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# EDDY CURRENTS IN LAMINATED RECTANGULAR CORES

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**Abstract**—A simplified expression for the eddy current loss in laminated rectangular core is obtained using linear electromagnetic field analysis. The treatment takes cognizance of current interruption phenomena, by considering capacitive effects of insulation regions. Analysis presented in this paper assumes identical field distribution in each lamination and ignores eddy currents in insulation regions.

## 1. INTRODUCTION

Eddy currents are induced in conductors subjected to transient electromagnetic fields [1–7]. Eddy currents are also produced due to periodically time-varying excitations [8–17]. Alternating magnetic flux in transformer cores induces eddy currents resulting in eddy current loss. Time-invariant magnetic flux is established in the poles of a synchronous machine operating under steady state conditions. The pole-shoes, however, are subjected to pulsating magnetic field due to slotted armature surface. To reduce eddy current loss, transformer cores and pole-shoes of synchronous machines are invariably laminated.

Eddy current phenomena in laminated cores have been studied [8–11] using electromagnetic field analysis. For this purpose the laminated core is substituted by an equivalent homogeneous core with anisotropic conductivity. The treatment is simple and results are concise. However, the values of conductivity in the directions parallel and perpendicular to laminations are to be selected rather empirically.

Theoretical and experimental investigations of tooth-ripple phenomena in laminated pole-shoes are reported [12–17] in literature. In their treatment for eddy currents in laminated pole-shoes, authors [15, 16] have considered infinite half-space filled with identical laminations of arbitrary thickness. It is assumed that insulation regions of negligible thickness, restrains eddy currents in one lamination from flowing into another. Resulting equations for eddy current loss are quite complicated. Simplified version of these equations has been also reported [17].

In an earlier paper [18], it has been indicated that if a high-resistive region is introduced in the middle of a conductive core, it amounts to insertion of distributed capacitance in eddy current path. This reduces eddy current and eddy current loss. The current-interruption phenomena, thus conceived, have been taken into cognizance in developing the expression for eddy current loss in rectangular laminated cores.

In a laminated core, the field distribution in a lamination depends on the position of the lamination in the stack. The variation in field distribution from lamination to lamination is due to the finite stack-thickness. In a core with large stack-thickness, this variation is small for laminations located near the middle of the stack. The general solution that takes cognizance of the finite stack-thickness effects is lengthy and the resulting equation for eddy current loss is indeed complicated. It can, however, be used for computer aided optimization studies.

A simplified treatment for eddy currents in laminated cores is presented here. It is assumed that the core consists of a large number of laminations so that the field distribution in each lamination is identical. Further, the simplified treatment ignores eddy currents in insulation regions by setting zero conductivity for these regions. The advantage of the analytical approach developed here is that it provides a better understanding of eddy current loss over larger range of parameter values.

## 2. FIELD EQUATIONS

Consider a rectangular core consisting of n insulated laminations, each of width $W$ and overall thickness $T$. Let the insulation thickness on each side of a lamination be $T_1/2$ and its iron thickness be $T_2$. Further, let the corners of the core be located at $(-W/2, 0)$, $(W/2, 0)$, $(-W/2, nT)$ and $(W/2, nT)$, as shown in Fig. 1. In this figure, insulation regions are indicated as Region-0', 1', 2', 3', $\ldots$, $m'$, $\ldots$, $n'$. The iron regions are indicated as Region-1, 2, 3,$\ldots$, $m$, $\ldots$, $n$. The exciting coil wound around the long rectangular core, carrying alternating current

$$i = Ie^{j\omega t} \tag{1}$$

is simulated by a surface current density with a peak value which is modulus of the complex quantity:

$$J_o = I \cdot N \tag{2}$$

where $N$ is the number of turns per unit length of the coil. The current carrying coil will produce time varying magnetic filed in the core, eddy currents in the conducting regions and displacement currents in the insulation regions of the core. The magnetic field outside the coil is neglected. For the long rectangular core with uniformly distributed current sheet, the magnetic field is entirely axial and independent of $z$-coordinate, along the axial direction. It is assumed that the permeability $\mu$ for the iron regions, permittivity $\varepsilon$, for the insulation regions, and conductivity ($\sigma$, $\sigma'$), for both types of regions, are constant. Thus from Maxwell's equations for harmonic fields, in charge-free regions:

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = \gamma^2 H_z \tag{3}$$

for iron regions, where

$$\gamma = \sqrt{(-j\omega\mu)(\sigma + j\omega\varepsilon_0)} \tag{3.1}$$

and

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = -\left(\gamma'\right)^2 H_z \tag{4}$$

for insulation regions, where

$$\gamma' = \sqrt{(-j\omega\mu_0)\left(\sigma' + j\omega\varepsilon\right)} \tag{4.1}$$

For perfect insulation, $\sigma$' is zero.

Solutions of Eqs. (3) and (4) can be used to determine the components of electric fields in iron and insulation regions, since for iron regions:

$$E_x = \frac{1}{\sigma + j\omega\varepsilon_0}\frac{\partial H_z}{\partial y} \tag{5.1}$$

and

$$E_y = -\frac{1}{\sigma + j\omega\varepsilon_0}\frac{\partial H_z}{\partial z} \tag{5.2}$$

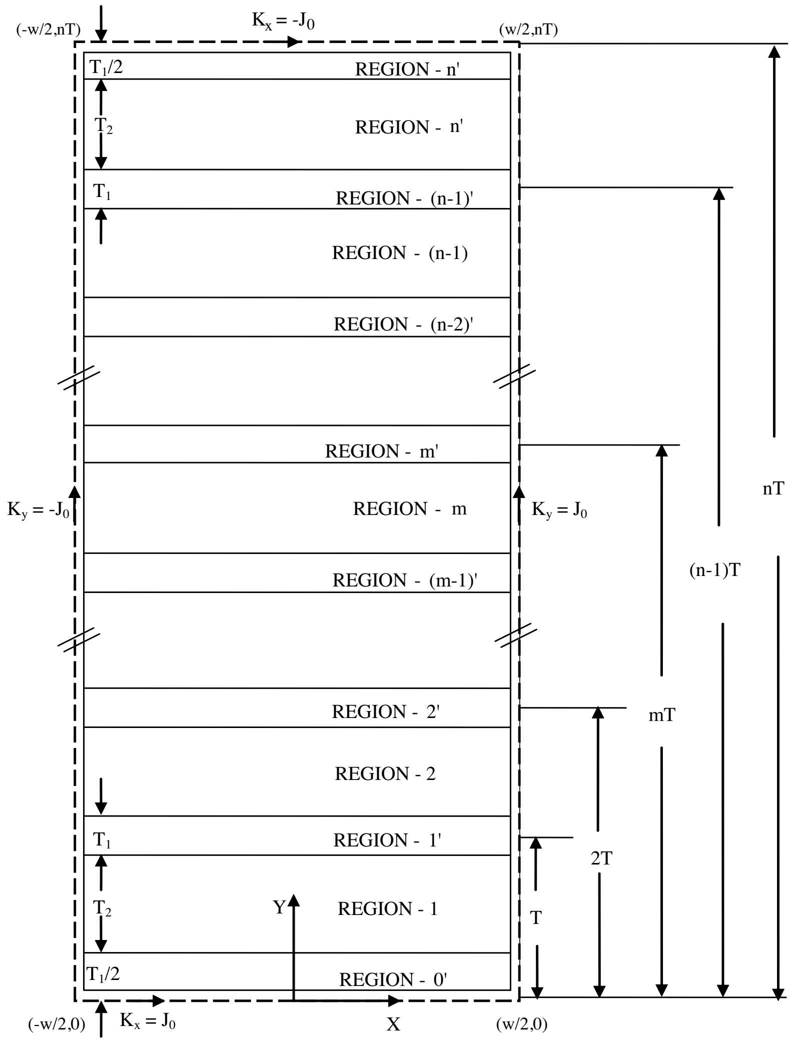


**Figure 1.** Cross-sectional view of a laminated rectangular core.

While, for insulation region:

$$E_x = \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} \tag{6.1}$$

and

$$E_y = -\frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} \tag{6.2}$$

## 3. FIELD DISTRIBUTIONS

Consider Fig. 1. In view of Eq. (3), the distribution of magnetic field in the conducting region-$m$ can be given as:

