

FDTD HARD SOURCE AND SOFT SOURCE REVIEWS AND MODIFICATIONS

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Abstract—We intend to study two kinds of sources known as Hard Source and Soft Source, which are the simplest kinds of sources used in FDTD. We introduce an exact method to control the direction of propagation of a plane wave which can be either a hard source or soft source. Also we show that a simple implementation of a soft source in the Yee algorithm causes an undesired propagation which can result in incorrect answers. To remove this error we suggest a method for different kinds of sources and modes.

1. INTRODUCTION

After the release of Yee's paper on FDTD in 1966 [1], a lot of effort has been paid to develop this novel method. Many people have strengthened the base of this method and lots of applications have been developed through the FDTD method [2, Ch. 1, 3, 4, 5, Ch. 1]. FDTD is an algorithm suitable for use with a computer and can be used as a real-time solver of electromagnetic problems [5, Ch. 1, 6, 7].

It is necessary that one define some basic elements in an analytical electromagnetic problem; in the FDTD method one should do the same, too. These elements are: 1) the basic equations which are Maxwell's equations, 2) spatial and temporal grids, 3) constitutive parameters which include permittivity, permeability, electric and magnetic conductivities, etc., and 4) sources [8, Sec. 2.1]. The first item is split to extract basic equations used in the Yee algorithm [1]. The second item is used to separate time and space in order to interleave Maxwell's equations in space and time. The third item defines the medium and boundary conditions [9, Ch. 1]. The last one, which we

are going to discuss, is used to simulate a physical source or excite a structure to obtain the desired response [5, Ch. 5].

There have been different kinds of sources introduced and analyzed (e.g., point wise $\vec{E}$ and $\vec{H}$ hard source, $\vec{J}$ and $\vec{M}$ current source, the plane-wave source and waveguide source) [5, Ch. 5].

The basic sources called *Hard Source* and *Soft Source* are the simplest kinds of source used in the FDTD method. These sources are formed by setting electric and magnetic fields to the amount of source¹ at proper positions [5, Ch. 5], [10, Ch. 1]. A hard source can simulate a physical source like a probe exciting a waveguide [5, Sec. 5.2]. A soft source may be used to excite a structure; for example to find the resonance frequencies of a cavity one can use a pulsed hard source to excite the cavity [5, Sec. 5.2], but this can be done easily by using a soft source instead of the pulsed hard source.

Although these kinds of sources are simple and applicable, there are some problems with them.

The first problem is the matter of determining the direction of propagation of a plane wave. We will introduce a method to specify the direction of plane wave propagation in a medium for any polarization (e.g., TEM, TM, TE, etc.). The propagation along any direction besides propagation along major axes will be discussed too. Note that our method is different from the *TF/SF* (Total Field and Scattered Field) method [11].

The second problem is the error existing in amplitude and shape of a soft source. The amplitude and shape of a soft source is dependent on the spatial and temporal grids used in the simulation. We show that using soft sources without correction leads to incorrect results. We suggest a simple method to eliminate this error.

To unify the results, we use the same basic Parameters necessary for the FDTD method in the scripts. This Parameters are available in Appendix A.

2. THE YEE ALGORITHM'S NOTATION

Throughout this paper we assume that the simulation's environment is free space ($\epsilon_r = 1$, $\mu_r = 1$, $\sigma = 0$, $\sigma^* = 0$). Also we use a notation which will ease the way we manipulate the equations.

Suppose there is a TEM plane wave propagating along the z axis and polarized in the x direction. The interleaved equations can be obtained from [8] as below:

¹ Hard source assignment means setting the field to the amount of source, whereas soft source means adding the field to the amount of source.

$$E_x^{n+\frac{1}{2}} \left(K + \frac{1}{2} \right) = E_x^{n-\frac{1}{2}} \left(K + \frac{1}{2} \right) + \frac{\Delta t}{\varepsilon_0 \Delta z} \left[H_y^n (K) - H_y^n (K+1) \right], \quad (1)$$

$$H_y^{n+1} (K+1) = H_y^n (K+1) + \frac{\Delta t}{\mu_0 \Delta z} \left[E_x^{n+\frac{1}{2}} \left(K + \frac{1}{2} \right) - E_x^{n+\frac{1}{2}} \left(K + \frac{3}{2} \right) \right]. \quad (2)$$

