Progress In Electromagnetics Research, PIER 83, 107–118, 2008

SUPPORT VECTOR REGRESSION MACHINES TO EVALUATE RESONANT FREQUENCIES OF ELLIPTIC SUBSTRATE INTEGRATED WAVEGUIDE RESONATORS

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Abstract—In this paper an efficient technique for the determination of the resonances of elliptic Substrate Integrated Waveguide (SIW) resonators is presented. The method is based on the implementation of Support Vector Regression Machines trained using a fast algorithm for the computation of the resonant frequencies of SIW structures. Results for resonators with a wide range of parameters will be presented. A comparison with results obtained with Multi Layer Perceptron Artificial Neural Network and with full wave simulations will show the effectiveness of the proposed approach.

1. INTRODUCTION

Substrate Integrated Waveguide (SIW) circuits [1] are a major subject of interest because they are an effective alternative to metallic waveguide structures. The simplicity of the realization process and its reduced costs make SIW suitable for mass scale production maintaining the advantages of metallic waveguides. SIWs are normally built in a board of laminate by realizing arrays of metallic via holes to create a waveguiding channel or a cavity. One of the major advantage of SIW technology is to combine waveguide and microstrip circuits keeping the possibility of integration typical of microstrip structures. Recently, a large number of SIW-based devices have been realized [2, 3]. In particular many waveguide based filter designs have been proposed.

In these cases one of the major advantege of SIW technology is the reduction of the structures size obtained without recurring to more complex configurations [4]. Very often the design of SIW filters is based on resonating structures [5, 6]. In all these cases, the accurate determination of the resonant frequencies is fundamental for a correct design process. In conventional metallic cavities resonances can be determined in closed form for canonical shapes or by falling back on numerical methods for more general configurations. Conversely, for SIW resonators the accurate determination of the resonant frequencies can be cumbersome even for canonically shaped structures. Indeed, a SIW resonator is bounded by the top and bottom metallic plates and by the walls of metallic vias and even when the vias fence forms a canonical geometry, resonances still depend on the cylinders radii and on their separation. As a consequence, any attempt to consider equivalent metallic cavities is doomed to fail if these two parameters are not properly considered. In this paper the characterization of the most important case of elliptic resonators is presented considering the mutual scattering by metallic via holes. To account for full wave effects, the scattered field is expanded in terms of vector eigenfunctions which represent fields TE and TM to $\hat{z}$. Once proper boundary conditions are enforced on the cylinders surfaces one obtains a system of equations where the coefficients of the expansions are unknowns. Resonant solutions can be found searching for frequencies for which the determinant of the system matrix is zero. However, this condition is valid only in theory. In fact, it is well known [7] that the computation of the matrix determinant is numerically unstable. For this reason, another approach has been recently proposed in [8] and has been applied in [9] where the basic case of SIW circular resonators has been studied. Specifically, the minimum singular value of the matrix is seen as a function of the frequency and resonances are defined as those frequencies where this newly defined function has a minimum. This approach allows for an efficient identification of the matrix singularities without recurring to the computation of the determinant. In this paper we use the method based on Singular Value Decomposition (SVD) to develop a Support Vector Regression Machine SVRM model of SIW resonators in order to provide an efficient tool for the analysis of SIW resonators. In the last years SVRMs have been extensively employed to develop fast CAD models of microwave active and passive devices, antennas and to SAR image classification [10–15]. It has been shown that SVRMs have better performance than Artificial Neural Networks. In the following, a brief description of the method used to compute the scattering from the ensemble of posts within the parallel plates waveguide will be presented and the theory behind the determination

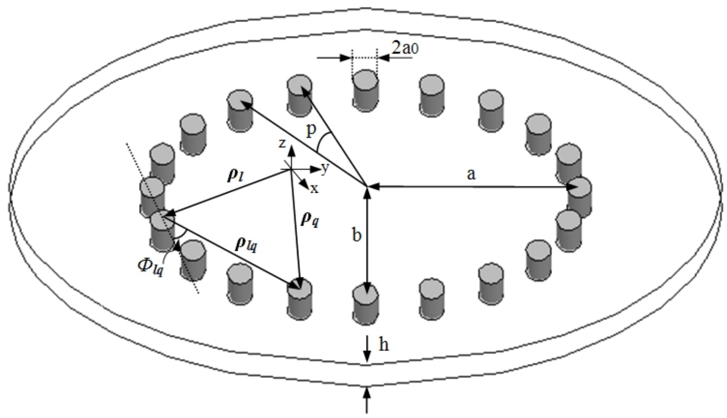

Figure 1. An elliptic SIW resonator, where in particular the angular pitch p, the via hole radius a_0, the semi major axis a, the semi minor axis b and the dielectric substrate height h are shown.