$$H_{zm} = J_0 \frac{\cos\left(\dfrac{p\pi}{W}x\right)}{\cos\left(\dfrac{p\pi}{W}x\right)} + \sum_{q=1}^{\infty} a_p \frac{\cosh \alpha_p (y - mT + T/2)}{\cosh(\alpha_p T_2/2)} \cos\left(\frac{p\pi}{W}x\right) \quad (7)$$

where,

$$p = 2q - 1, \quad (7.1)$$

indicating odd integer numbers, and

$$\alpha_p = \sqrt{\left(\frac{p\pi}{W}\right)^2 - \gamma^2} \quad (7.2)$$

while $a_p$ indicates a set of arbitrary constants.

The components of electric field in this region are

$$E_{xm} = \frac{1}{\sigma + j\omega\varepsilon_0} \sum_{q=1}^{\infty} a_p \alpha_p \frac{\sinh \alpha_p (y - mT + T/2)}{\cosh(\alpha_p T_2/2)} \cos\left(\frac{p\pi}{W}x\right) \quad (8.1)$$

and

$$E_{ym} = \frac{J_0 \gamma}{\sigma + j\omega\varepsilon_0} \frac{\sin(\gamma x)}{\cos(\gamma W/2)} + \sum_{q=1}^{\infty} \frac{p\pi/W}{\sigma + j\omega\varepsilon_0} a_p \frac{\cosh \alpha_p (y - mT + T/2)}{\cosh(\alpha_p T_2/2)} \sin\left(\frac{p\pi}{W}x\right) \quad (8.2)$$

The magnetic field distribution in the non-conducting region-$m'$, in view of Eq. (4), can be given as:

$$H_{zm'} = J_0 \frac{\cos(\gamma' x)}{\cos(\gamma' W/2)} + \sum_{q=1}^{\infty} b_p \frac{\cosh \alpha'_p (y - mT)}{\cosh\left(\alpha'_p T_1/2\right)} \cos\left(\frac{p\pi}{W}x\right) \quad (9)$$

where,

$$\alpha'_p = \sqrt{\left(\frac{p\pi}{W}\right)^2 - (\gamma')^2} \quad (9.1)$$

while $b_p$ indicates a set of arbitrary constants. The components of electric field in this region are given as:

$$E_{xm'} = \sum_{q=1}^{\infty} \frac{\alpha'_p}{j\omega\varepsilon} b_p \frac{\sinh \alpha'_p (y - mT)}{\cosh(\alpha'_p T_1/2)} \cos\left(\frac{p\pi}{W} x\right) \tag{10.1}$$

and

$$E_{ym'} = J_0 \frac{\gamma'}{j\omega\varepsilon} \frac{\sin(\gamma' x)}{\cos(\gamma' W/2)} + \sum_{q=1}^{\infty} \frac{p\pi/W}{j\omega\varepsilon} b_p \frac{\cosh \alpha'_p (y - mT)}{\cosh\left(\alpha'_p T_1/2\right)} \sin\left(\frac{p\pi}{W} x\right) \tag{10.2}$$

Now, since

$$H_{zm} = H_{zm'} \quad \text{at } \; y = mT - T_1/2, \quad \text{over} - W/2 \le x \le W/2 \tag{11.1}$$

and

$$E_{xm} = E_{xm'} \quad \text{at } \; y = mT - T_1/2\,, \; \text{over} - W/2 \le x \le W/2 \tag{11.2}$$

Therefore one gets:

$$a_p - b_p = \frac{4}{\pi} J_0 \sin\left(\frac{p\pi}{2}\right) \left[\frac{(\gamma' W/\pi)}{p^2 - (\gamma' W/\pi)^2} - \frac{(\gamma\, W/\pi)}{p^2 - (\gamma\, W/\pi)^2}\right] \tag{12.1}$$

and

$$\frac{\alpha_p}{\sigma + j\omega\varepsilon_0} \tanh(\alpha_p T_2/2)\, a_p = \frac{\alpha'_p}{j\omega\varepsilon} \tanh(\alpha'_p T_1/2)\, b_p \tag{12.2}$$

