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DESIGN AND APPLICATION OF A BEAM SHIFTER BY TRANSFORMATION MEDIA

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Abstract—A set of beam shifter which can effectively control the propagation of the beam is proposed. The permittivity and permeability of the beam shifter can be obtained by applying form-invariant, spatial coordinate transformations to Maxwell's equations. We show that the beam is smoothly guided to avoid hitting some irremovable objects, which could be useful in the practical application. Besides, inspired by some phenomenon from the above application, an interesting utilization has been found that by placing a set of beam shifters, electromagnetic detectors can be misled and make mistakes about where the target is located, which is very useful in the anti-detection. All our ideas are verified by numerical simulations with finite element method.

1. INTRODUCTION

Recently the use of coordinate transformations to produce material specification that control electromagnetic fields in interesting and useful ways has been discussed. Pendry et al. reported a methodology based on continuous form-invariant coordinate transformations of Maxwell's equation which allows for the manipulation of electromagnetic fields [1]. The transformation method provides great freedom to design complex electromagnetic devices, among which the invisibility is one that has attracted more and more attention [2–13].

Coordinate transformation is a powerful method that we can realize devices with unique properties in a much easier way. It can

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be widely used in the practical environment. To freely control the behavior of electromagnetic beams is quite significant and attracts the interests of many researchers [14–21]. A recent paper has shown how to design a beam shifter by coordinate transformation [2]. Utilizing the similar idea proposed in [2], here we consider the case where there are some irremovable obstacles in the original route of a beam, which might be required to reach some destination. In order that the beam will not be disturbed by this obstacle, we should steer the paths of the beam and making it round the obstacle before propagating the original path. It's quite useful if we design a set of beam shifters to control the beam.

Based on this motivation, in this paper, we report how to use coordinate transformation theory to design a beam shifter and compose a set of beam shifters to solve the above problem. We find that other than guiding the beams to avoid hitting some irremovable object, it's interesting that the shifter can also be applied in misleading the electromagnetic detectors. By applying a set of beam shifter, an electromagnetic detector will make the misjudgment in localizing the position of the target.

2. THEORETICAL MODELS

For a given coordinate transformation $x^{\alpha'}(x^{\alpha}) = \Lambda_{\alpha}^{\alpha'} x^{\alpha} (\Lambda_{\alpha}^{\alpha'}$: Jacobi matrix, $\alpha = 1 \ldots 3$), the relative electric permittivity and the magnetic permeability of the resulting material are given by [1–3]

$$\begin{aligned} \Lambda_{\alpha}^{\alpha'} &= \frac{\partial x^{\alpha'}}{\partial x^{\alpha}} \\ (\varepsilon_r)^{i'j'} &= \left[\det(\Lambda_i^{i'})\right]^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \\ (\mu_r)^{i'j'} &= \left[\det(\Lambda_i^{i'})\right]^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \end{aligned} \tag{1}$$

in which $\det(\Lambda_i^{i'})$ is the determinant of the Jacobi matrix.

Consider a beam shifter with thickness d starting from x_0, a general coordinate transformation is in the form of

$$\begin{cases} x'(x,y,z) = x \\ y'(x,y,z) = f(x,y) \quad x_0 < x < x_0 + d \\ z'(x,y,z) = z \end{cases} \tag{2}$$

As we consider the 2D condition, the new component $y'(x,y,z)$ is independent of z. The Jacobi matrix of the transformation and its determinant are

$$\Lambda_i^{i'} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\det(\Lambda_i^{i'}) = a_{22} \tag{4}$$

in which

$$\begin{aligned} a_{21} &= \partial f(x,y)/\partial x \\ a_{22} &= \partial f(x,y)/\partial y \end{aligned} \tag{5}$$

According to Eq. (1), we have

$$(\varepsilon_r)^{i'j'} = (\mu_r)^{i'j'} = \frac{1}{a_{22}} g^{ij} \tag{6}$$

in which

$$g^{ij} = \begin{bmatrix} 1 & a_{21} & 0 \\ a_{21} & a_{21}^2 + a_{22}^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

As depicted in Fig. 1(a), a linear case is the best and easiest to calculate and simulate. The linear coordinate transformation is given by

$$\begin{cases} x'(x,y,z) = x \\ y'(x,y,z) = b(x - x_0) + y \quad x_0 < x < x_0 + d \\ z'(x,y,z) = z \end{cases} \tag{8}$$

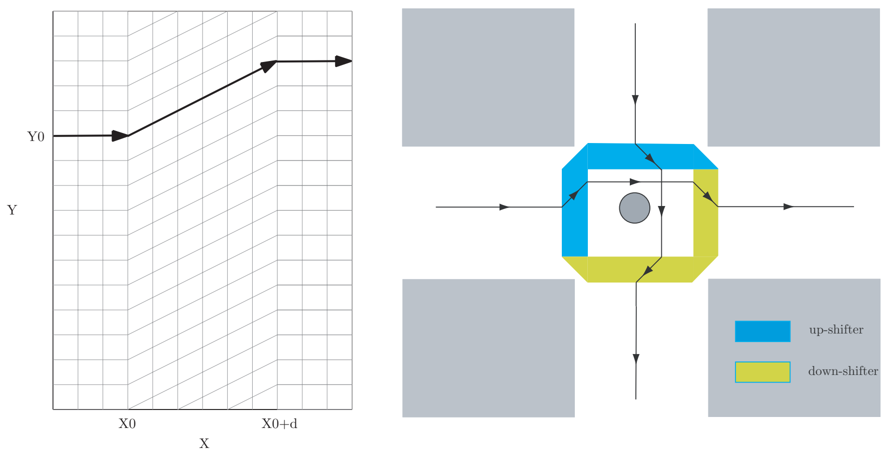
in which a positive parameter b is introduced. We can adjust the shift of the beam by controlling the b. The transformation in Eq. (8) can shift the beam upwards, so we call it up-shifter. From Eq. (6) the constitutive relation of a up-shifter is

$$(\varepsilon_r)_u^{i'j'} = (\mu_r)_u^{i'j'} = \begin{bmatrix} 1 & b & 0 \\ b & b^2 + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

