

## PHOTONIC BAND GAPS IN ONE-DIMENSIONAL METALLIC STAR WAVEGUIDE STRUCTURE

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**Abstract**—In the present communication, we investigate theoretically and study a different type of photonic structure called metallic star waveguide (SWG) structure. The proposed structure, having single homogenous metallic material, is composed of a backbone (or substrate) waveguide along which finite side branches grafted periodically. In order to obtain the dispersion relation and hence the photonic band gaps (PBGs) of the SWG structure the Interface Response Theory (IRT) have been applied. Such types of structures show the band gaps without the contrast in the refractive index of the constituent materials. We also show that the range of forbidden bands can be tuned to different value by varying the number grafted branches of the SWG structures, without changing the other parameters. Moreover, the effects of variation of absorption of metals and plasma frequency on the band gaps of the proposed structures have been investigated.

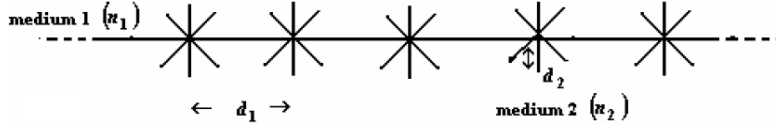
### 1. INTRODUCTION

During the past two decades photonic crystal structures have generated a great deal of interest due to their peculiar optical properties such as absence of electromagnetic modes, inhibition of spontaneous emission and absence of zero point fluctuations in the band gaps. This unique feature of the photonic crystal structures alters dramatically the flow

of light and manipulation of photons within the structure and can lead to many potential applications in field of photonics [1–9].

In recent past a different types of photonic structure called star waveguide (SWG) structure or comblike waveguide (CWG) structure have been investigated and studied. These types of structure have a narrow pass bands separated by a large forbidden bands. The CWG structure or SWG structure is composed of a backbone (or substrate) waveguide along which finite side branches grafted periodically. The existence of photonic bands in such types of structures has been experimentally confirmed in the frequency range up to 500 MHz by Dobrzynski et al., [10]. However, this type of waveguide structure can be used any region of the electromagnetic spectrum. The existence of band gaps in such type of structures do not require the contrast in the refractive indices of the constituent materials which is unlike the usual photonic crystal structure where the contrast in the refractive index is must for the existence band gaps. In other words, the photonic structure is tailored within a single homogeneous medium, although the boundary conditions impose that the electromagnetic waves only propagate in the interior of the waveguides. The one-dimensional nature of the proposed model retains its validity to any region of the electromagnetic spectrum. Indeed, it must be emphasized that the diameter of the guide should be much smaller than the wavelength in order to allow the propagation of a single mode guide and also the diameter of guide being much smaller than its period [10–12]. Recently a waveguide structure is used to design the compact band pass filter by using different materials in different region of the wavelength [14–16].

In the present paper we study the photonic band structure in the metallic star waveguide structure composed of one-dimensional continuous branches grafted on same substrates. In recent past some authors have studied such types of structure using dielectric material [10–12]. The reason to choose the metallic structure is that introduction of metals to photonic crystals has many advantages. These include reduction in size and weight, easier fabrications and comparatively lower costs. Also metallic photonic structures have tolerance to temperature fluctuations and are applicable for the applications at high temperature above 1000°C. In addition to the band structure we have seen the effect of absorption of the material on the band gaps of metallic SWG structure. Further, the variation of plasma frequency and its influence on the forbidden bands has also been investigated.



**Figure 1.** Schematic representation of periodic star waveguide structure with infinite number of stars.

## 2. THEORETICAL MODEL

The propagation of EM wave and hence the dispersion relation of the star waveguide structure can be obtained with the help of interface response theory (IRT) [10,11]. The proposed structure under study is shown in the Fig. 1. The dispersion relation of the proposed structure for an infinite number of sites ( $N \rightarrow \infty$ ) can be obtained by choosing the two different boundary conditions i.e., by vanishing of either electric field ( $E = 0$ ) or magnetic field ( $H = 0$ ) at the extremities of the side branches. If  $n_1$  and  $n_2$  be the refractive indices of the backbone and side branches and  $d_1$  (periodicity of the system) and  $d_2$  (length of the grafted branches) be the length scale of the system, then the characteristics equations of the infinite star waveguide with the boundary conditions  $E = 0$  or  $H = 0$  is given by