Knowing the definition of the speed of light in free space ($c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$), the intrinsic impedance of free space ($\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$) [9], and the Courant number² ($S = \frac{c \Delta t}{\Delta z}$) [5, Ch. 4], we can write the following equations:

$$\frac{\Delta t}{\varepsilon_0 \Delta z} = \frac{\Delta t \sqrt{\mu_0}}{\sqrt{\mu_0} \sqrt{\varepsilon_0} \sqrt{\varepsilon_0} \Delta z} = \frac{c \Delta t}{\Delta z} \eta = S \eta, \quad (3)$$

$$\frac{\Delta t}{\mu_0 \Delta z} = \frac{\Delta t \sqrt{\varepsilon_0}}{\sqrt{\varepsilon_0} \sqrt{\mu_0} \sqrt{\mu_0} \Delta z} = \frac{c \Delta t}{\Delta z} \frac{1}{\eta} = \frac{S}{\eta}. \quad (4)$$

Substituting (3) and (4) in (1) and (2) respectively, results in:

$$E_x^{n+\frac{1}{2}} \left(K + \frac{1}{2} \right) = E_x^{n-\frac{1}{2}} \left(K + \frac{1}{2} \right) + S \eta \left[H_y^n (K) - H_y^n (K+1) \right], \quad (5)$$

$$H_y^{n+1} (K+1) = H_y^n (K+1) + \frac{S}{\eta} \left[E_x^{n+\frac{1}{2}} \left(K + \frac{1}{2} \right) - E_x^{n+\frac{1}{2}} \left(K + \frac{3}{2} \right) \right]. \quad (6)$$

Looking at (5) and (6), it is clear that to write the equations for the electric fields one should multiply the expressions of the magnetic fields by $S\eta$, and to write the equations for the magnetic fields one should multiply the expressions of the electric fields by $\frac{S}{\eta}$. We will use the same notation throughout this paper. We can apply the same notation for higher order dimensions too, but for simplicity when dealing with higher order dimensions, we suppose $\Delta x = \Delta y = \Delta z$. For example a TM_z case [10] will be:

² This definition is correct for one dimension case.

$$E_z^{n+\frac{1}{2}}\left(i - \frac{1}{2}, j + \frac{1}{2}\right) = E_z^{n-\frac{1}{2}}\left(i - \frac{1}{2}, j + \frac{1}{2}\right) + S\eta \left[\begin{aligned} &H_y^n\left(i, j + \frac{1}{2}\right) - H_y^n\left(i - 1, j + \frac{1}{2}\right) + \\ &H_x^n\left(i - \frac{1}{2}, j\right) - H_x^n\left(i - \frac{1}{2}, j + 1\right) \end{aligned} \right], \quad (7)$$

$$H_x^{n+1}\left(i - \frac{1}{2}, j + 1\right) = H_x^n\left(i - \frac{1}{2}, j + 1\right) + \frac{S}{\eta} \left[\begin{aligned} &E_z^{n+\frac{1}{2}}\left(i - \frac{1}{2}, j + \frac{1}{2}\right) \\ &- E_z^{n+\frac{1}{2}}\left(i - \frac{1}{2}, j + \frac{3}{2}\right) \end{aligned} \right], \quad (8)$$

$$H_y^{n+1}\left(i, j + \frac{1}{2}\right) = H_y^n\left(i, j + \frac{1}{2}\right) + \frac{S}{\eta} \left[\begin{aligned} &E_z^{n+\frac{1}{2}}\left(i + \frac{1}{2}, j + \frac{1}{2}\right) \\ &- E_z^{n+\frac{1}{2}}\left(i - \frac{1}{2}, j + \frac{1}{2}\right) \end{aligned} \right]. \quad (9)$$

3. DIRECTION OF PROPAGATION OF A PLANE WAVE HARD SOURCE

3.1. TEM Mode Plane Wave Hard Source

We begin with a TEM case whose equations were written in the new notation (see (5) and (6)). Suppose there is a hard source emitting an electric field at the point K_s propagating in the positive z direction³.

To show the accuracy of propagation and to compare the results, we also perform a simultaneous simulation propagating bilaterally in both $-z$ and $+z$ directions. For simplicity suppose the source is a unit step which is zero for $n < n_s$ and is one for $n > n_s$. Figure 1 shows the field configuration for different time steps after applying the source.

If there has been no source and excitation for $n < n_s$ thus:

$$\begin{aligned} E_x^{n+\frac{1}{2}}\left(K + \frac{1}{2}\right) &\equiv 0, \quad \text{for } n < n_s \\ H_y^{n+1}(K + 1) &\equiv 0 \quad \text{every } K. \end{aligned} \quad (10)$$

According to the FDTD algorithm, after applying (5), one should assign the electric sources, and then should apply (6), and finally assign the magnetic sources. This cycle is repeated again and again as

³ Note that the index s refers to the source.

