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NATURAL FREQUENCY EXTRACTION USING GENERALIZED PENCIL-OF-FUNCTION METHOD AND TRANSIENT RESPONSE RECONSTRUCTION

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Abstract—A technique for natural frequency extraction without a prior knowledge of the number of natural frequencies is proposed. The proposed scheme is based on the GPOF method with the overestimated number of natural frequencies, and it has been shown from simulation result that the proposed method is superior to the GPOF method. The method is applied to the extraction of the natural frequencies of the thin wires whose exact natural frequencies are known. While the absence of the true natural frequency has much effect on the transient response reconstruction, the absence of the spurious natural frequency has little effect on the transient response reconstruction. Using the above property, true natural frequencies and spurious natural frequencies can be discriminated.

1. INTRODUCTION

Many radar target discrimination techniques utilizing late time transient response of radar target have been published [1–5]. According to the singularity expansion method (SEM) [6], the late time electromagnetic field scattered from a finite sized conducting body is represented as a sum of damped sinusoids. Thus, identification of radar targets based on their late time natural resonance requires a method of finding the natural frequencies using the transient response of scatterer. Any algorithm for the natural frequency extraction should be as noise insensitive and accurate as possible. To obtain the natural frequencies from a transient response of a target, a generalized pencil-of-function (GPOF) method is proposed by Hua et al. [7–9]. The GPOF method requires a prior knowledge of the number of natural frequencies contained in the transient data. Theoretically, the number of natural frequencies can be determined with transient data [10–13]. But in low signal-to-noise ratio (SNR) environment, it is difficult to determine it correctly. In this study, natural frequency extraction method using the GPOF method without a prior knowledge of the number natural frequencies is considered. The number of natural frequencies is overestimated and the resulting spurious natural frequencies are discriminated from the true natural frequencies.

2. GENERALIZED PENCIL OF FUNCTION METHOD

In this section, the GPOF method is described briefly [7]. Late time transient response sampled with time interval δt can be expressed by

$$y_k = \sum_{i=1}^{M} b_i \exp(s_i\, \delta t\, k) \qquad k = 0, \ldots, N-1 \tag{1}$$

where s_i are the natural frequencies, b_i are the residues, M is the number of natural frequencies.

For brevity, let us define

$$z_i = \exp(s_i \delta t). \tag{2}$$

The natural frequencies can be extracted from target measurement data y_k using the GPOF method. We define the matrices Y_1 and Y_2 as

$$Y_1 = [\mathbf{y}_0,\ \mathbf{y}_1,\ \ldots,\ \mathbf{y}_{L-1}] \tag{3}$$

$$Y_2 = [\mathbf{y}_1,\ \mathbf{y}_2,\ \ldots,\ \mathbf{y}_L] \tag{4}$$

where matrix element vector $\mathbf{y}_i$ is given by

$$\mathbf{y}_i = [y_i,\ y_{i+1},\ \ldots,\ y_{i+N-L-1}]^T. \tag{5}$$

Then $\{z_i\}_{i=1}^{M}$ are the eigenvalues of Z.

$$Z = D^{-1} U^H Y_2 V \tag{6}$$

where D, U and V are given by the singular value decomposition of Y_1.

$$Y_1 = \sum_{i=1}^{M} \sigma_i \mathbf{u}_i \mathbf{v}_i^H = UDV^H \tag{7}$$

$$Y_1^+ = VD^{-1}U^H \tag{8}$$

The superscript H denotes the conjugate transpose of a matrix. $\{s_i\}_{i=1}^M$ can be obtained from $\{z_i\}_{i=1}^M$ with (2).

3. PROCEDURE FOR EXTRACTING THE TRUE NATURAL FREQUENCIES

Let M_0 be the number of the true natural frequencies which should be known a priori in the GPOF method. It is known that natural frequencies occur in complex conjugate pairs for real valued transient response. Thus, the true natural frequencies can be represented by $\{s_i\}_{i=1}^{M_0/2}$ and $\{s_i{}^*\}_{i=1}^{M_0/2}$ where M_0 is the number of the true natural frequencies. In the first place, M_1 and M_2 which are sufficiently larger than the expected M_0 are chosen. Note that it is not necessary to know M_0 exactly to determine M_1 and M_2. The GPOF method is applied with $M = M_1 > M_0$, $M = M_2 > M_0$ and the extracted natural frequencies are denoted by s_i^a $(i = 1, \cdots, M_1)$, s_i^b $(i = 1, \cdots, M_2)$, respectively. In $\{s_i^a\}_{i=1}^{M_1}$ and $\{s_i^b\}_{i=1}^{M_2}$, there exist spurious natural frequencies because more natural frequencies than actually exist is assumed in the GPOF method. The objective is to select M_0 true natural frequencies out of $\{s_i^a\}_{i=1}^{M_1}$ and $\{s_i^b\}_{i=1}^{M_2}$. As previously stated, natural frequencies occur in complex conjugate pairs. In addition to it, the real part of the natural frequencies must be negative for the physical constraint of power balance. From $\{s_i^a\}_{i=1}^{M_1}$, the natural frequencies which do not satisfy the above requirement are excluded. The remaining natural frequencies are denoted by $\{s_i^A,\ s_i^{A*}\}_{i=1}^{M_A/2}$, and the corresponding natural frequencies for $\{s_i^b\}_{i=1}^{M_2}$ are denoted by $\{s_i^B,\ s_i^{B*}\}_{i=1}^{M_B/2}$ where M_A and M_B are the number of the natural frequencies satisfying the above requirement for $\{s_i^a\}$, $\{s_i^b\}$, respectively. Here M_A and M_B are less than or equal to M_1 and M_2, respectively. Let $\{s_i^c,\ s_i^{c*}\}_1^{M_3/2}$ denote the common natural frequencies of $\{s_i^A,\ s_i^{A*}\}_{i=1}^{M_A/2}$ and $\{s_i^B,\ s_i^{B*}\}_{i=1}^{M_B/2}$, where M_3 is the number of common natural frequencies.

