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IMPROVED SPECTRAL ITERATION TECHNIQUE FOR THE SCATTERING BY THIN METAL PLATES

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Abstract—The problem of electromagnetic scattering by thin metal plates is formulated in terms of Electric Field Integral Equation and solved by an improved form of the Spectral Iteration Technique. The local solution at the edges of the plate is chosen as initial guess for the unknown surface current in order to guarantee and enhance the convergence of the iterative scheme. Numerical simulations on a square conducting plate are presented to validate the proposed approach.

1. INTRODUCTION

The computational efficiency is a primary concern in the numerical solution of electromagnetic scattering [1–3] problems. Factors such as time and storage becomes more and more difficult to control when considering bodies of arbitrary size and geometry. As well known, the Moment Method [4–7] relies on matrix solutions which can be computationally too expensive when considering complex structures [8, 9]. A large interest has then been addressed to iterative methods [10–13], mostly based on the Conjugate Gradient algorithm [14–16], which guarantees monotonic convergence for arbitrary structures [17, 18]. However, numerical difficulties are also encountered in this scheme, as it suffers from machine round-off errors causing loss of orthogonality and linear independence [19], so the global error can decrease very slowly, resulting in a large number of iterations. Moreover, an erroneous behavior is observed in the current density even when using expansion functions for improving the rate of convergence [20].

An iterative method alternative to the Conjugate Gradient is given by the Spectral Iteration Technique (SIT) [20, 21], which manipulates the Electric Field Integral Equation (EFIE) in the spectral domain to derive an iterative equation for the current density on the scatterer. SIT approach is advantageous for two primary reasons. First of all, it circumvents the problem due to matrices of prohibitively large dimensions; in addition, the adoption of Fourier transform converts the original integral equation into an algebraic one, which is easier to manipulate. Nevertheless, the convergence of the SIT is not always guaranteed, and even in successful applications it could require a great number of iterations. In recent works [22, 23], the authors investigated the convergence properties of the SIT for the problem of scattering by strips and proposed an improved form which guarantees the achievement of the solution in the presence of bodies having arbitrary size. The strong enhancement of convergence is obtained by simply choosing the local solution at the edges of the scatterer as initial guess of the unknown current density. This leads to a regularization of the integral equation, as the singular part of the current (local-edge solution) is extracted and the scheme is iterated with respect to the remaining regular part. In this paper, the Improved Spectral Iteration Technique (ISIT) is applied to the analysis of three-dimensional scattering by metallic plates. An iterative scheme is derived which makes use of the scalar Green's function in the spectral domain. The assumption of the approximated local edge solution as initial estimate of the unknown surface current enforces the boundary condition, so assuring the exact solution for plates of arbitrary size in a relatively small number of steps. Numerical simulations on a square metallic plate are presented in order to check the efficiency of the method, which can be easily extended to more complex structures such as microstrip and stacked antennas.

2. FORMULATION

Let us consider the geometry in Fig. 1, where a perfectly conducting thin plate is illuminated by an obliquely incident plane wave. For the sake of simplicity, an incident electric field of unitary amplitude is assumed, which is oriented along the x-axis and expressed as:

$$\underline{E}^i = \widehat{x} \cdot e^{-jk_o(ysin\theta_i - zcos\theta_i)} \tag{1}$$

where k_o is the free-space wavenumber.

The EFIE for the surface current $\underline{J}_S$ induced on the plate is obtained by requiring the tangential component of the total electric field (incident plus scattered) to vanish at the plate surface S, so having [24]:

$$\underline{E}^i = k_o^2 \iint_S \underline{J}_S(\rho')G(\rho-\rho')d\rho' + \nabla \iint_S \nabla \cdot \underline{J}_S(\rho')G(\rho-\rho')d\rho' \tag{2}$$

The terms $\rho' = \sqrt{x'^2 + y'^2}$ and $\rho = \sqrt{x^2 + y^2}$ into Equation (2) give the radial coordinates of the source and the observation points, respectively, while

$$G(\rho - \rho') = \frac{e^{-jk_o\sqrt{(x-x')^2+(y-y')^2}}}{4\pi\sqrt{(x-x')^2 + (y-y')^2}}$$

denotes the three-dimensional Green function confined to the xy plane.