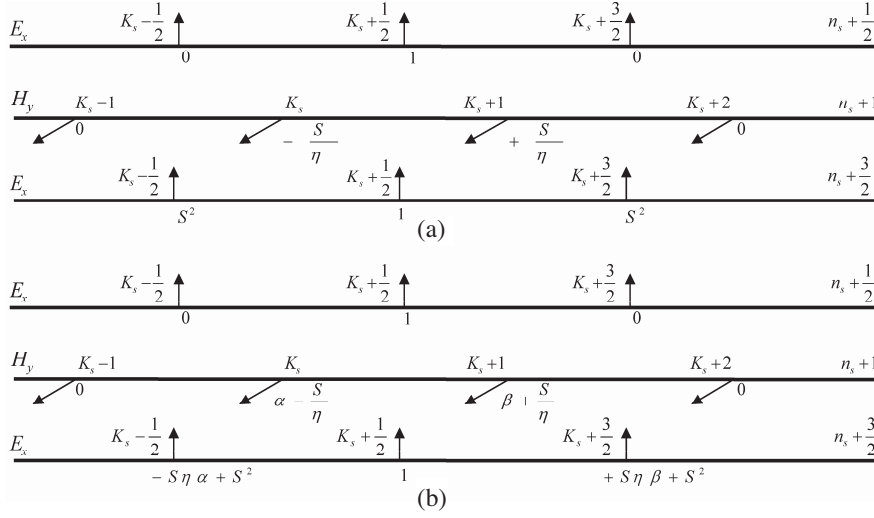


Figure 1. Fields configuration for one dimensional propagation of a hard source at point K_s for different time steps after applying the source (TEM mode) (a) Hard source propagating bilaterally in both $-z$ and $+z$ directions (b) Hard source propagating in the $+z$ direction.

desired [10, Ch. 1]. With this concept, after applying the electric source at the point K_s and the time step n_s , the algorithm of the bilaterally propagating source is as below:

$$E_x^{n_s+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = 0. \quad (11)$$

Source assignment is

$$E_x^{n_s+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = 1. \quad (12)$$

Relation (11) is obtained by substituting the parameters into (5). We use (6) to calculate magnetic fields at the positions of K_s and $K_s + 1$ and the time step of $n_s + 1$:

$$H_y^{n_s+1} (K_s) = -\frac{S}{\eta}, \quad (13)$$

$$H_y^{n_s+1} (K_s + 1) = \frac{S}{\eta}. \quad (14)$$

Calculating the electric fields at $K_s - \frac{1}{2}$, $K_s + \frac{1}{2}$ and $K_s + \frac{3}{2}$ gives:

$$E_x^{n_s+\frac{3}{2}}\left(K_s - \frac{1}{2}\right) = S^2, \quad (15)$$

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{1}{2}\right) = 1 - 2S^2, \quad (16)$$

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{3}{2}\right) = S^2. \quad (17)$$

And source assignment is

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{1}{2}\right) = 1. \quad (18)$$

The fields calculated during these steps are illustrated in Fig. 1(a). Note that we did not calculate the fields at other positions because they are definitely zero.

Now we do the same procedure to the algorithm of oriented propagation, for $n > n_s$:

$$E_x^{n_s+\frac{1}{2}}\left(K_s + \frac{1}{2}\right) = 0, \quad (19)$$

And source assignment is

$$E_x^{n_s+\frac{1}{2}}\left(K_s + \frac{1}{2}\right) = 1. \quad (20)$$

We apply the magnetic source just after and before the position of the electric source (i.e., we apply the magnetic sources at K_s and $K_s + 1$). The reason will be clear very soon. We apply theses sources as below:

$$H_y^{n_s}(K_s) = \alpha \times 1 = \alpha, \quad (21)$$

$$H_y^{n_s}(K_s + 1) = \beta \times 1 = \beta. \quad (22)$$

where α and β are unknown coefficients to be decided. Note the time step index; we have not already calculated the magnetic fields at $n_s + 1$, so we choose the time index n_s ⁴.

⁴ This notation does not cause any consistency in our work. One can use $n_s + 1$ but the magnetic source assignments must be done before calculating the magnetic field by (6) and both the electric and magnetic source assignments must be done at the same time.

We calculate the magnetic fields at the positions K_s and $K_s + 1$ and the time $n_s + 1$:

$$H_y^{n_s+1}(K_s) = \alpha - \frac{S}{\eta}, \quad (23)$$

$$H_y^{n_s+1}(K_s + 1) = \beta + \frac{S}{\eta}. \quad (24)$$

If we follow the algorithm, the electric field will be:

$$E_x^{n_s+\frac{3}{2}}\left(K_s - \frac{1}{2}\right) = -S\alpha + S^2, \quad (25)$$

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{1}{2}\right) = 1 - 2S^2 + S(\alpha - \beta), \quad (26)$$

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{3}{2}\right) = S\beta + S^2. \quad (27)$$