For sufficiently large M_1 and M_2, the true natural frequencies will be extracted for both M_1 and M_2. But spurious natural frequencies extracted for M_1 may not be extracted for M_2. Thus, all of the true natural frequencies will belong to $\{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$. But the spurious natural frequencies may or may not belong to $\{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$. Consequently, $\{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$ contains at least all of the true natural frequencies. There are $M_3 - M_0$ spurious natural frequencies in $\{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$. How to determine M_0 and how to choose M_0 true natural frequencies out of M_3 natural frequencies will be explained in Section 4–Section 6.

Using the method in Section 4, it can be determined whether $\{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$ contains all of the true natural frequencies. If it does not include all of the true natural frequencies, this is because M_1 and M_2 used for the GPOF method are too small to include all of the true natural frequencies. So we must stop and increase M_1 and M_2. If it does include at least all of the true natural frequencies, the method in Section 5 is used with $p = 0$, $m = M_3$, $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2} = \{s_i^c,\ s_i^{c*}\}_{i=1}^{M_3/2}$ to check if there are spurious natural frequencies where $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ denotes the undetermined m natural frequencies. If there are no spurious natural frequencies, $M_0 = m$ and the true natural frequencies $\{s_i,\ s_i{}^*\}_{i=1}^{M_0/2}$ are given by the remaining natural frequencies of $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$. When there are spurious natural frequencies, the method in Section 6 is used. One natural frequency pair is excluded and m decreases by two. If a condition for the true natural frequencies which will be stated later is satisfied, the natural frequencies are selected as the true natural frequencies and excluded. Each time natural frequency pair is excluded, m decreases by two. Whenever the true natural frequency pair is excluded, p increases by two. Initial value for p is zero. Thus, p denotes the number of the selected true natural frequencies. The p selected true natural frequencies when there are m undetermined natural frequencies are denoted by $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$. Another natural frequency pair is excluded in the same manner until there are no spurious natural frequencies. The true natural frequencies are given by the p selected natural frequencies and finally remaining natural frequencies $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$.

4. TRANSIENT RESPONSE RECONSTRUCTION

Whether the specific natural frequencies $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$ contains at least all of the true natural frequencies $\{s_i,\ s_i^*\}_{i=1}^{M_0/2}$, can be determined using transient response reconstruction. The transient response of a scatterer with natural frequencies of $\{s_i,\ s_i^*\}_{i=1}^{M_0/2}$ can be represented as

$$y_k = \sum_{i=1}^{M_0/2} b_i \exp[s_i \delta t k] + \sum_{i=1}^{M_0/2} b_i{}^* \exp[s_i{}^* \delta t k] \quad k = 0, \ldots, N-1 \tag{9}$$

Given $\{y_k\}_0^{N-1}$ and $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$, $\{\bar{b}_i,\ \bar{b}_i^*\}_{i=1}^{m/2}$ is the least squares solution of (10).

$$y_k = \sum_{i=1}^{m/2} \bar{b}_i \exp[\bar{s}_i \delta t k] + \sum_{i=1}^{m/2} \bar{b}_i^* \exp[\bar{s}_i^* \delta t k] \quad k = 0, \ldots, N-1 \tag{10}$$

Usually the number of transient late time data N is greater than the number of natural frequencies m in (10). So (10) is an overdetermined system. Because $\{\bar{b}_i,\ \bar{b}_i^*\}_{i=1}^{m/2}$ is the least squares solution, it does not exactly satisfy equations in (10).

Using the obtained $\{\bar{b}_i,\ \bar{b}_i^*\}_{i=1}^{m/2}$, the transient response is reconstructed [5].

$$y_{(\mathrm{recon})k} \equiv \sum_{i=1}^{m/2} \bar{b}_i \exp[\bar{s}_i \delta t k] + \sum_{i=1}^{m/2} \bar{b}_i^* \exp[\bar{s}_i^* \delta t k] \quad k = 0, \ldots, N-1 \tag{11}$$

The following factor is defined with y_k and $y_{(\mathrm{recon})k}$;

$$\rho = 1 - \frac{\sum_{k=0}^{N-1} [y_k - y_{(\mathrm{recon})k}]^2}{\sum_{k=0}^{N-1} [y_k]^2} \tag{12}$$

Comparing (10) and (11), if there is no error in the least squares solution $\{\bar{b}_i,\ \bar{b}_i^*\}_{i=1}^{m/2}$, $y_{(\mathrm{recon})k}$ and y_k will coincide exactly and ρ is equal to unity. But because (10) is an overdetermined system, there is an inevitable error in the least squares solution $\{\bar{b}_i,\ \bar{b}_i^*\}_{i=1}^{m/2}$, $y_{(\mathrm{recon})k}$ and y_k will not coincide exactly, which means ρ can not be unity. If $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$ includes $\{s_i,\ {s_i}^*\}_{i=1}^{M_0/2}$, there will be little error and ρ is nearly unity. But if $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$ does not include $\{s_i,\ {s_i}^*\}_{i=1}^{M_0/2}$, there will be much error in the least squares solution and ρ will be much less than 1. A threshold value of $\rho_{\mathrm{threshold}}$ is chosen. If $\rho > \rho_{\mathrm{threshold}}$, it is assumed that $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$ includes $\{s_i,\ {s_i}^*\}_{i=1}^{M_0/2}$, Similarly if $\rho < \rho_{\mathrm{threshold}}$, it is assumed that $\{\bar{s}_i,\ \bar{s}_i^*\}_{i=1}^{m/2}$ does not include all of the true natural frequencies.

