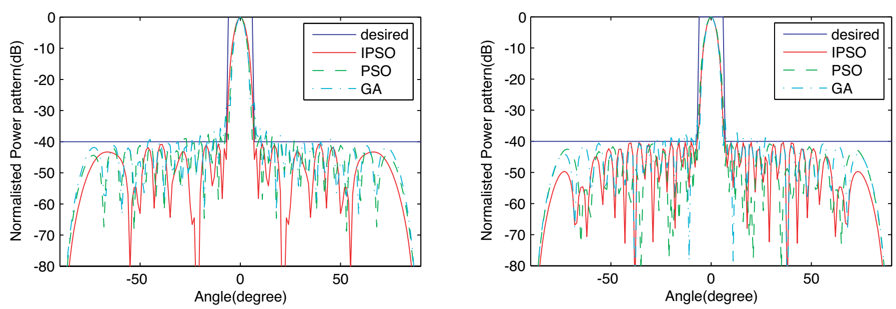

Figure 3. (a) Radiation pattern for the planar antenna array in the x-z plane, (b) radiation pattern for the planar antenna array in the y-z plane.

4.2. Synthesis of a Planar Array

The second experiment is concerned with a 32×32 half-wavelength-spaced rectangular planar array. It is desired that the sidelobe level should be lower than -40 dB. In applying IPSO, initially, 50 individuals are randomly generated in a population. The rest parameters are the same as those in the first example. Fig. 3(a) shows the normalized absolute power patterns in the x-z plane in dB. Fig. 3(b) shows the normalized absolute power patterns in the y-z plane in dB. The maximum sidelobe level obtained by IPSO in x-z plane is -40.6721 dB, and the maximum sidelobe level in y-z plane is -40.1442 dB. While the maximum sidelobe level obtained by PSO in x-z plane and y-z plane is -31.8530 dB and -40.7734 dB, respectively. The pattern obtained

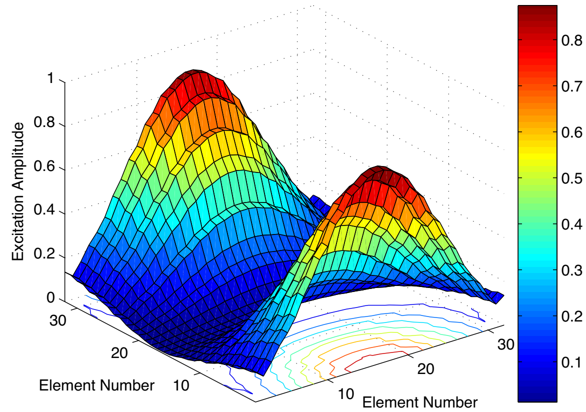

Figure 4. Excitation amplitude distributions for the planar antenna array by amplitude-only synthesis.

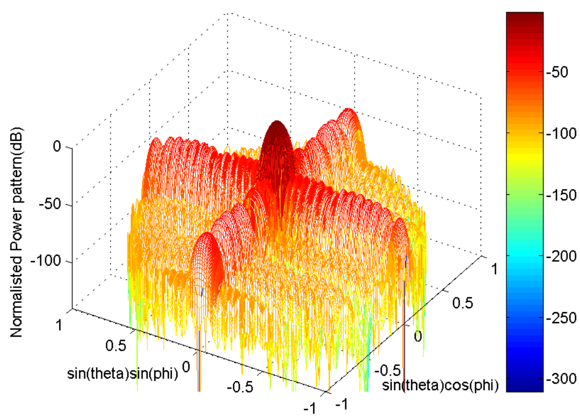

Figure 5. Three-dimensional radiation pattern for the planar antenna array.

by GA is the worst, the maximum sidelobe of which is -27.5543 dB and -32.7536 dB. Compared with the same uniform planar array, the maximum sidelobe of which is -13.27 dB, the result obtained by IPSO has been reduced by more than 27 dB. Fig. 4 shows the optimized distribution by IPSO. Fig. 5 shows the normalized absolute three-dimensional power pattern in dB. It can be seen that there is no high sidelobe in other directions and the width of mainlobe is not widened. It can also be concluded that the proposed algorithm can give optimal solution within a faster convergent speed.

The results shown in Figs. 1–5 confirm that the IPSO presented in this work is able to approach the desired pattern by controlling only the element amplitudes of the antenna array. While the standard PSO and GA show their limitations and inefficiency to some extent, respectively. Since the evaluation of the cost function tends to dominate the overall computation budget for electromagnetic optimization, a key factor is the number of iterations and function evaluations. From the tables 1–2, it is obvious that the proposed algorithm has less cost function evaluations compared with other algorithms. Hence, the efficiency of the proposed algorithm is proved.

Table 2. Performance comparisons for different methods of planar antenna array.

Method	Best fitness value	Cost function evaluations
GA	0.9853	33750
PSO	0.8149	35500
IPSO	0.0000	13300

5. CONCLUSIONS

This paper illustrates the use of improved particle swarm optimization algorithm in the pattern synthesis of phased arrays for the purpose of suppressed sidelobe in certain regions and null placement in prescribed directions. By adopting some approaches, the global search ability and the refined pinpointing search ability of available PSO have been enhanced. The numerical results, as reported in this paper, suggest that the proposed algorithm is successful in synthesizing linear array and planar array. By comparing with PSO and GA, it demonstrates the superiority of IPSO in higher convergence accuracy and fewer cost function evaluations. As for the future work, the authors will strive to develop an adaptive algorithm to use the information of the design problem acquired during the course of the search process for tuning the parameters automatically and apply the algorithm to other electromagnetic problems.

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