of the resonant frequencies will be also outlined. Notice that lossless resonator will be considered. The effects of copper and dielectric losses have been studied in [16, 17] and they will be included in the model in a forthcoming paper. The basic theory of SVRM will be briefly reported and its application to the structures analyzed in this paper will be described. Results relevant to elliptic resonators will be given for a range of parameters of engineering interests. A comparison with results obtained with Multilayer Perceptron Artificial Neural Network (MLPANN) and with full wave simulations will be also presented. It will be shown that SVRM performances are superior to MLPANN.

2. SCATTERING FROM METALLIC POSTS

The analysis of a parallel plates waveguide with metallic posts has been presented in [18] and a similar formulation has been applied to SIW structures in [19]. In those papers the magnetic dyadic Green's function is expressed in terms of circular vector eigenfunctions. Here a similar procedure is adopted and the electric field scattered by the metallic posts is expanded as follows

$$\mathbf{E}_s(\mathbf{r})=\sum_l \sum_{n,m} \left[\mathbf{M}_n^H(k_{\rho m}, k_{zm}, \rho-\rho_l, z)+\mathbf{N}_n^H(k_{\rho m}, k_{zm}, \rho-\rho_l, z) A_{m,n,l}^{TM}\right] \tag{1}$$

with

$$\mathbf{M}_n^H(k_{\rho m}, k_{zm}, \rho-\rho_l, z) = \nabla\times(H_n^{(2)}(k_{\rho m}|\rho-\rho_l|)e^{-jn\phi}\sin(k_{zm}z)\hat{\mathbf{z}}) \tag{2}$$

$$\mathbf{N}_n^H(k_{\rho m}, k_{zm}, \rho-\rho_l, z) = \frac{1}{k}\nabla\times\nabla\times(H_n^{(2)}(k_{\rho m}|\rho-\rho_l|)e^{-jn\phi}\cos(k_{zm}z)\hat{\mathbf{z}}) \tag{3}$$

where $\mathbf{r} = \rho\hat{\rho} + z\hat{\mathbf{z}}$, $k_{zm} = \frac{m\pi}{h}$, $k_{\rho_m} = \sqrt{k^2 - k_{zm}^2}$, $k = \omega\sqrt{\epsilon_r\epsilon_0\mu_0}$, ϕ is the angle of $\rho - \rho_{\mathbf{l}}$, l is an index spanning over the cylinders, m and n are integers relevant to vertical and azimuthal dependencies and $\rho_{\mathbf{l}}$ is the position of the center of the cylinder l. Coefficients $A_{m,n,l}^{TE}$, $A_{m,n,l}^{TM}$ are determined making the total tangential electric field vanish on the surface of the cylinders. In particular, the incident field is considered for any cylinder, given by the field scattered by the remaining cylinders, and the field scattered by the cylinder under consideration. The application of the boundary condition leads to two systems of equations, one for the TM scattering and the other for TE scattering, from which the coefficients are calculated. For a given m one has

$$\Gamma_{q,r,m}^{TM,TE} = \sum_{l=1 l\neq q}^{N}\sum_{n} L_{q,r,m,l,n}^{TE,TM} A_{m,n,l}^{TE,TM} + A_{m,r,q}^{TE,TM} \qquad \forall q, r \tag{4}$$

where

$$L_{q,r,m,l,n}^{TE} = -\frac{J_r'(k_{\rho m}a)}{H_r^{(2)'}(k_{\rho m}a)} H_{n-r}^{(2)}(k_{\rho m}\rho_{lq})e^{-j(n-r)\phi lq}$$
$$L_{q,r,m,l,n}^{TM} = -\frac{J_r(k_{\rho m}a)}{H_r^{(2)}(k_{\rho m}a)} H_{n-r}^{(2)}(k_{\rho m}\rho_{lq})e^{-j(n-r)\phi lq} \tag{5}$$

and $\Gamma_{q,r,m}^{TM,TE}$ represents the excitation, a is the radius of the metallic posts and ρ_{lq} is the distance between cylinders q and l (Fig. 1). As is well known, resonances are the frequencies at which system (4) admits non-trivial solutions for $\Gamma_{q,r,m}^{TM,TE} = 0$.