5. HOW TO CHECK THE EXISTENCE OF SPURIOUS NATURAL FREQUENCIES

Given the response of a target $\{y_k\}_{k=0}^{N-1}$, m undetermined natural frequencies $\{\hat{s}_i(m/2),\ \hat{s}_i{}^*(m/2)\}_{i=1}^{m/2}$, and previously selected true natural frequencies $\{s_i(m/2), s_i{}^*(m/2)\}_{i=1}^{p/2}$, it can be determined whether all of the undetermined natural frequencies $\{\hat{s}_i(m/2),\ \hat{s}_i{}^*(m/2)\}_{i=1}^{m/2}$ are the true natural frequencies.

Let $\{\hat{b}_i(m/2),\ \hat{b}_i{}^*(m/2)\}_{i=1}^{m/2}$, $\{b_i(m/2),\ b_i{}^*(m/2)\}_{i=1}^{p/2}$ denote the least squares solution of (13).

$$\begin{aligned} y_k \; = \; & \sum_{i=1}^{m/2} \hat{b}_i(m/2) \exp[\hat{s}_i(m/2)\,\delta t\, k] \\ & + \sum_{i=1}^{m/2} \hat{b}_i{}^*(m/2) \exp[\hat{s}_i{}^*(m/2)\,\delta t\, k] \\ & + \sum_{i=1}^{p/2} b_i(m/2) \exp[s_i(m/2)\,\delta t\, k] \\ & + \sum_{i=1}^{p/2} b_i{}^*(m/2) \exp[s_i{}^*(m/2)\,\delta t\, k] \quad k = 0, \ldots, N-1 \quad (13) \end{aligned}$$

Similarly, $\{\hat{b}_i(m/2, j),\ \hat{b}_i{}^*(m/2, j)\}_{i=1}^{m/2}$, $\{b_i(m/2, j),\ b_i{}^*(m/2, j)\}_{i=1}^{p/2}$ is the least squares solution of (14)

$$\begin{aligned} y_k \; = \; & \sum_{i=1,\, i \neq j}^{m/2} \hat{b}_i(m/2, j) \exp[\hat{s}_i(m/2)\,\delta t\, k] \\ & + \sum_{i=1,\, i \neq j}^{m/2} \hat{b}_i{}^*(m/2, j) \exp[\hat{s}_i{}^*(m/2)\,\delta t\, k] \\ & + \sum_{i=1}^{p/2} b_i(m/2, j) \exp[s_i(m/2)\,\delta t\, k] \\ + & \sum_{i=1}^{p/2} b_i{}^*(m/2, j) \exp[s_i{}^*(m/2)\,\delta t\, k] \quad k = 0, \ldots, N-1 \quad (14) \end{aligned}$$

As in Section 4, the reconstructed responses are defined.

$$\begin{aligned} y_{(\mathrm{recon})k}(m/2) \equiv & \sum_{i=1}^{m/2} \hat{b}_i(m/2) \exp[\hat{s}_i(m/2)\,\delta t\, k] \\ & + \sum_{i=1}^{m/2} \hat{b}_i{}^*(m/2) \exp[\hat{s}_i{}^*(m/2)\,\delta t\, k] \end{aligned}$$

$$+\sum_{i=1}^{p/2} b_i(m/2)\exp[s_i(m/2)\,\delta t\,k]$$
$$+\sum_{i=1}^{p/2} b_i{}^*(m/2)\exp[s_i{}^*(m/2)\delta t\,k]\;\; k=0,\ldots,N\text{–}1 \quad (15)$$

$y_{(\mathrm{recon})k}(m/2,\,j)$ is defined by the following equations for each $j = 1,\ldots,m/2$.

$$y_{(\mathrm{recon})k}(m/2,j) \equiv \sum_{i=1,\,i\neq j}^{m/2} \hat{b}_i(m/2,\,j)\exp[\hat{s}_i(m/2)\,\delta t\,k]$$
$$+\sum_{i=1,\,i\neq j}^{m/2} \hat{b}_i{}^*(m/2,\,j)\exp[\hat{s}_i{}^*(m/2)\,\delta t\,k]$$
$$+\sum_{i=1}^{p/2} b_i(m/2,j)\exp[s_i(m/2)\,\delta t\,k]$$
$$+\sum_{i=1}^{p/2} b_i{}^*(m/2,j)\exp[s_i{}^*(m/2)\delta tk]\;\; k=0,\ldots,N\text{–}1 \quad (16)$$

To discriminate the correlation factors of $y_{(\mathrm{recon})k}(m/2)$ and $y_{(\mathrm{recon})k}(m/2,\,j)$, $\rho(m/2)$ and $\rho(m/2,\,j)$ are defined.

$$\rho(m/2) = 1 - \frac{\sum_{k=0}^{N-1}[y_k - y_{(\mathrm{recon})k}(m/2)]^2}{\sum_{k=0}^{N-1}[y_k]^2} \quad (17)$$

$$\rho(m/2,j) = 1 - \frac{\sum_{k=0}^{N-1}[y_k - y_{(\mathrm{recon})k}(m/2,j)]^2}{\sum_{k=0}^{N-1}[y_k]^2} \quad j=1,\ldots,m/2. \quad (18)$$

Note that $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$ denotes the selected true natural frequencies when there are m undetermined natural frequencies. Thus, if $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ are the true natural frequencies, $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$ and $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ includes all of the true natural frequencies. According to Section 4, $\rho(m/2)$ is larger than $\rho_{\text{threshold}}$ because $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$, $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ contains all of the true natural frequencies. But $\rho(m/2, j)$ which is calculated without $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ will be less than $\rho_{\text{threshold}}$ because $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$, $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ without $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ does not include all of the true natural frequencies.