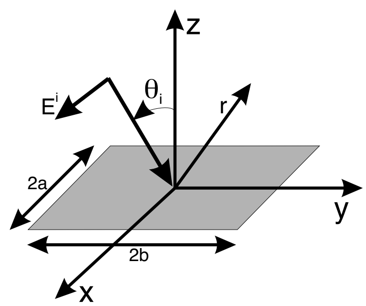

Figure 1. Thin conducting plate illuminated by an obliquely incident plane wave.

Integro-differential Equation (2), describing the scattering by metal plates of arbitrary shape, has a useful convolution structure which can be efficiently handled by Fourier analysis [25]. As a matter of fact, the following scalar expressions can be derived (see Appendix A) from (2), which are valid for all $(x, y) \in S$:

$$E_x^i = \left(k_o^2 + \frac{\partial^2}{\partial x^2}\right) J_x * G + \frac{\partial^2 J_y}{\partial x \partial y} * G \tag{3}$$

$$0 = \left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) J_y * G + \frac{\partial^2 J_x}{\partial y \partial x} * G \tag{4}$$

where $\underline{J}_S = \widehat{x}J_x + \widehat{y}J_y$ and the symbol $(*)$ denotes the convolution operator.

To obtain an extended form of Equations (3) and (4), valid all over the space, let us define the truncation operator as follows:

$$\theta\left(\underline{A}(\underline{r})\right) = \begin{cases} \underline{A}(\underline{r}), & \underline{r} \in S \\ 0, & \underline{r} \ni S \end{cases} \tag{5}$$

with:

$$\widehat{\theta}(\underline{A}) = \underline{A} - \theta(\underline{A}) \tag{6}$$

After applying relations (5) and (6) into expressions (3) and (4), the following equations are derived (see Appendix A), which are valid on the entire $z = 0$ plane:

$$\theta\left(E_x^i\right) + \widehat{\theta}\left[\left(k_o^2 + \frac{\partial^2}{\partial x^2}\right) J_x * G + \frac{\partial^2}{\partial x \partial y} J_y * G\right] = \\ = \left(k_o^2 + \frac{\partial^2}{\partial x^2}\right) J_x * G + \frac{\partial^2}{\partial x \partial y} J_y * G \quad (7)$$

$$0 = \left(k_o^2 + \frac{\partial^2}{\partial y^2}\right) J_y * G + \frac{\partial^2}{\partial y \partial x} J_x * G \quad (8)$$

Observe that Equation (8) remains unchanged due to the particular choice of $\widehat{x}$-polarized incident electric field.

The convolution form of Equations (7) and (8) leads to apply the convolution theorem [26] to rapidly perform computations in terms of Fourier transform on the following spectral domain expressions (see Appendix A):

$$\widetilde{\theta}\left(E_x^i\right) + \mathcal{F}\{\mathcal{F}^{-1}\{\widetilde{G}_1 \widetilde{J}_x \widetilde{G}\}\} - \widetilde{\theta}\left(\mathcal{F}^{-1}\{\widetilde{G}_1 \widetilde{J}_x \widetilde{G}\}\right) = \widetilde{G}_1 \widetilde{J}_x \widetilde{G} \quad (9)$$

$$0 = \left(k_o^2 - k_y^2\right) \widetilde{J}_y \widetilde{G} - k_y k_x \widetilde{J}_x \widetilde{G} \quad (10)$$

where:

$$\widetilde{G}_1 = \frac{k_o^2 (k_o^2 - k_x^2 - k_y^2)}{k_o^2 - k_y^2} \quad (11)$$

and:

$$\widetilde{G} = \frac{\eta}{2k_o} \cdot \frac{1}{k_o^2 - k_x^2 - k_y^2} \quad (12)$$

An iterative expression for the current component J_x can be easily derived from Equation (9) as:

$$J_x^{(n+1)} = \mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1} \widetilde{G}^{-1} \widetilde{\theta}\left(E_x^i\right)\right\} + \\ + \mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1} \widetilde{G}^{-1}\left(\mathcal{F}\left\{\mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1} \widetilde{J}_x^{(n)} \widetilde{G}^{-1}\right\}\right\} \right.\right. \\ \left.\left. - \widetilde{\theta}\left(\mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1} \widetilde{J}_x^{(n)} \widetilde{G}^{-1}\right\}\right)\right)\right\} \quad (13)$$