And source assignment is

$$E_x^{n_s+\frac{3}{2}}\left(K_s + \frac{1}{2}\right) = 1. \quad (28)$$

The fields calculated during these steps are illustrated in Fig. 1(b). We can decide coefficients α and β according to the direction of propagation. If the plane wave is propagating in the positive z direction any fields for $K < K_s + \frac{1}{2}$ must be zero. It is true when $\alpha = \frac{S}{\eta}$, and to avoid disturbing non-source produced wave propagating in the negative z direction, (22) should not be applied. If propagation in the negative z direction is desired, then we must set $\beta = -\frac{S}{\eta}$ and (21) should not be applied. The results of simulation of both cases are compared to the case of bilateral propagation in Fig. 2.

Of course one can decide the source definitions by knowing the directions of field components. For example, suppose propagation along positive z is desired. In this case the magnetic component before the source position is in $-y$ direction or we can interpret that it has a negative value. So to eliminate the propagation along negative z , one can set this component to positive value of what it is expected to be. To explain more, after setting the electric field to 1, the magnetic field just before it, would be $-\frac{S}{\eta}$. So setting this magnetic field to $\frac{S}{\eta}$ is enough to eliminate the propagation along negative z .

Note the usage of a sinusoidal source in the simulation. Although we approach the results with the concept that the source is unit step, these results are correct for any kinds of sources.

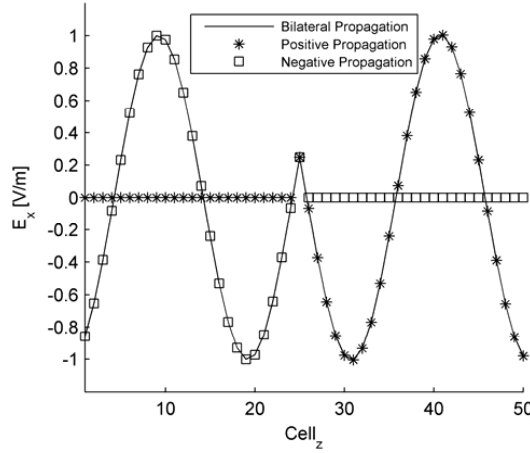


Figure 2. Propagation of a hard source in the bilateral, positive and negative directions (Courant number = 0.6, TEM mode).

3.2. Procedure of Orienting a Plane Wave Hard Source

To explain a general case, where we want to control a plane wave propagation direction; assume propagation along the $\vec{u}$ direction is desired. The source assignment can be $E_u(P_s) = Source$. To control the direction of propagation, the magnetic sources must be assigned just before or after the point P_s (see Fig. 3(a)). One can write them as $H_u(P_{s-1}) = \alpha \times Source$ or $H_u(P_{s+1}) = \beta \times Source$.

For example in the TM_z case, when the propagation of a plane wave with z electric field component along the x direction is needed, the magnetic component would be the y magnetic component Fig. 3(b).

To decide the direction of propagation one can follow Yee's algorithm for some time steps, then by arranging the electric field before or after⁵ the point P_s and setting it to zero, one can find the source assignments while the other source assignments to avoid interference with other propagations will be ignored. This can be done more easily by using the analytical method explained in Section 3.1.

This procedure is correct for any polarization and direction. For example, if we do the same procedure to a plane wave propagating in the x direction in TM_z mode, the z electric field component and y magnetic field component are as below:

⁵ Note that according to the desired direction of propagation one of these magnetic fields must be chosen. For example if the propagation in the positive (negative) u direction is desired $H_u(P_{s-1})$ ($H_u(P_{s+1})$) is chosen.

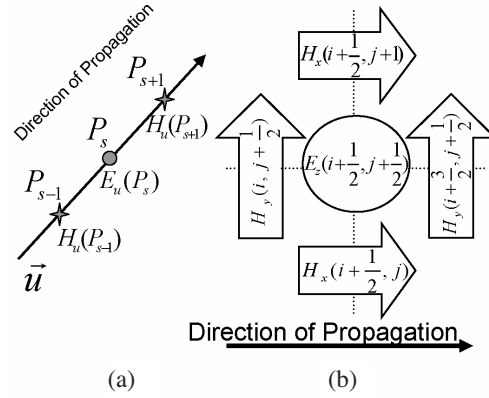


Figure 3. Field configuration used to explain the basic idea of determining the direction of propagation (a) Electric source and adjacent magnetic fields laid along the direction of propagation (b) Electric source and adjacent x magnetic field component which lay along the direction of propagation (TM_z mode).