3. LOCALIZATION OF RESONANCES

System (4) can be more conveniently expressed in a matrix form:

$$[\mathbf{L}]^{\text{TE, TM}}[\mathbf{A}]^{\text{TE, TM}} = [\mathbf{\Gamma}]^{\text{TE, TM}} \tag{6}$$

Theoretically SIW resonances are the complex frequencies f_r for which

$$\det([\mathbf{L}]^{\text{TE, TM}}(\omega_r)) = 0 \tag{7}$$

with $\omega_r = 2\pi f_r$. However, from a practical point of view, $[\mathbf{L}]^{\text{TE, TM}}$ can be efficiently tested to be singular only by the Singular

Value Decomposition (SVD). In numerical algebra, SVD permits an arbitrarily $n \times n$ complex matrix $[\mathbf{L}]^{\text{TE, TM}}$ to be factorized in the form $\mathbf{U\Sigma W}^*$, where $\mathbf{U}$ and $\mathbf{W}$ are n-by-n unitary matrices, * denotes the conjugated transposition and $\mathbf{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$ is an n-by-n diagonal matrix, whose entries $\sigma_1, \sigma_2, \ldots, \sigma_n$ are non negative real numbers (namely, *singular values*), rearranged in such a way that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$ [20]. Then, in order to prove whether $[\mathbf{L}]^{\text{TE, TM}}$ is singular or not, first of all we recall that the smallest singular value σ_n represents the actual distance between $[\mathbf{L}]^{\text{TE, TM}}$ and the set of all matrices whose rank is $\leq n-1$. Hence, provided a threshold $\epsilon > 0$, we state $[\mathbf{L}]^{\text{TE, TM}}$ is ϵ-singular, whenever $\sigma_n \leq \epsilon$. Starting from this condition, the set Ω of the complex resonant frequencies $\omega_r = \omega_{re} + j\omega_{im}$ (for which the matrix $[\mathbf{L}]^{\text{TE, TM}}$ is ϵ-singular) can be determined seeking for the minima of the two variable function $\sigma_n = \sigma_n(\omega_{re}, \omega_{im})$ in a given frequency band $[\omega_{min}, \omega_{max}]$. In [8] an effective search strategy is discussed, basically consisting of an approximate computation of σ_n rather than a direct complete SVD factorization. The search strategy first considers the estimated set of N as a function of real frequency only. Once the minima on the real axis are located they are used as starting points of a Muller search routine in the complex plane [21]. The results of this procedure are the complex resonant frequencies of the structure over the prescribed band.

4. SUPPORT VECTOR REGRESSION MACHINES

Support Vector Machines (SVMs) are learning machines performing pattern recognition tasks. Originally introduced by Vapnik and co-workers [22], they are getting more and more popular for overcoming the limitations typical to ANNs (see [23] and references within). This is because the Structural Risk Minimization principle embodied by SVMs has been proved to be more effective than the traditional Empirical Risk Minimization principle employed by ANNs (see [22] and references within), hence equipping the former with a greater ability to generalize, when compared with the latter. By means of non linear transformations they map the n-dimensional input space into a higher dimensional space where the data can actually be linearly separated [24]. SVMs can also be employed to solve regression problems specializing in Support Vector Regression Machines (SVRMs). In this case also, as for standard SVMs. non linear transformations are adopted to map incoming data into a higher dimensional space, where a linear regression can be carried out. In order to explain the mathematical framework in which SVRMs are defined, let us consider the problem of the approximation of the set of data $\mathcal{D} = \{(x^1, y^1), ..., (x^l, y^l)\}, \mathbf{x} \in \mathcal{X}, y \in \Re$ by a linear function $f(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle + b$, where $\langle , \rangle$ denotes the inner product in $\mathcal{X}$, i.e., the space of the input patterns. This problem can be solved selecting the optimal regression function as the minima of the functional [25]

$$\Phi(\mathbf{w}, \xi) = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_i(\xi_i^- + \xi_i^+) \tag{8}$$