The current component J_y is then obtained from Equation (10) as:

$$J_y^{(n+1)} = \mathcal{F}^{-1}\left\{\frac{k_y k_x}{k_o^2 - k_y^2} \cdot \widetilde{J}_x^{(n+1)}\right\} \quad (14)$$

The iterative Equation (13) can be expressed in operator notation as follows [22]:

$$J_x = h + L(J_x) \tag{15}$$

where:

$$h = \mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1}\widetilde{G}^{-1}\widetilde{\theta}\left(E_x^i\right)\right\} \tag{16}$$

$$\begin{aligned} L(J_x) \;=\; & \mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1}\widetilde{G}^{-1}\left(\mathcal{F}\left\{\mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1}\widetilde{J}_x\widetilde{G}^{-1}\right\}\right\} + \right.\right. \\ & \left.\left. -\widetilde{\theta}\left(\mathcal{F}^{-1}\left\{\widetilde{G}_1^{-1}\widetilde{J}_x\widetilde{G}^{-1}\right\}\right)\right)\right\} \end{aligned} \tag{17}$$

The solution of Equation (15) are the fixed points [27] of the linear integral operator L, so convergence properties of SIT scheme (13) can be analyzed in terms of the Banach principle of contraction mapping [27]. Recall that an operator L acting in a Banach space B is said to be a contraction when, for any sequences x, y, the distance between their images is closer than the distance between the objects x, y in the domain L, that is [27]:

$$\|L(x-y)\| \leq \|x-y\| \tag{18}$$

A transformation L with the contractive property (18) is responsible for clustering the sequence $\{J_x^{(n)}\}$ of the iterative process (13) toward a limit point. As a matter of fact, the Banach-Caccioppoli theorem assures that, if the contraction operator L maps a complete metric space $M \subset B$ into itself, than it has a unique fixed point to which the sequence $\{J_x^{(n)}\}$ converges from any initial point [27].

As yet proved in [23] for the problem of diffraction by strips and strip gratings, the operator L of SIT scheme is not always a contraction mapping. As a matter of fact, it is shown in [23] that the norm of this operator results to be greater than one for small scatterers, as compared to the wavelength of the excitation field. Nevertheless, the existence and uniqueness of solutions for operator Equation (15) is strictly related to the choice of a proper Banach space B, so a crucial point is related to the finding of an initial guess $\{J_x^{(0)}\}$ which gives the best approximation of the unknown in the sense that it has smallest distance from the exact solution [27]. Our approach is based on the consideration that the current distribution on the scatterer can be regarded as the sum of a singular part, taking into account the static solution, and a regular part, which models the correct oscillating behavior. If assuming the local edge solution as initial guess for the unknown current, a regularization is performed on the original operator Equation (15), as the scheme (13) is iterated with respect to the regular part of the unknown, so the convergence is assured for scatterers of arbitrary size and the rate of convergence is accelerated itself. As a further advantage, the proposed method does not suffer from inaccuracy problems in the presence of near grazing incidence, when standard SIT scheme using asymptotic initial guess [20] gives unsatisfactory results.

3. NUMERICAL RESULTS

SIT scheme is applied to the analysis of electromagnetic scattering by thin metal plates of dimensions $2a = 2b = D$ (Fig. 1). First of all, convergence properties of iterative process (13) are investigated by computing the spectral norm of operator L into Equation (15) as given by the maximum singular value obtained by a Singular Value Decomposition (SVD) [28] algorithm. A plot of this norm as a function of the plate side D is reported under Fig. 2. According to the Banach-Caccioppoli theorem [27], L is a contraction operator if and only if its norm is less than one, so numerical results in Fig. 2 show that convergence of SIT scheme (13) is not theoretically guaranteed for plates of small size ($D < 0.5\lambda$).