$$\cos(Kd_1) = \cos\left(\frac{n_1\omega d_1}{c}\right) - \frac{N'}{2} \left(\frac{n_2}{n_1}\right) \frac{\sin\left(\frac{n_1\omega d_1}{c}\right) \sin\left(\frac{n_2\omega d_2}{c}\right)}{\cos\left(\frac{n_2\omega d_2}{c}\right)} \quad (1)$$

or

$$\cos(Kd_1) = \cos\left(\frac{n_1\omega d_1}{c}\right) + \frac{N'}{2} \left(\frac{n_2}{n_1}\right) \frac{\sin\left(\frac{n_1\omega d_1}{c}\right) \cos\left(\frac{n_2\omega d_2}{c}\right)}{\sin\left(\frac{n_2\omega d_2}{c}\right)} \quad (2)$$

Here ' $K$ ' is the propagation vector along the guide, ' $\omega$ ' is the angular frequency ' $c$ ' is velocity of light and  $N'$  is the number of grafted branches.

The dispersion relation of the infinite star waveguide structure

with the boundary conditions  $E = 0$  or  $H = 0$  is given by

$$K(\omega) = \frac{1}{d_1} \cos^{-1} \left[ \cos \left( \frac{n_1 \omega d_1}{c} \right) - \frac{N'}{2} \left( \frac{n_2}{n_1} \right) \frac{\sin \left( \frac{n_1 \omega d_1}{c} \right) \sin \left( \frac{n_2 \omega d_2}{c} \right)}{\cos \left( \frac{n_2 \omega d_2}{c} \right)} \right] \quad (3)$$

or

$$K(\omega) = \frac{1}{d_1} \cos^{-1} \left[ \cos \left( \frac{n_1 \omega d_1}{c} \right) + \frac{N'}{2} \left( \frac{n_2}{n_1} \right) \frac{\sin \left( \frac{n_1 \omega d_1}{c} \right) \cos \left( \frac{n_2 \omega d_2}{c} \right)}{\sin \left( \frac{n_2 \omega d_2}{c} \right)} \right] \quad (4)$$

For the metallic conductor refractive index of the material is complex and frequency dependent. The frequency-dependent dielectric function for the metals is governed by Drude dispersion model [13], which provides an excellent fit to the measured data over a wide frequency range, and is defined as

$$n(\omega) = \sqrt{\varepsilon(\omega)} = \left( 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \right)^{1/2} = n_r + in_i \quad (5)$$

where  $\omega_p$  is the plasma frequency of the conduction electron and  $\gamma$  is the damping frequency and is related to the absorption of the conducting materials. Also, for the conducting material propagation vector  $K$  is complex one and can be written as  $K = K_r + iK_i$ . In order to obtain the dispersion relation of the proposed waveguide structure composed of single homogeneous conducting material (i.e.,  $n_1 = n_2 = n$ ) we substitute the value of Eq. (5) in Eq. (1) and simplify. Thus the resultant dispersion relation for the metallic SWG structure can be written as

$$\begin{aligned} \cos(K(\omega)d_1) &= \cos(K_r(\omega)d_1 + iK_i(\omega)d_1) \\ &= U + \frac{N'}{2} \frac{(V + W)}{X} - i \left[ P - \frac{N'}{2} \frac{(Q + R)}{X} \right] \end{aligned} \quad (6)$$

The abbreviations of the symbols  $U$ ,  $V$ ,  $W$ ,  $P$ ,  $Q$ ,  $R$  and  $X$  are as follows

$$\begin{aligned} U &= \cos \left( \frac{n_r \omega d_1}{c} \right) \cosh \left( \frac{n_i \omega d_1}{c} \right); \\ V &= \sin \left( \frac{n_r \omega d_1}{c} \right) \cosh \left( \frac{n_i \omega d_1}{c} \right) \cos \left( \frac{n_r \omega d_2}{c} \right) \sin \left( \frac{n_r \omega d_2}{c} \right); \end{aligned}$$

$$\begin{aligned}
W &= \cos\left(\frac{n_r \omega d_1}{c}\right) \sinh\left(\frac{n_i \omega d_1}{c}\right) \sinh\left(\frac{n_i \omega d_2}{c}\right) \cosh\left(\frac{n_i \omega d_2}{c}\right); \\
P &= \sin\left(\frac{n_r \omega d_1}{c}\right) \sinh\left(\frac{n_i \omega d_1}{c}\right); \\
Q &= \cos\left(\frac{n_r \omega d_1}{c}\right) \sinh\left(\frac{n_i \omega d_1}{c}\right) \cos\left(\frac{n_r \omega d_2}{c}\right) \sin\left(\frac{n_r \omega d_2}{c}\right); \\
R &= \sin\left(\frac{n_r \omega d_1}{c}\right) \cosh\left(\frac{n_i \omega d_1}{c}\right) \sinh\left(\frac{n_i \omega d_2}{c}\right) \cos\left(\frac{n_i \omega d_2}{c}\right); \\
X &= \sin^2\left(\frac{n_r \omega d_2}{c}\right) + \sinh^2\left(\frac{n_i \omega d_2}{c}\right)
\end{aligned}$$