For propagation in the negative x direction, it must be:

$$\begin{aligned} E_z^{n+\frac{1}{2}}\left(i_s + \frac{1}{2}, j + \frac{1}{2}\right) &= Source^n, \quad \text{for every } n \\ H_y^n\left(i_s + 1, j + \frac{1}{2}\right) &= +\frac{S}{\eta} Source^n, \quad \text{and } j. \end{aligned} \quad (29)$$

For propagation in the positive x direction, it must be:

$$\begin{aligned} E_z^{n+\frac{1}{2}}\left(i_s + \frac{1}{2}, j + \frac{1}{2}\right) &= Source^n, \quad \text{for every } n \\ H_y^n\left(i_s, j + \frac{1}{2}\right) &= -\frac{S}{\eta} Source^n, \quad \text{and } j. \end{aligned} \quad (30)$$

Note that these source assignments should be done at the same time.

3.3. Orienting an Oblique Plane Wave Hard Source

The method of orienting a plane wave is correct even when the direction of propagation is oblique. The first step is finding the source position indices so that they provide an equiphase and smooth wave front. The second step is setting the electric fields at the source positions to the amount of the source. The last step is setting the magnetic

sources adjacent to the electric source in an appropriate way so that the appropriate direction of propagation results.

Consider a TM_z oblique plane wave which is propagating at the angle $0 \leq \phi \leq 90$ relative to the positive x axis. To avoid propagation in the area behind the source positions one must set both H_x and H_y fields to proper amounts. It means that the procedure of orienting the source propagation (discussed in Section 3.2) must be done to H_x and H_y , separately. If the indices of the source positions are expressed by $j_s = f(i_s)$, the electric and magnetic source assignments will be:

$$E_s^{n+\frac{1}{2}} \left(i_s + \frac{1}{2}, j_s + \frac{1}{2} \right) = Source^n, \quad \text{for every } n \quad (31)$$

$$H_x^n \left(i_s + \frac{1}{2}, j_s \right) = +\frac{S}{\eta} Source^n, \quad \text{and } j$$

$$H_y^n \left(i_s, j_s + \frac{1}{2} \right) = -\frac{S}{\eta} Source^n, \quad \text{and } j_s = f(i_s).$$

We perform a TM_z plane wave propagation at angle $\phi = 55$. The source positions are chosen so that the resulted wave front would be smooth. The method *Look-Up Table* [5, Sec. 5.7.2] is used to define the source. This method beside *Interpolation of Look-Up Table Data* [5, Sec. 5.7.2] can improve the resulting wave. Figure 4 shows the result of the simulation. The existing wave behind the source positions is because of the leakage at the interface of PMLs [12].

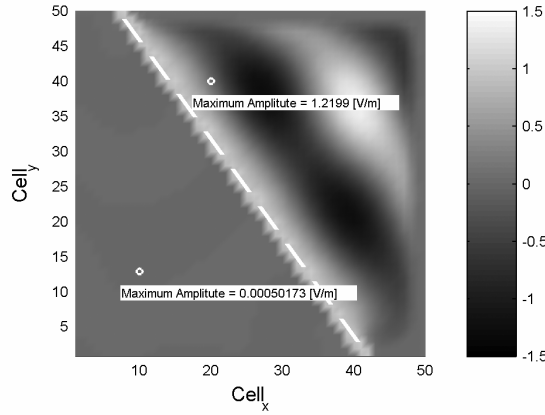


Figure 4. Simulation result of a directed oblique plane wave at the angle of propagation $\phi = 55$ and $S = 0.5$; electric field amount distributed over the simulation area (TM_z mode).

If the hard source is a magnetic source, one can follow the instructions discussed, considering that source assignments must be done just after calculating the electric fields.

4. PLANE WAVE SOFT SOURCE

4.1. TEM Mode Plane Wave Soft Source

We show that a simple soft source assignment does not result in a correct answer and then suggest a method to get rid of this problem. Consider TEM propagation in the positive z direction and polarized in the x direction. The soft source assignment is as below:

$$E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) + Source^n. \quad (32)$$

A script is written to simulate this sort of source and sample the electric field at the source position. The source is a sinusoidal one. The Courant number is changed from 0.1 to 1. In this range the simulation is stable [5, Ch. 4, 13]. Figure 5 shows the amplitude of the sampled field with respect to the Courant number.

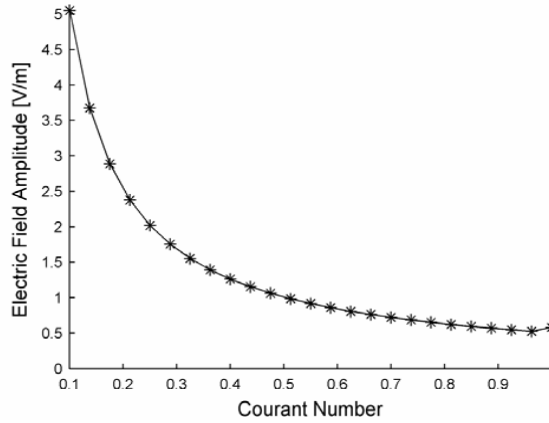


Figure 5. Amplitude of the sampled electric field at the position of a soft source as a function of the Courant number (TEM mode).