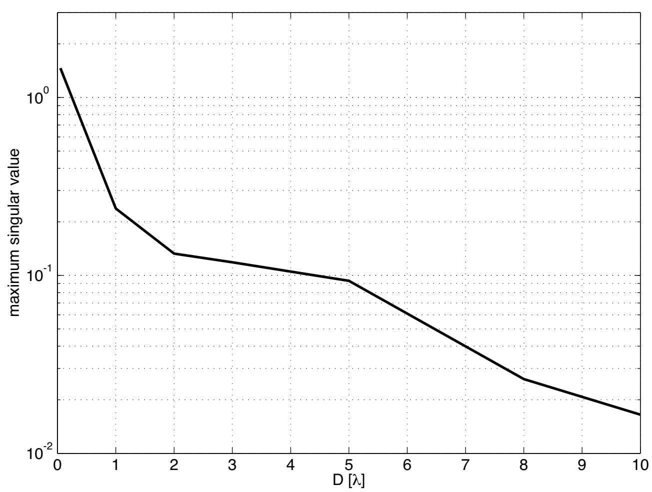

Figure 2. Norm (maximum singular value) of operator L as a function of plate side D.

As remarked in the previous section, SIT divergence problems can be solved by properly choosing the initial guess for the unknown current as strictly localized near the exact solution. This approach is validated by computing the root mean square error for different

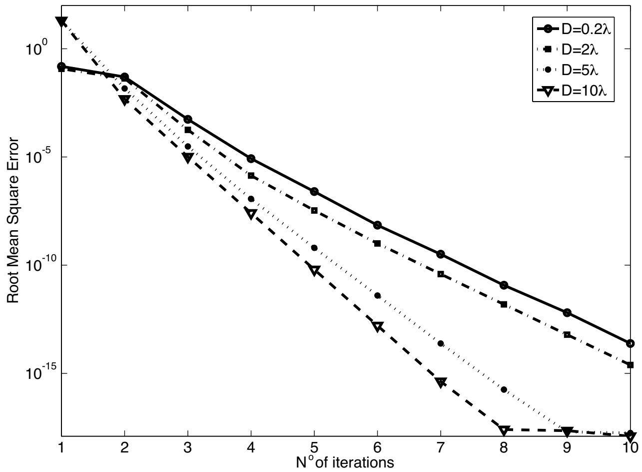

Figure 3. Root mean square error for different values of the plate side D (local-edge solution as initial guess for the current).

values of the plate side D (Fig. 3) when assuming as initial guess the approximated local solution at the plate edges, which is given by the expression:

$$J_x^{(0)} = \sqrt{\frac{1 - \left(\dfrac{x}{D/2}\right)^2}{1 - \left(\dfrac{y}{D/2}\right)^2}}, \quad -\frac{D}{2} < x < \frac{D}{2}, \quad -\frac{D}{2} < y < \frac{D}{2} \qquad (19)$$

A monotonic error decrease can be observed in Fig. 3 even for a small length $D = 0.2\lambda$, so demonstrating that the use of local-edge solution as first-order approximation of the current assures the convergence of SIT scheme (13) for plates of arbitrary size. A rapid decrease of the error is obtained with ISIT, while a much larger number of iterations is required by the Conjugate Gradient method, as shown in Fig. 4 for the case $D = \lambda$ [29].

The fast convergence rate of the proposed approach is tested by computing the current distribution on a square $2\lambda \times 2\lambda$ metal plate illuminated by a normally incident ($\theta_i = 0°$) $\widehat{x}$-directed electric field (Fig. 1). Equations (13) and (14) are applied and the correct edge behavior (19) is assumed as initial guess for the dominant current J_x, whose 3D plot is reported under Fig. 4. Comparisons between results obtained with ISIT and those provided by NEC-Win Moment Method code are shown in Figs. 5–6 for the cut $x = 0$ and $y = 0$,

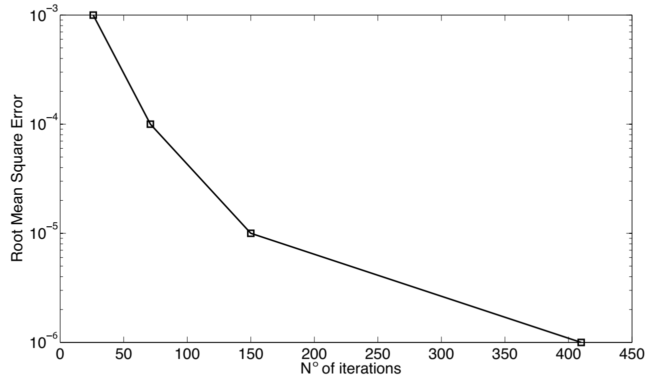

Figure 4. Convergence rate of the Conjugate Gradient method for a square plate of side $D = \lambda$ (from [29]).