where C is a user defined value, and ξ_i^-, ξ_i^+ are slack variables representing upper and lower constraints on the output of the system. However, the set $\mathcal{D}$ often is not linearly separable: the separation can be obtained by means of a suitable non-linear mapping witch project $\mathcal{D}$ into a high dimensional space where the linear regression can be carry out. This non-linear mapping is accomplished by a *kernel function* $K(\mathbf{x}_i, \mathbf{x}_j)$ [25]. Loosely speaking, like in canonical regression problems, the relationship between dependent and independent variables is supposed to be the sum of an unknown smooth function $f(\mathbf{x})$ plus some additive noise. The main task is to find an analytical closed form for $f(\mathbf{x})$ granting for the possibility to predict the behavior of brand new cases the SVM has never been presented with. This can be achieved by training the SVM by a suitable training set. The overall process involves sequential optimization of an error (or loss) function [24]. According to the specific application, the proposed heuristic approach can "learn" the relation between measurable physical (inputs) and modelling (outputs) quantities from the experience; so, experimental datasets are necessary to train our heuristic estimative system for each considered application. In order to implement a SVRM-based estimator, a loss function must be used; in this case, the approach has been carried out by means of the following Vapnik ε-insensitive loss function:

$$L_\varepsilon(y) = \begin{cases} 0 & for \;\; |f(\mathbf{x}) - y| < \varepsilon \\ |f(\mathbf{x}) - y| & otherwise \end{cases} \tag{9}$$

performing a tuning, carried out according to [25], in order to obtain an acceptable setting for C and ε parameters.

5. NUMERICAL RESULTS

In Figure 1 is shown the layout of the structures under consideration. The algorithm described in Section 3 to search the resonances frequencies was firstly tested. In Figure 2 the estimated σ_N as a function of the real frequency for two SIW elliptical resonators are

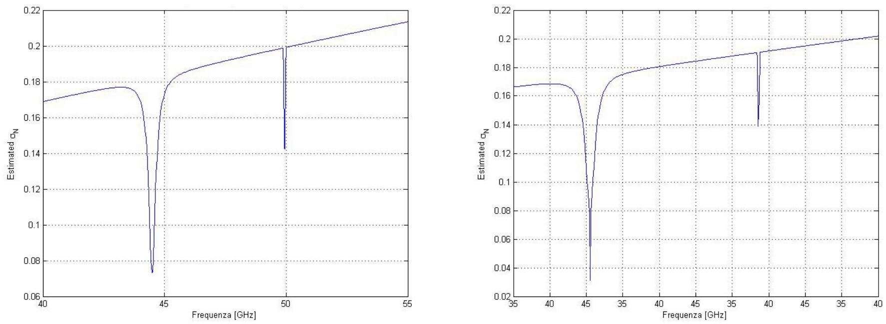

Figure 2. The estimated σ_N as a function of the real frequency for two SIW elliptical resonators. (left: $a = 3.125\,\text{mm}$, $b = 2.500\,\text{mm}$; right: $a = 3.800\,\text{mm}$, $b = 3.040\,\text{mm}$; $\epsilon_r = 2.2$).

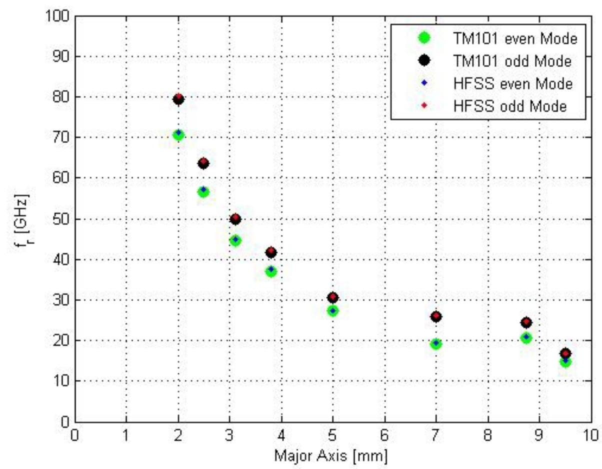

Figure 3. Real part of the resonant frequencies of TM_{101} even and odd modes as a function of ellipse major axes of an elliptical resonator with eccentricity $e = 0.8$.

reported. As it can be observed, the curve has evident minima corresponding to two resonances, which can be located with a reduced number of computer runs. In a preceding paper [9] the estimated values have been used as initial guess for a Muller search in the complex plane aimed to obtain an accurate determination of resonant frequencies. However it has been observed that, for the lossless case, the imaginary part, accounting for the power leaking out from the vias' fence, is

Table 1. Ranges of considered input for SVRM models (a and c are in millimeters).