It is evident that only when $S = 0.5$, the amplitude is almost correct and in other cases the amplitude is not correct. To correct this problem we use the following source assignment instead of (32):

$$E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) + \alpha \times S \times Source^n, \quad (33)$$

where α is the unknown parameter. We follow Yee's algorithm and assume the source is a unit step applied at $n = 0$. We used a script which is able to manipulate the symbolic variables. We calculated the electric field at the source position after 5 time steps. The calculated electric field is:

$$E_z^{5+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = \alpha \times S \left(5S - 40S^3 + 126S^5 - 160S^7 + 70S^9 \right). \quad (34)$$

Note that the polynomial obtained from the script is a function of the Courant number (S). Expression (34) is the electric field after source assignment. The electric field is to be equal to the source, so the following equation must be held:

$$E_z^{5+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = \alpha \times S \left(5S - 40S^3 + 126S^5 - 160S^7 + 70S^9 \right) = 1. \quad (35)$$

It gives:

$$\alpha = \frac{1}{S \left(5S - 40S^3 + 126S^5 - 160S^7 + 70S^9 \right)}. \quad (36)$$

Note that the range of S is between 0 and 1. Calculating the average of the fraction in (36), the result is $\alpha = 1.9998$. We performed the same simulation, but this time the parameter n is changed. The results are illustrated in Fig. 6(a).

As we see, when n increases, α approaches 2, so the coefficient can be 2. To make sure, this time a script is written to calculate this coefficient for different amount of Courant numbers. A sinusoidal source is applied and the electric field is sampled at the source point. The result is shown in Fig. 6(b). The numerical result proves the conclusion obtained by the symbolic analysis. Although we used a step or sinusoidal source to get these results, using other sources. Simulating the plane wave in a 2-D or 3-D medium does not change the coefficients.

4.2. Oriented Plane Wave Soft Source

The only remaining point is to control the direction of propagation of a plane wave soft source. After performing analyses as mentioned in Section 4.1, both symbolic and numeric, the results will be the same as Fig. 7.

Note that the source is supposed to be propagating in the positive z direction so the source assignments would be:

$$E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) = E_x^{n+\frac{1}{2}} \left(K_s + \frac{1}{2} \right) + \alpha \times S \times Source^n \quad (37)$$

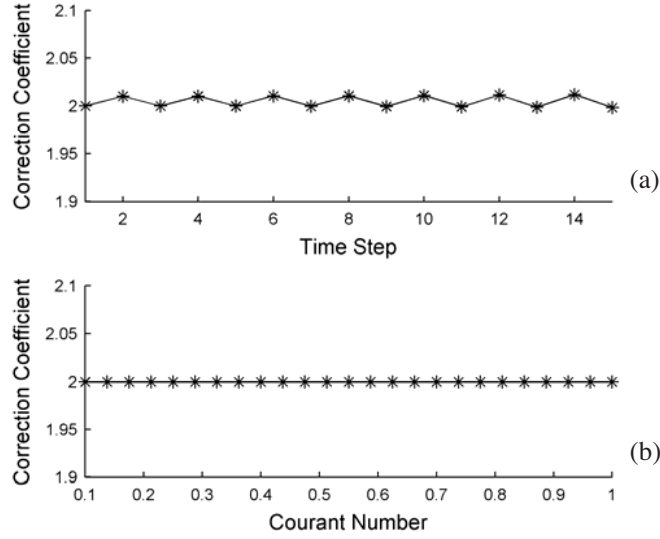


Figure 6. Analyzed results of the correction coefficient for a bilateral soft source (TEM mode) (a) Result of a symbolic analysis of the correction coefficient α with respect to the time step number (b) Result of a symbolic analysis of the correction coefficient with respect to the Courant number.

$$H_y^n(K_s) = H_y^n(K_s) + \alpha \times \frac{S}{\eta} \times Source^n.$$

To change the direction of propagation, we simply use the following relations:

$$E_x^{n+\frac{1}{2}}\left(K_s + \frac{1}{2}\right) = E_x^{n+\frac{1}{2}}\left(K_s + \frac{1}{2}\right) + \alpha \times S \times Source^n \quad (38)$$

$$H_y^n(K_s + 1) = H_y^n(K_s + 1) - \alpha \times \frac{S}{\eta} \times Source^n.$$

It is clear from Fig. 7 that the coefficient needed to correct the propagation is half of that obtained in previous case where the wave was propagating bilaterally. Again these results are correct for 2-D and 3-D cases too. Note that the source types will not change the results.