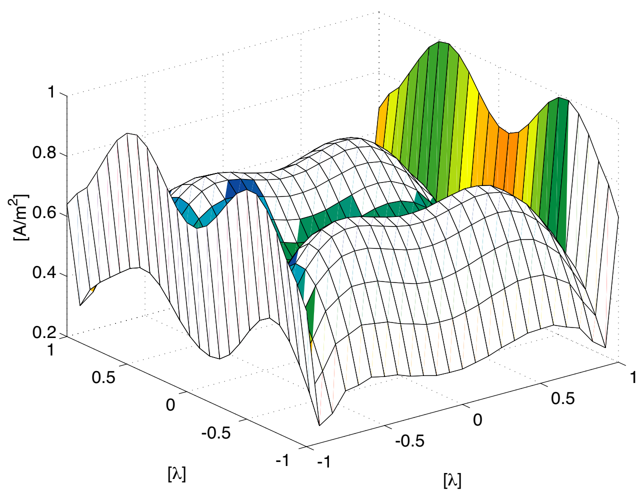

Figure 5. 3D plot of the dominant current J_x on a $2\lambda \times 2\lambda$ metal plate.

respectively. A more pronounced correct behavior at the edges of the plate can be observed in the ISIT solution, which has required 10 iterations performed in 30 seconds on a Pentium III processor. For the simulation with NEC-Win code, a mesh of 62 wires, 30 segments for each wire and a spacing $\Delta = \lambda/15$ between segments is considered. The required computational time has been equal to 8 minutes on the same processor. Finally, the scattered field is computed form the known current distribution on the plate. A good agreement with NEC-Win

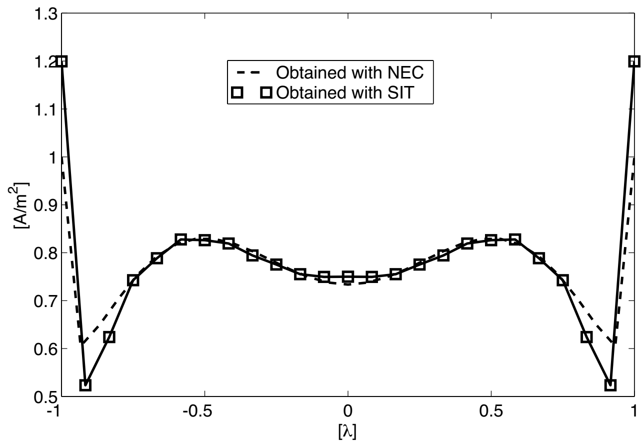

Figure 6. Dominant current J_x along the y-axis.

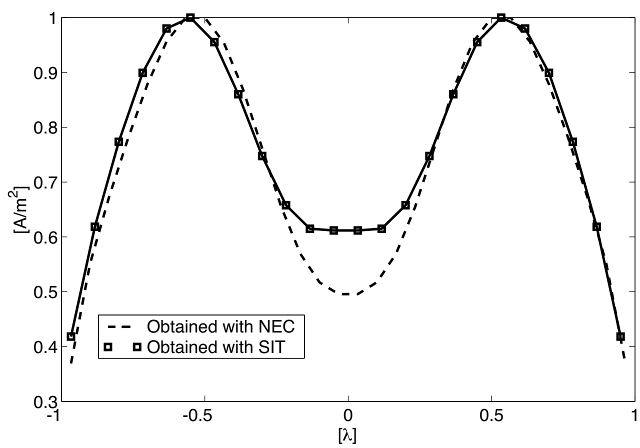

Figure 7. Dominant current J_x along the x-axis.

result can be observed in Fig. 7 for the E_ϕ component at $\phi = 0°$.