On the contrary, suppose there are spurious natural frequencies as well as the true natural frequencies in $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$. Even though there are spurious natural frequencies in $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$, $\rho(m/2)$ is greater than $\rho_{\text{threshold}}$ because there are all of the true natural frequencies in $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$ and $\{s_i(m/2),\ s_i{}^*(m/2)\}_{i=1}^{p/2}$. The number of the spurious natural frequencies is $m + p - M_0$. If $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ is the true natural frequency, $\{s_i,\ s_i{}^*\}_{i=1}^{p/2}$ and $\{\hat{s}_i(m/2),\ \hat{s}_i{}^*(m/2)\}_{i=1}^{m/2}$ without $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ does not include $\{s_i,\ s_i{}^*\}_{i=1}^{M_0/2}$. But if $\hat{s}_j$ is not the true natural frequency, $\{s_i,\ s_i{}^*\}_{i=1}^{p/2}$ and $\{\hat{s}_i(m/2),\ \hat{s}_i{}^*(m/2)\}_{i=1}^{m/2}$ without $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ does include $\{s_i,\ s_i{}^*\}_{i=1}^{M_0/2}$. Because of the spurious natural frequencies, there will be some j whose $\rho(m/2, j)$ is larger than $\rho_{\text{threshold}}$ since transient response can be reconstructed without one spurious natural frequency pair.

6. HOW TO EXCLUDE SPURIOUS NATURAL FREQUENCY PAIR AND TRUE NATURAL FREQUENCY PAIR

As previously stated, if $\rho(m/2, j) > \rho_{\text{threshold}}$ for certain j, there are spurious natural frequencies in $\{\hat{s}_i(m/2),\ \hat{s}_i^*(m/2)\}_{i=1}^{m/2}$. To select true natural frequency pair and spurious natural frequency pair, the following equation is defined using $\hat{b}_i(m/2)$, $\hat{b}_i(m/2, j)$, $b_i(m/2)$ and $b_i(m/2, j)$ of (13) and (14).

$$d(m/2,j) = \sum_{i=1,i\neq j}^{m/2}\left|\frac{\hat{b}_i(m/2)-\hat{b}_i(m/2,\,j)}{\hat{b}_i(m/2)}\right| + \sum_{i=1}^{p/2}\left|\frac{b_i(m/2)-b_i(m/2,\,j)}{b_i(m/2)}\right| \quad j=1,\ldots,m/2 \tag{19}$$

$d(m/2, j)$ represent the sum of the difference of the obtained least squares solution with and without $\hat{s}_j(m/2)$ and $\hat{s}_j^*(m/2)$.

Thus, large $d(m/2, j)$ means $\{\hat{s}_j(m/2),\ \hat{s}_j^*(m/2)\}$ has much effect on the least squares solution. Let $j_{\min}$ and $j_{\max}$ denote j whose $d(m/2,\, j)$ is the minimum and the maximum of all $\{d(m/2,\, j)\}_{i=1}^{m/2}$ respectively. $\{\hat{s}_{j_{\min}}(m/2),\ \hat{s}_{j_{\min}}^*(m/2)\}$ has the least effect on least squares solution and $\{\hat{s}_{j_{\max}}(m/2),\ \hat{s}_{j_{\max}}^*(m/2)\}$ has the greatest effect on least squares solution. It is assumed that $\{\hat{s}_{j_{\min}}(m/2),\ \hat{s}_{j_{\min}}^*(m/2)\}$ is spurious natural frequency pair and is excluded from $\{\hat{s}_i(m/2),\ \hat{s}_i{}^*(m/2)\}_{i=1}^{m/2}$. If $d(m/2, j_{max})$ is much larger than $d(m/2, j_{min})$, $\{\hat{s}_{j_{\max}}(m/2),\ \hat{s}_{j_{\max}}^*(m/2)\}$ is considered to be the true natural frequency pair. As a rule of thumb, whether $d(m/2, j_{max})$ is larger than ten times $d(m/2, j_{min})$ or not is used as a criterion. The flowchart for extraction of the natural frequencies is given in Fig. 1.

7. NUMERICAL RESULTS

To justify the proposed scheme for natural frequency extraction, computer simulation was performed. Scattering data were generated using a frequency-domain method-of-moments solution. A piecewise-sinusoidal basis function is employed and thin-wire approximation was used. The backscattering complex field values were calculated at 64 and 128 equally spaced frequencies. The frequency step is 7.8 MHz. An inverse Fourier transform was subsequently applied to these results to obtain the simulated transient response. The Gaussian random noise is added to the transient response for noise simulation. Each point of the thin wire transient response is perturbed with a Gaussian noise. The signal to noise ratio is defined as follows.

$$S/N\ (\text{dB}) = 10 \log \frac{1}{\sigma^2} \sum_{i=0}^{\gamma-1} \frac{|y_i|^2}{\gamma} \tag{20}$$

where σ^2 is the variance of Gaussian noise and $\{y_i\}_{i=0}^{\gamma-1}$ are the late time transient data and γ is the number of data considered. The angular frequency range covered with 64 frequency responses and 128 frequency responses are

$$7.8 \times 10^6 \times 63 \times 2\pi \,=\, 3.088 \times 10^9\ (\text{rad}) \tag{21}$$

$$7.8 \times 10^6 \times 127 \times 2\pi \,=\, 6.224 \times 10^9\ (\text{rad}) \tag{22}$$