5. TM_z MODE SINUSOIDAL SOFT SOURCE

We show that using a simple soft source assignment in two dimensions simulations, leads to wrong answers. Then, we suggest some correction

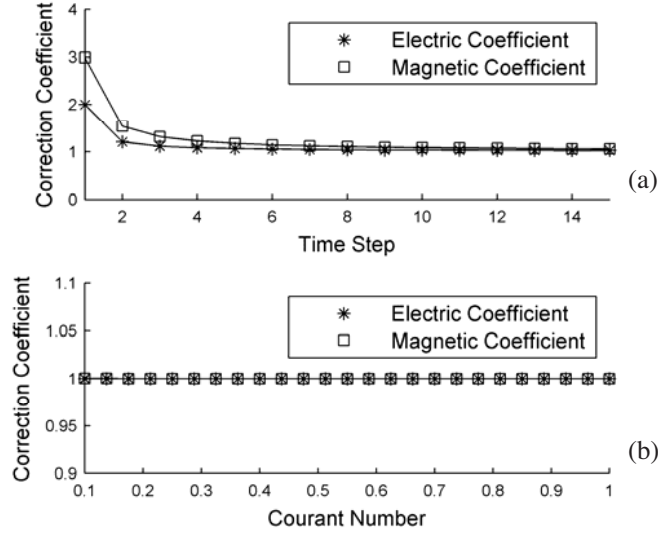


Figure 7. Analysis results of the correction coefficient for a positive direction soft source (TEM mode) (a) Result of symbolic analysis of the correction coefficient α with respect to the time step number (b) Result of a symbolic analysis of the correction coefficient with respect to the Courant number.

to remove this inaccuracy. We begin with a sinusoidal source located at the center of a 2-D area surrounded by PMLs. The TM_z mode is considered. The electric field is sampled at the source position. The source assignment is as below:

$$E_z^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) = E_z^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) + Source^n. \quad (39)$$

One can see the amplitude of sampled field with respect to the Courant number curve in Fig. 8(a). Note that the range of S is between 0.1 and $\frac{1}{\sqrt{2}}$, so that the simulation would be stable [5, Ch 4, 13].

The procedure used here is the same as that used for a plane wave. The only difference is, that because of PMLs, we split the E_z field component into E_{zx} and E_{zy} , and the source assignments are as below:

$$E_{zx}^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) = E_{zx}^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) + \frac{\alpha}{2} \times S \times Source^n \quad (40)$$

$$E_{zy}^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) = E_{zy}^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) + \frac{\alpha}{2} \times S \times Source^n.$$

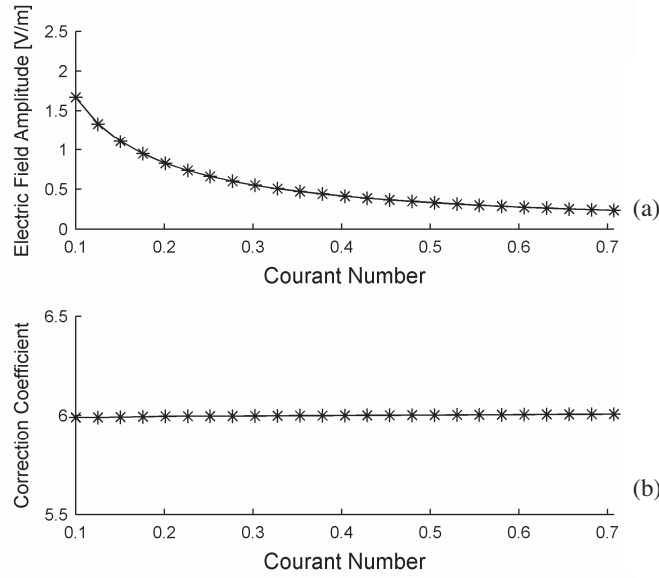


Figure 8. Analysis results for a cylindrical sinusoidal soft source (TM_z mode) (a) Amplitude of the sampled electric field with respect to the Courant number (b) Correction coefficient α with respect to the Courant number.

We just performed the numerical simulation. The coefficient with respect to the Courant number is shown in Fig. 8(b). It is clear that $\alpha = 6$ and we note that if one does not want to split the E_z field component, he should use the source assignment as below:

$$E_z^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) = E_z^{n+\frac{1}{2}}\left(i_s - \frac{1}{2}, j_s + \frac{1}{2}\right) + \alpha \times S \times Source^n. \quad (41)$$

Note that this result is correct only for a sinusoidal source.