4. CONCLUSIONS

The problem of three-dimensional scattering by thin metal plates is formulated in terms of EFIE and solved by an Improved form of the SIT. A strong enhancement in the convergence rate is obtained by imposing the correct edge behavior at the edges of the plate as initial guess of the unknown current distribution. The efficiency of the

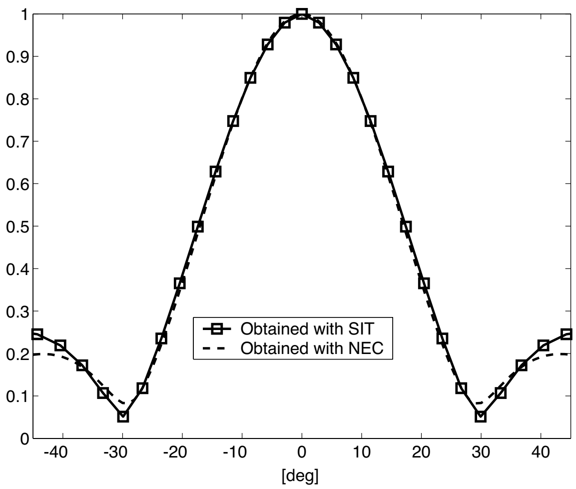

Figure 8. Normalized amplitude of scattered field (E_ϕ at $\phi = 0°$).

method is tested on square plates and a reduced computation time is demonstrated whit respect to a commercial Moment Method code.

APPENDIX A.

A compact form of Equation (2) in terms of convolution operator ($*$) can be written as:

$$\widehat{x}E_x^i = k_o^2 \left(\widehat{x}J_x + \widehat{y}J_y\right) * G + \nabla \left\{\left[\nabla \cdot \left(\widehat{x}J_x + \widehat{y}J_y\right)\right] * G\right\} \tag{A1}$$

By developing the term $\nabla\nabla \cdot (\widehat{x}J_x + \widehat{y}J_y)$, the following expression is derived from Equation (A1):

$$\widehat{x}E_x^i = k_o^2 \left(\widehat{x}J_x + \widehat{y}J_y\right) * G + \\ + \left[\widehat{x}\left(\frac{\partial^2 J_x}{\partial x^2} + \frac{\partial^2 J_y}{\partial x \partial y}\right) + \widehat{y}\left(\frac{\partial^2 J_x}{\partial y \partial x} + \frac{\partial^2 J_y}{\partial y^2}\right)\right] * G \tag{A2}$$

Equations (3) and (4) are then obtained by projection of Equation (A2) along x and y axes.
After application of truncation operator (5), expressions (7) and (8) are manipulated in the Fourier transform domain to have:

$$\widetilde{\theta}\left(E_x^i\right) + \mathcal{F}\left\{\mathcal{F}^{-1}\left\{(k_o^2 - k_x^2)\widetilde{J}_x\widetilde{G} - k_x k_y \widetilde{J}_y \widetilde{G}\right\}\right\} +$$

$$-\widetilde{\theta}\left(\mathcal{F}^{-1}\left\{(k_o^2-k_x^2)\widetilde{J}_x\widetilde{G}-k_xk_y\widetilde{J}_y\widetilde{G}\right\}\right)=(k_o^2-k_x^2)\widetilde{J}_x\widetilde{G}-k_xk_y\widetilde{J}_y\widetilde{G} \tag{A3}$$

$$0=(k_o^2-k_y^2)\widetilde{J}_y\widetilde{G}-k_yk_x\widetilde{J}_x\widetilde{G} \tag{A4}$$

The term:

$$(k_o^2-k_x^2)\widetilde{J}_x\widetilde{G}-k_xk_y\widetilde{J}_y\widetilde{G} \tag{A5}$$

can be properly simplified when combined with Equation (A4) to have:

$$(k_o^2-k_x^2)\widetilde{J}_x\widetilde{G}-k_xk_y\cdot\frac{k_yk_x}{k_o^2-k_y^2}\widetilde{J}_x\widetilde{G}=\widetilde{G}_1\widetilde{J}_x\widetilde{G} \tag{A6}$$

with the spectral domain functions $\widetilde{G}_1$ and $\widetilde{G}$ defined by Equations (11) and (12).

By inserting expression (A6) into relation (A3), Equation (9) is finally derived.

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