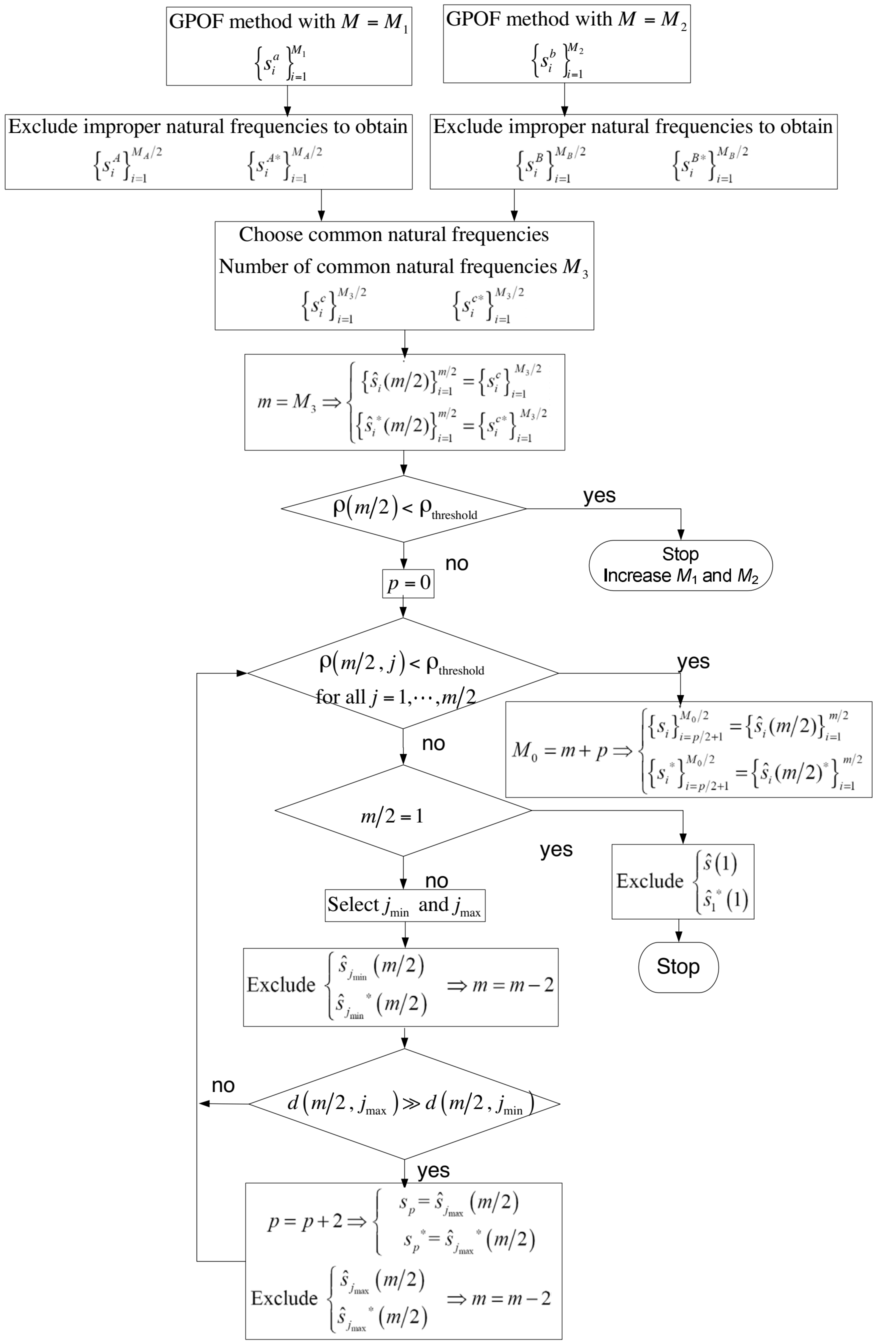

Figure 1. Flowchart for extraction of the natural frequencies.

Table 1. First ten natural frequencies pairs of thin wire($L/a = 200$).

	normalized	unnormalized (L=1m) ($\times 10^9$)
1	-0.0828 +j 0.9251	-0.07803 +j 0.8719
	-0.0828 -j 0.9251	-0.07803 -j 0.8719
2	-0.1212 +j 1.912	-0.1142 +j 1.802
	-0.1212 -j 1.912	-0.1142 -j 1.802
3	-0.1491 +j 2.884	-0.1405 +j 2.718
	-0.1491 -j 2.884	-0.1405 -j 2.718
4	-0.1713 +j 3.874	-0.1614 +j 3.651
	-0.1713 -j 3.874	-0.1614 -j 3.651
5	-0.1909 +j 4.854	-0.1799 +j 4.574
	-0.1909 -j 4.854	-0.1799 -j 4.574
6	-0.2080 +j 5.845	-0.1960 +j 5.509
	-0.2080 -j 5.845	-0.1960 -j 5.509
7	-0.2240 +j 6.829	-0.2111 +j 6.436
	-0.2240 -j 6.829	-0.2111 -j 6.436
8	-0.2383 +j 7.821	-0.2245 +j 7.371
	-0.2383 -j 7.821	-0.2245 -j 7.371
9	-0.2522 +j 8.807	-0.2376 +j 8.301
	-0.2522 -j 8.807	-0.2376 -j 8.301
10	-0.2648 +j 9.800	-0.2495 +j 9.237
	-0.2648 -j 9.800	-0.2495 -j 9.237

Table 1 shows the first ten natural frequencies pairs of thin wire with length/radius = 200 [1]. In Table 1, it can be seen that the imaginary part of the first three natural frequencies pairs are below 3.088×10^9 (rad) and the imaginary part of the first six natural frequencies pairs are below 6.224×10^9 (rad). Thus, the first three natural frequencies pairs are contained in the simulated transient response when using 64 frequency responses, and the first six natural frequencies pairs are contained in the simulated transient response when using 128 frequency responses. From sampling theorem, the sampling interval must be not more than π/ω_m apart where ω_m is the highest angular frequency component contained in the signal. Thus, from (21) and (22), sampling interval must be shorter than 10.17×10^{-10} for the extraction of the first three natural frequencies pairs and 5.048×10^{-10} for the extraction of the first six natural frequencies pairs.