Arbitrary constants found by solving these equations are:

$$a_p = \frac{F_p \alpha'_p}{j\omega\varepsilon} \tanh(\alpha'_p T_1/2) \tag{13.1}$$

and

$$b_p = \frac{F_p \alpha_p}{\sigma + j\omega\varepsilon_0} \tanh(\alpha_p T_2/2) \tag{13.2}$$

where,

$$F_p = \frac{\dfrac{4}{W} J_0 \sin\left(\dfrac{p\pi}{2}\right) \left[\dfrac{\gamma'}{(\alpha'_p)^2} - \dfrac{\gamma}{(\alpha_p)^2}\right]}{\dfrac{\alpha'_p \tanh(\alpha'_p T_1/2)}{j\omega\varepsilon} - \dfrac{\alpha_p \tanh(\alpha_p T_2/2)}{\sigma + j\omega\varepsilon_0}} \tag{14}$$

## 4. EDDY CURRENT LOSS

Using Poynting theorem, the complex power input per unit core-length, for $n$-number of laminations is:

$$P_c = -n\left[\int\limits_{mT-T_1/2-T_2}^{mT-T_1/2} E_{ym}\cdot H^*_{zm}dy\right]_{x=W/2} +n\left[\int\limits_{W/2}^{-W/2} E_{xm}\cdot H^*_{zm}dx\right]_{y=mT-T_1/2} \quad (15)$$

Therefore, the expression for the complex power $P_c$, found is:

$$\begin{aligned} P_c \;=\; & -nJ^*_0\left[\frac{J_0\gamma}{\sigma+j\omega\varepsilon_0}\tan(\gamma W/2)T_2 \right. \\ & \left. +\frac{1}{\sigma+j\omega\varepsilon_0}\sum_{q=1}^{\infty}a_p\frac{p\pi}{W}\tanh(\alpha_pT_2/2)\sin\left(\frac{p\pi}{2}\right)\frac{2}{\alpha_p}\right] \\ & +\left[\begin{array}{l}\dfrac{nJ^*_0}{\sigma+j\omega\varepsilon_0}\displaystyle\sum_{q=1}^{\infty}a_p\alpha_p\tanh(\alpha_pT_2/2)\sin\left(\frac{p\pi}{2}\right)\frac{2\gamma^*}{(\alpha^*_p)^2} \\ +\dfrac{n}{\sigma+j\omega\varepsilon_0}\displaystyle\sum_{q=1}^{\infty}a_pa^*_p\alpha_p\tanh(\alpha_pT_2/2)\frac{W}{2}\end{array}\right] \end{aligned} \quad (16)$$

The eddy current loss per unit core-length is the real part of this complex power, i.e.,

$$P_e = \Re e[P_c] \quad (17)$$

## 5. APPROXIMATIONS

In view of common numerical values for various parameters, one may find further simplified, although approximate, expressions for $P_c$ and $P_e$. Since $\sigma$, the conductivity of iron is a large quantity, and $\gamma$' is very small, in view of Eqs. (13.1) and (14):

$$a_p \approx -\frac{4}{W}J_0\sin\left(\frac{p\pi}{2}\right)\frac{\gamma}{\alpha_p^2} \quad (18)$$

Further, from Eqs. (7.1) and (7.2), it may be seen that for large values of $p$ (i.e., for $q > Q$, say):

$$\alpha_p \approx \frac{p\pi}{W} \quad (19.1)$$

and

$$\tanh(\alpha_p T_2/2) \approx 1 \tag{19.2}$$

Further, for small values of $p$, (i.e., for $q \leq Q$, say):

$$\alpha_p \approx j\gamma \tag{20.1}$$

The value of Q should be chosen so as to satisfy Eqs. (19.1) and (20.1), as best as possible, for a given set of parameter values. Now $T_2$ being small:

$$\tanh(\alpha_p T_2/2) \approx \alpha_p T_2/2 \approx j\gamma T_2/2 \tag{20.2}$$