From Fig. 1(b), to restore the position of the beam, we also need a down-shifter to shift the beam downwards. We have its coordinate transformation and constitutive relation as following,

$$\begin{cases} x'(x,y,z) = x \\ y'(x,y,z) = -b(x - x_0) + y \quad x_0 < x < x_0 + d \\ z'(x,y,z) = z \end{cases} \tag{10}$$

$$(\varepsilon_r)_d^{i'j'} = (\mu_r)_d^{i'j'} = \begin{bmatrix} 1 & -b & 0 \\ -b & b^2+1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$



(a) (b)

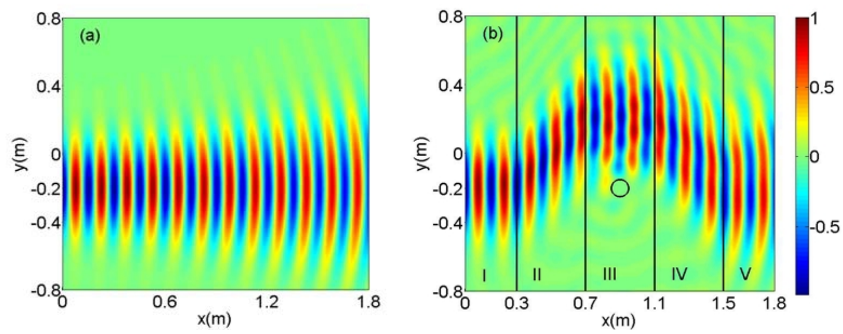
Figure 1. Linear coordinate transformation of a beam shifter and its application. (a) Linear coordinate transformation (b) Sketch map to show how to compose a set of beam shifters.

3. SIMULATION AND DISCUSSION

In Fig. 1(b), the only route of the beam is occupied by an irremovable wire in the center. So a set of beam shifters is needed to make the beam deviated from the wire. We compose a set of beam shifters to force the beam steer clear of the wire. As shown in Fig. 2(b), an up-shifter(region II) and a down-shifter(region IV) are placed in free space. When a beam is impinging, the up-shifter shifts up the beam for a certain distance, while the down-shifter shifts down the beam for the same distance to restore the beam to the original vertical position. As the two shifters have the same thickness, the parameter b of them should be equal.

A 2D full wave simulation was taken to see how the beam shifter works. The region was bounded by perfectly matched layers in order to prevent reflections, and the polarization of the wave was set to be perpendicular to the x-y plane. A Gaussian beam at 2 GHz is chosen as the source [22, 23] and a copper rod is placed in the center of Fig. 2(b).

The parameter b of the beam shifters is set to be 1. Our numerical results in Fig. 2 displays the normalized electric field distribution when the source is embedded at the left boundary. In Fig. 2(b), the set of beam shifters shift the beam up in region II and then shift down in region IV, avoiding the beam to hit the copper rod. Compared with Fig. 2(a), the transmitted beam in Fig. 2(b) remains undisturbed. So this set of beam shifters can force the beam steer clear of the rod.



(a) (b)

Figure 2. Normalized electric field distribution in (a) free space as reference (b) a set of beam shifters with $b = 1$. Regions I, III, V are free space; region II is an up-shifter; region IV is a down-shifter. The Gaussian beam was centered at $y = -0.2\,\text{m}$.

Then we test the performances of beam shifters with different parameters by varying the parameter b. In Figs. 3(a) and 3(b), we changed b to 1.2 and 1.5, and then set their new constitutive relations respectively. The geometry is the same as in Fig. 2(b). We see that the bigger b is, the more the beam is bent. It can be explained by Eq. (8) that the beam shifts more when b increases. That is, a beam shifter with bigger b can cause larger shift. So if we want to shift the beam with a certain amount, we can decrease the width of the slab by increasing its parameter b. However, it's not easy to increase b. According to Eq. (11), the permittivity and the permeability have components lower than -1. What's more, if a more complicated transformation is chosen, the difficulty in the fabrication of the materials will increase. So the methodology of coordinate transformation put forward new tasks in the study of metamaterials.

The fact that the wavefronts remain undisturbed gives us a new idea. If we use a set of beam shifters, the position of the rod could be misjudged. A simulation was carried out to show this (Fig. 4). In free space, we place a rod in the center; while in a set of beam shifters

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4. CONCLUSION

In conclusion, we have presented how to design electromagnetic devices using coordinate transformation, and designed a set of beam shifters to guide the beams to avoid hitting some irremovable object. We found that by our beam shifter, we can mislead the EM detector to make mistakes about the location of the target.

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