a	e	c	ϵ_r
$2.000 \div 9.500$	$0.141 \div 0.714$	$1.300 \div 6.800$	$2.000 \div 10.000$

negligible and that the real part is very close to the estimated value. In Figure 3 is shown the real part of the resonant frequencies of TM_{101} even and odd modes as a function of ellipse major axes of an elliptical resonator with eccentricity $e = 0.8$. The structure is realized on a substrate with with $\epsilon_r = 2.2$, $h = 0.5$ millimeter. As it can be seen the difference between full wave HFSS simulations and results obtained with the method presented in this paper is very small and always below 1%. Two SVR machines, one for the even mode TMe_{101} and the other one for the odd mode TMo_{101}, were implemented using the set of estimated values. The via hole radius a_0, the angular pitch p, the eccentricity e, the focal distance c and the dielectric constant ϵ_r were used as input parameters while the resonant frequency f_r was the output. Ranges of variation of engineering interest for the inputs were chosen and are shown in Table 1. After some numerical experiments carried out by using the Spider Toolbox, a two degree polynomial kernel setting with $C = 10$ and $\varepsilon = 0.01$, has been selected. Figures 4 and 5 reports the scattering plots of the fundamental resonant modes obtained with SVRMs compared with HFSS simulations. These graphs confirm a very good agreement between computed and estimated values of f_r over the whole range of the geometrical and electrical parameters considered giving an excellent indication of the ability of SVRMs to capture the input-output relationship present in the data. Furthermore, two MLPANN having the same input and output of SVRMs and two hidden layers (nine hidden nodes in the first layer, three hidden nodes in the second one), have been implemented. They have been trained in the supervised mode using the Levenberg-Marquadt backpropagation learning rule. The size of the hidden layer was determined carried out extensive numerical simulations by means of the Matlab Neural Network Toolbox by selecting the number of hidden nodes resulting from the lowest training error while maintaining adequate generalization. In Table 2 is reported the comparison of performances between SVRM and MLPANN model in term of RMSPE. It can be observed that SVRMs have better performances than MLPANN. Notice that the training time for MLPANNs was thirty times larger than the time needed for training SVRMs.

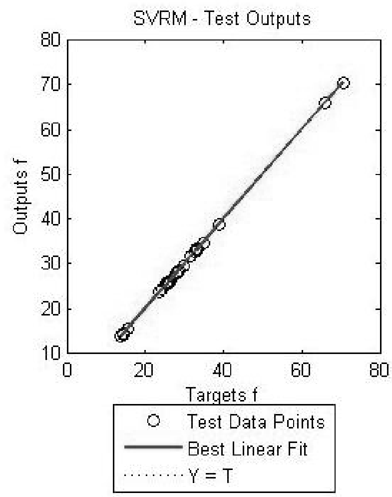

Figure 4. Scatter plot: Full wave computed vs SVRM predicted $\mathrm{TM}_{e,101}$ resonances.

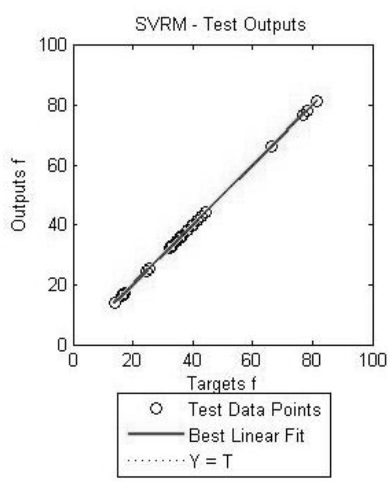

Figure 5. Scatter plot: Full wave computed vs SVRM predicted $\mathrm{TM}_{o,101}$ resonances.

Table 2. Performances comparison in terms of RMSEP between MLPANN and SVRM model.

	SVRM	MLPANN
$\text{TM}_{e,101}$	0.02%	0.21%
$\text{TM}_{o,101}$	0.01%	0.27%

6. CONCLUSIONS

In this work an efficient algorithm for the estimation of the resonant frequencies of elliptic SIW resonators has been presented. The estimated values have been used to implement a SVRM model. The results obtained have been compared with the ones found with a MLPANN model and with full -wave simulations. It has been observed that SVRM shows better RMSEP and shorter training time than MLPANN. In particular, experimental results suggest that SVRMs can be profitably employed to model SIW devices in accurate way.

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