Therefore, one gets an approximate expression for $P_c$ as:

$$\begin{aligned} P_c \approx\ & n\frac{J_0 J_0^*}{\sigma}\left[\sum_{q=1}^{\infty}\frac{8}{\pi}\frac{(\gamma W/\pi)}{p^2-(\gamma W/\pi)^2}+\sum_{q=1}^{Q}\frac{8}{\gamma W}\right] \\ & -n\frac{J_0 J_0^*}{\sigma}\frac{T_2}{W}\left[(\gamma W)\tan(\gamma W/\pi)+\sum_{q=1}^{Q}\frac{4\pi}{\gamma W}p\right] \end{aligned} \tag{21}$$

On summing up the two finite series and the infinite series [19] in the above equation, the expression for $P_c$ found is:

$$\begin{aligned} P_c \approx\ & n\frac{J_0 J_0^*}{\sigma}2\left[\tan(\gamma W/2)+\frac{4Q}{\gamma W}\right] \\ & -n\frac{J_0 J_0^*}{\sigma}\frac{T_2}{W}\left[(\gamma W)\tan(\gamma W/2)+\frac{4\pi Q^2}{\gamma W}\right] \end{aligned} \tag{22}$$

Therefore, using Eqs. (3.1) and (17) one gets:

$$\begin{aligned} P_e \approx\ & n\frac{J_0 J_0^*}{\sigma}2\left[\frac{\sin\theta}{\cos\theta+\cosh\theta}+\frac{2Q}{\theta}\right] \\ & -n\frac{J_0 J_0^*}{\sigma}\frac{T_2}{W}\left[\theta\frac{\sin\theta-\sinh\theta}{\cos\theta+\cosh\theta}+2\pi\frac{Q^2}{\theta}\right] \end{aligned} \tag{23}$$

where, the parameter $\theta$ is the ratio of the core width to the classical depth of penetration:

$$\theta = W\sqrt{\frac{\omega\mu\sigma}{2}} \tag{23.1}$$

From Eq. (23), for small values of $\theta$, one gets:

$$P_e \approx n \frac{J_0 J_0^*}{\sigma} \left[ 4Q - \frac{T_2}{W} 2\pi Q^2 \right] \frac{1}{\theta} \tag{24}$$

And for large values of $\theta$:

$$P_e \approx n \frac{J_0 J_0^*}{\sigma} \frac{T_2}{W} \theta \tag{25}$$

Since the insulation thickness $T_1$ is usually very small, i.e., $T_1 \ll T_2$, Eqs. (24) and (25) can be rewritten, respectively as:

$$P_e \approx n \frac{J_0 J_0^*}{\sigma} \left[ 4Q - \frac{T}{W} 2\pi Q^2 \right] \frac{1}{\theta} \tag{24.1}$$

for small $\theta$, and

$$P_e \approx n \frac{J_0 J_0^*}{\sigma} \frac{T}{W} \theta \tag{25.1}$$

for large $\theta$, where the lamination thickness $T$ is given as:

$$T = T_1 + T_2 \tag{26}$$

## 6. CONCLUSION

Using linear electromagnetic field theory, simple expressions for eddy current loss in laminated rectangular cores have been derived. These expressions can be readily adapted for cores made of left-handed materials [20–22].

In view of Eqs. (8.1), (8.2) and (13.1), if the insulation thickness $T_1$ is zero (i.e., in the absence of any insulation), there will be only $y$-component of eddy current density. The presence of insulation layers interrupts the eddy current path. As a result, the $y$-component of eddy current density is modified and an $x$-component of eddy current density appears.

It may be seen from Eqs. (13.1), (14) and (16), that due to high conductivity of iron $\sigma$, eddy current loss in a lamination is only mildly sensitive to the value of the thickness of insulation layers, $T_1$.

Equation (23) shows that the eddy current loss can be approximately expressed as a function of a core parameter $\theta$, which is the ratio of the core width to the classical depth of penetration for iron. For large values of $\theta$, the eddy current loss in a lamination, vide Eqs. (25) and (25.1), is linearly proportional to the lamination thickness. However, for small values of $\theta$, as shown by Eqs. (24) and (24.1), there are two components in the expression for the eddy current loss in a lamination. One component is independent of lamination thickness, while the other is proportional to the lamination thickness.

As shown by Eqs. (24) and (24.1), it is possible that the eddy current loss in a laminated core may increase if the lamination thickness is reduced. This is because a reduced eddy current damping results deeper field penetration in the lamination.

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