Using the transient response of SNR of 20 dB obtained from the 64 frequency-responses, the first three natural frequencies pairs can be extracted. In these examples, $\rho_{\text{threshold}}$ of 0.9 is used and sampling interval is 4×10^{-10} which is in the range of 11.5×10^{-11}. Here we choose M_1 of 20 and M_2 of 18. The GPOF method is applied with $L = 20$, $M_1 = 20$, $N = 40$ and $L = 20$, $M_2 = 18$, $N = 40$. In this case, M_A, M_B, and M_3 happen to be 20, 16 and 14, respectively. Fourteen common natural frequencies and the subsequent discrimination procedure is illustrated in Table 2. In Table 2, only the natural frequencies whose imaginary part is positive are shown. With $m = M_3 = 14$, $\rho(7)$ is larger than $\rho_{\text{threshold}}$ and $\{\rho(7,i)\}_{i=1}^{7}$ are not less than $\rho_{\text{threshold}}$ for all $i = 1, \ldots, 7$. Thus, according to Section 5, it is assumed that there are spurious natural frequencies. To exclude natural frequency pairs, $\{d(7,i)\}_{i=1}^{7}$ are calculated and the minimum and the maximum of all $\{d(7,i)\}_{i=1}^{7}$ are chosen. From Table 2, it is observed that $d(7,3)$ is the minimum and $d(7,1)$ is the maximum. $\hat{s}_3(7)$ and $\hat{s}_3^*(7)$ are excluded and m decreases by 2 from 14 to 12. Since $d(7,1)$ is larger than ten times $d(7,3)$, $\{\hat{s}_1(7),\ \hat{s}_1^*(7)\}$ is selected as $\{s_1,\ s_1{}^*\}$ and excluded. p increases by two from zero to two and m decreases by two. With the remaining ten natural frequencies of $\{\hat{s}_i(5),\ \hat{s}_i{}^*(5)\}_{i=1}^{5}$, $\{d(5,i)\}_{i=1}^{5}$ are calculated. Also $\rho(5)$ is larger than $\rho_{\text{threshold}}$ and $\{\rho(5,i)\}_{i=1}^{5}$ are not less than $\rho_{\text{threshold}}$ for all $i = 1, \ldots, 5$, implying the existence of spurious natural frequencies. The minimum and the maximum of $\{d(5,i)\}_{i=1}^{5}$ are chosen. The minimum is $d(5,4)$ and the fourth natural frequency pair is excluded and m decreases from ten to eight. Condition for true natural frequency is satisfied and $\{\hat{s}_1(5),\ \hat{s}_1^*(5)\}$ is selected as $\{s_2,\ s_2{}^*\}$ and excluded. p increases from two to four and m decreases from eight to six. It is repeated until $\{d(m/2,i)\}_{i=1}^{m/2}$ are less than $\rho_{\text{threshold}}$ for all $i = 1, \cdots, m/2$. In this example, $\{\rho(1,i)\}_{i=1}^{1}$ is less than $\rho_{threshold}$. Thus, $\{\hat{s}_1(1),\ \hat{s}_1^*(1)\}$ is the true natural frequency pair. Fig. 2 illustrates the exact natural frequencies, natural frequencies extracted using the GPOF method with $L = 20$, $M_0 = 6$, $N = 40$ and natural frequencies extracted using our method. It is shown that the natural frequencies extracted by our method is more accurate. With the same L, M_1, M_2, M_0 and N, the corresponding results for SNR of 30 dB are presented in Table 3 and Fig. 3.

If more frequency responses which are obtained by method-of-moments solution are used, the number of the extracted natural frequencies increases. As previously stated, using the transient response obtained from the 128 frequencies responses, the first six

Table 2. Selecting true natural frequencies (SNR = 20 dB, three natural frequencies pairs).

$\rho(7)$		0.994938		state
i	$\hat{s}_i(7)$	$\rho(7,i)$	$d(7,i)$	
1	-0.0906607 +j 0.911136	0.891326	26.1098	true (s_1)
2	-0.133584 +j 1.88521	0.982303	10.22956	undetermined
3	-0.0923101 +j 5.85415	0.994347	1.17856	spurious
4	-0.161559 +j 4.74416	0.993366	2.40442	undetermined
5	-0.137720 +j 3.76205	0.989987	4.58826	undetermined
6	-0.0489817 +j 3.28859	0.986676	1.19432	undetermined
7	-0.154516 +j 2.89268	0.987316	4.70505	undetermined
$\rho(5)$		0.994347		state
i	$\hat{s}_i(5)$	$\rho(5,i)$	$d(5,i)$	
1	-0.133584 +j 1.88521	0.982046	10.00019	true (s_2)
2	-0.161559 +j 4.74416	0.993227	1.30021	undetermined
3	-0.137720 +j 3.76205	0.989625	4.17326	undetermined
4	-0.0489817 +j 3.28859	0.986168	0.891189	spurious
5	-0.154516 +j 2.89268	0.986575	4.66208	undetermined
$\rho(3)$		0.986168		state
i	$\hat{s}_i(3)$	$\rho(3,i)$	$d(3,i)$	
1	-0.161559 +j 4.74416	0.984985	1.04358	spurious
2	-0.137720 +j 3.76205	0.977552	2.88703	undetermined
3	-0.154516 +j 2.89268	0.973224	4.40101	undetermined
$\rho(2)$		0.984985		state
i	$\hat{s}_i(2)$	$\rho(2,i)$	$d(2,i)$	
1	-0.137720 +j 3.76205	0.975040	1.80875	spurious
2	-0.154516 +j 2.89268	0.968638	2.39563	undetermined
$\rho(1)$		0.975040		state
i	$\hat{s}_i(1)$	$\rho(1,i)$	$d(1,i)$	
1	-0.154516 +j 2.89268	0.456913	1.41963	true (s_3)

natural frequencies pairs can be extracted. The results for SNR of 20 dB with $M_1 = 20$, $M_2 = 18$ are shown in Table 4 and Fig. 4. The GPOF methods with $L = 20$, $M_1 = 20$, $N = 40$ and $L = 20$, $M_2 = 18$, $N = 40$ are used. The procedure for true natural frequency selection is shown in Table 4. The conventional GPOF method is applied with $L = 20$, $M_0 = 12$, $N = 40$ and the result is compared with the natural frequencies extracted using our method in Fig. 4. The corresponding