6. TM_z GENERALIZED SOFT SOURCE

In previous section we studied a simple method which corrects soft source problem. But that method works well with sinusoidal sources. In this section we want to introduce a method which is applicable for all kinds of sources.

In this method there are two distinct source propagation simulations. One is the reference propagation, in which a hard source is placed at source position.

The other one is the main simulation where the boundary conditions are assigned as electromagnetic problem dictates. But instead of placing a source at source position, the effect of source existence is placed.

Consider a Gaussian pulse propagating in TM_z mode. The source is applied to electric component. We perform two distinct simulations mentioned before. To apply the effect of source in the second simulation, we should consider roles of H_x and H_y in calculating E_z . Relation (7) shows these roles.

Assume that soft source assignment is done as below:

$$E_z^{n+\frac{1}{2}}\left(i-\frac{1}{2}, j+\frac{1}{2}\right) = E_z^{n-\frac{1}{2}}\left(i-\frac{1}{2}, j+\frac{1}{2}\right) + \alpha S \eta \left[\hat{H}_y^n\left(i, j+\frac{1}{2}\right) - \hat{H}_y^n\left(i-1, j+\frac{1}{2}\right) + \hat{H}_x^n\left(i-\frac{1}{2}, j\right) - \hat{H}_x^n\left(i-\frac{1}{2}, j+1\right) \right] \quad (42)$$

where $\hat{H}_x$ and $\hat{H}_y$ denotes the fields obtained in reference propagation simulation. The only unknown parameter would be α . Setting the source to unit step for all times, one can decide this coefficient by

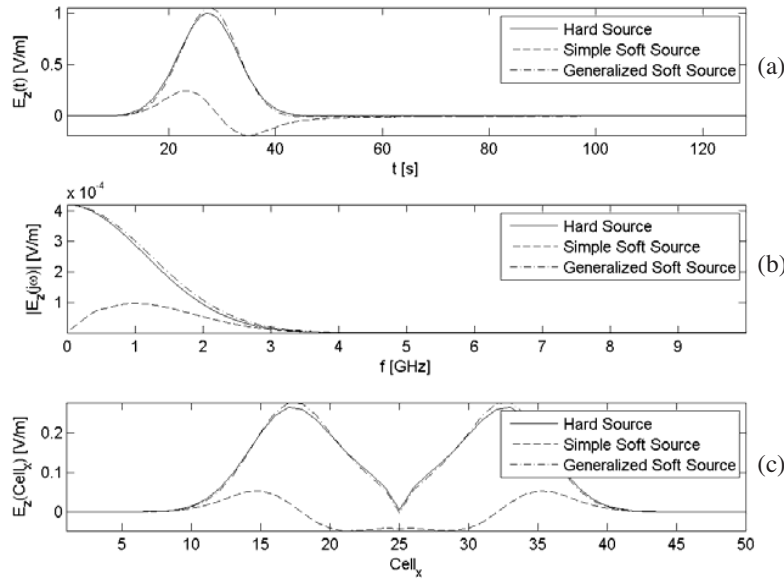


Figure 9. Gaussian pulse propagation (TM_z mode) (a) time domain sample of electric field (b) frequency domain sample of electric field (c) spatial domain sample of electric field.

performing either a symbolic simulation or a numerical simulation. Anyway the results will not be different and $\alpha = -1$. Figure 9 shows the results of simulation for different kinds of sources in time, frequency and spatial domains.

Note that to save memory and time, the two dimensional propagation reference simulation can be changed to one dimensional simulation. The procedure can be found in [5, Ch. 12].

7. CONCLUSIONS

We suggested an exact method to control the direction of propagation of a TEM plane wave in any polarizations and directions. This was done by assigning an electric and magnetic source at the same time at specific positions. We showed that the amplitude of soft sources, either a TEM plane wave or a cylindrical wave, varies according to the Courant number unless a correction is performed. A simple method to obtain this correction was suggested for TEM mode and, although we used this method for a step unit pulse, the result is correct for every kind of signal. We also suggested a general method to solve this problem for any modes and kinds of sources.

APPENDIX A.

The basic parameters used in the scripts are listed in this appendix.

FDTD Basic Parameters:

Number of Time Steps = 2^{15}

Number of Cells along each axis = 50

$\Delta x = \Delta y = \Delta z = 0.05\lambda_{\min}$

$\Delta t = \frac{S\Delta z}{c}$, $0 < S \leq 1$ (One-Dimensional)

$\Delta t = \frac{S\Delta x}{c}$, $0 < S \leq \frac{1}{\sqrt{2}}$ (Two-Dimensional)

Step Source:

Amplitude = $1 \frac{V}{m}$

Sinusoidal Source:

Amplitude = $1 \frac{V}{m}$

Frequency = 1 GHz

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