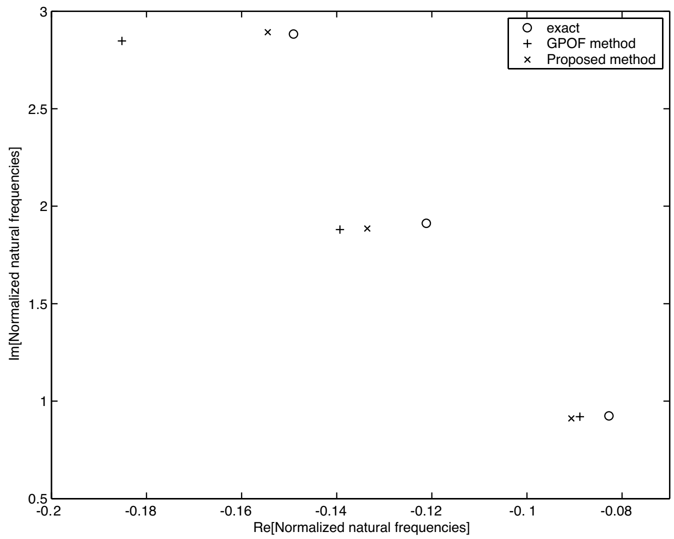

Figure 2. Extracted natural frequencies (SNR = 20 dB, three true natural frequencies pairs).

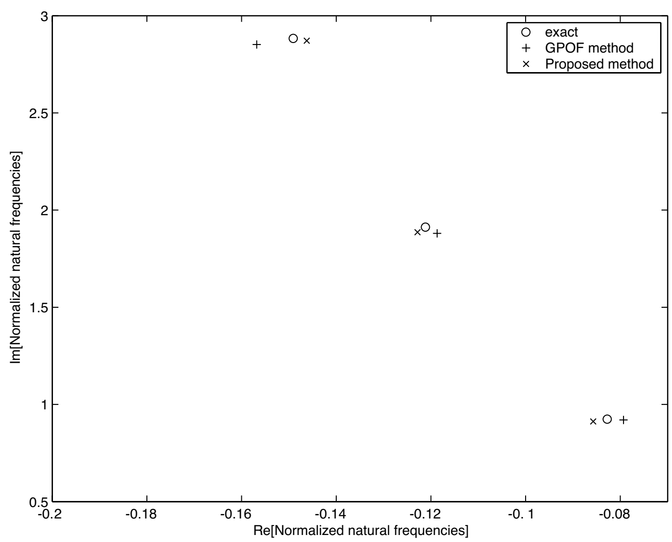

Figure 3. Extracted natural frequencies (SNR = 30 dB, three true natural frequencies pairs).

Table 3. Selecting true natural frequencies (SNR = 30 dB, three true natural frequencies pairs).

$\rho(5)$		0.989796		state
i	$\hat{s}_i(5)$	$\rho(5,i)$	$d(5,i)$	
1	-0.0857292+j 0.912428	0.581495	73.9304	true (s_1)
2	-0.122845 +j 1.88618	0.989253	3.86233	undetermined
3	-0.122662+j 4.77972	0.989695	1.74639	spurious
4	-0.143351+j 4.08567	0.989334	1.86280	undetermined
5	-0.146231 +j 2.87137	0.844520	66.1954	undetermined
$\rho(3)$		0.989695		state
i	$\hat{s}_i(3)$	$\rho(3,i)$	$d(3,i)$	
1	-0.122845 +j 1.88618	0.989253	21.3006	undetermined
2	-0.143351 +j 4.08567	0.989216	1.03916E-01	spurious
3	-0.146231 +j 2.87137	0.819969	32.0684	true (s_2)
$\rho(1)$		0.989216		state
i	$\hat{s}_i(1)$	$\rho(1,i)$	$d(1,i)$	
1	-0.122845 +j 1.88618	0.503264	1.19583	true (s_3)

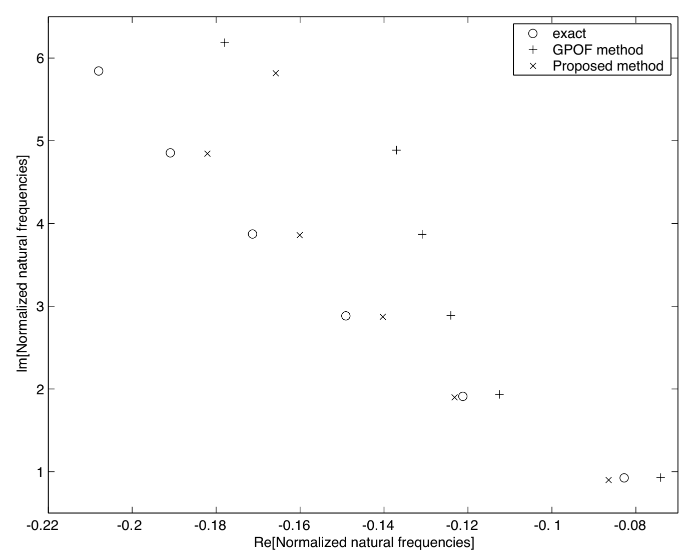

Figure 4. Extracted natural frequencies (SNR = 20 dB, six true natural frequencies pairs).

Table 4. Selecting true natural frequencies (SNR = 20 dB, six true natural frequencies pairs).

$\rho(8)$		0.995117		state
i	$\hat{s}_i(8)$	$\rho(8,i)$	$d(8,i)$	
1	-0.0864837 +j 0.899732	0.780497	6.67516	undetermined
2	-0.123205 +j 1.89848	0.710686	13.9417	undetermined
3	-0.140275 +j 2.87221	0.744482	17.4737	undetermined
4	-0.160085 +j 3.85850	0.789838	20.2802	true(s_1)
5	-0.182086 +j 4.84443	0.856405	14.4648	undetermined
6	-0.165811 +j 5.81579	0.989899	6.90813	undetermined
7	-0.0314307 +j 6.64646	0.965921	5.57765	undetermined
8	-0.144739 +j 7.56264	0.993917	1.30921	spurious
$\rho(6)$		0.993917		state
i	$\hat{s}_i(6)$	$\rho(6,i)$	$d(6,i)$	
1	-0.0864837 +j 0.899732	0.780482	3.67084	undetermined
2	-0.123205 +j 1.89848	0.709198	7.78125	undetermined
3	-0.140275 +j 2.87221	0.739141	9.56850	undetermined
4	-0.182086 +j 4.84443	0.846527	10.08546	true (s_2)
5	-0.165811 +j 5.81579	0.891441	2.78013	undetermined
6	-0.0314307 +j 6.64646	0.960090	0.647117	spurious
$\rho(4)$		0.960090		state
i	$\hat{s}_i(4)$	$\rho(4,i)$	$d(4,i)$	
1	-0.0864837 +j 0.899732	0.742198	0.978050	true (s_3)
2	-0.123205 +j 1.89848	0.663337	2.15259	true (s_4)
3	-0.140275 +j 2.87221	0.686860	2.38625	true (s_5)
4	-0.165811 +j 5.81579	0.838084	1.62200	true (s_6)

results for SNR of 30 dB are shown in Table 5 and Fig. 5.

Here it is shown that in the GPOF method, the true M_0 natural frequencies can be extracted more accurately with $M = \bar{M} > M_0$ than $M = M_0$. It is assumed that M_0 true natural frequencies can be selected from $\bar{M}$ natural frequencies when $M = \bar{M} > M_0$ is used. Thus, the accuracy of M_0 natural frequencies selected from $\bar{M}$ natural

Table 5. Selecting true natural frequencies (SNR = 30 dB, six true natural frequencies pairs).

$\rho(8)$		0.999701		state
i	$\hat{s}_i(8)$	$\rho(8,i)$	$d(8,i)$	
1	-0.0787733 +j 0.910284	0.773537	15.8967	undetermined
2	-0.119753 +j 1.88676	0.698721	35.6887	undetermined
3	-0.147986 +j 2.86483	0.742614	45.4355	true (s_1)
4	-0.169511 +j 3.84825	0.810425	44.8951	undetermined
5	-0.190466 +j 4.83495	0.874322	31.0695	undetermined
6	-0.197847 +j 5.81651	0.997197	15.3090	undetermined
7	-0.0185108 +j 6.64673	0.968546	18.8467	undetermined
8	-0.211369 +j 7.57315	0.999638	0.359863	spurious
$\rho(6)$		0.999638		state
i	$\hat{s}_i(6)$	$\rho(6,i)$	$d(6,i)$	
1	-0.0787733 +j 0.910284	0.772598	4.0551	undetermined
2	-0.119753 +j 1.88676	0.692282	9.01392	undetermined
3	-0.169511 +j 3.84825	0.794539	12.0660	undetermined
4	-0.190466 +j 4.83495	0.861544	10.21415	true (s_2)
5	-0.197847 +j 5.81651	0.904400	3.72376	undetermined
6	-0.0185108 +j 6.64673	0.963971	0.662052	spurious
$\rho(4)$		0.963971		state
i	$\hat{s}_i(4)$	$\rho(4,i)$	$d(4,i)$	
1	-0.0787733 +j 0.910284	0.732482	0.969457	true (s_3)
2	-0.119753 +j 1.88676	0.643776	2.29281	true (s_4)
3	-0.169511 +j 3.84825	0.736562	2.51375	true (s_5)
4	-0.197847 +j 5.81651	0.849300	1.72013	true (s_6)

frequencies obtained using the GPOF method with $M = \bar{M}$ and the accuracy of the M_0 natural frequencies obtained using the GPOF method with $M = M_0$ are compared.

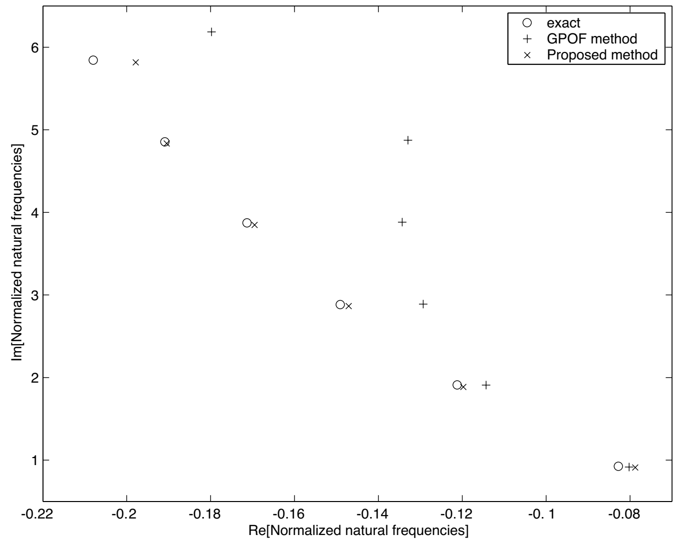

Figure 5. Extracted natural frequencies (SNR = 30 dB, six true natural frequencies pairs).

8. CONCLUSION

A scheme for natural frequency extraction is proposed in this paper. In the proposed scheme, the exact number of the true natural frequencies need not be known a priori because a GPOF method is applied with the overestimated number of natural frequencies. A GPOF method is applied twice with the overestimated number of natural frequencies to obtain two natural frequencies sets. Natural frequencies satisfying complex conjugate constraint with negative real part are selected to obtain two new natural frequencies sets. The common natural frequencies of two new natural frequencies sets are selected. From the common natural frequencies, spurious natural frequency pair and true natural frequency pair are excluded one after another using the suggested method until all the spurious natural frequencies are removed. The true natural frequencies are given by the excluded true natural frequencies and finally remaining natural frequencies. The spurious natural frequencies can be discriminated since the absence of the spurious natural frequencies has little effect on the transient response reconstruction. In the proposed scheme, the transient response reconstruction with all natural frequencies and without one specific natural frequency pair is used.

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