If  $f_1(\omega)$  and  $f_2(\omega)$  represents and imaginary parts of  $\cos(K(\omega)d_1)$ , then Eq. (6) can be written as

$$f_1(\omega) + i f_2(\omega) = U + \frac{N'}{2} \frac{(V+W)}{X} - i \left[ P - \frac{N'}{2} \frac{(Q+R)}{X} \right] \quad (7)$$

Now comparing the real and imaginary parts of both sides of Eq. (7) we can write  $f_1(\omega)$  and  $f_2(\omega)$  explicitly in the forms.

Real Part of  $\cos(K(\omega)d_1)$

$$\cos(K_r(\omega)d_1) = f_1(\omega) = U + \frac{N'}{2} \frac{(V+W)}{X}; \quad (8)$$

Imaginary Parts  $\cos(K(\omega)d_1)$

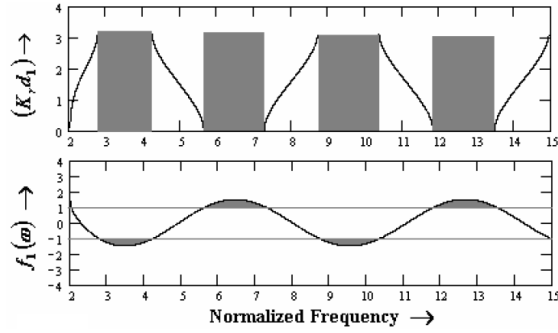
$$\cos(K_i(\omega)d_1) = f_2(\omega) = P - \frac{N'}{2} \frac{(Q-R)}{X} \quad (9)$$

We are interested only in the real parts of the Eq. (7) i.e., in Eq. (8) because only this part of equation shows the real propagation of electromagnetic waves through the structure. Now, in the next section we will compute the band structure of metallic SWG structure with the help of Eq. (9), numerically for different values of absorption constituent materials and also, we will see the effect of variation of grafted braches and effect of variation of plasma frequency on the reflection bands.

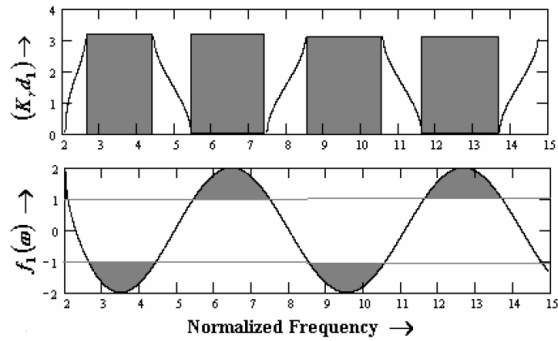
### 3. RESULTS AND DISCUSSIONS

Figures 2(a)–(d) show the photonic band structure obtained for the metallic SWG structure with zero absorption. For the sake of numerical computations we choose  $\omega_p = 1.5 \times 10^{16}$  Hz,  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $\gamma = 0$

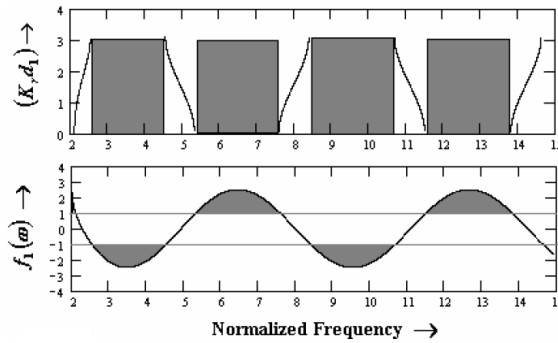
and the number of grafted branches ( $N'$ ) are taken as  $N' = 1, 2, 3$  and 4 respectively. From the study of these figures, it is found that the proposed structure has total four forbidden bands in a given range of normalized frequency and the width of each successive bands increase as one move towards the higher frequency side. The width of the



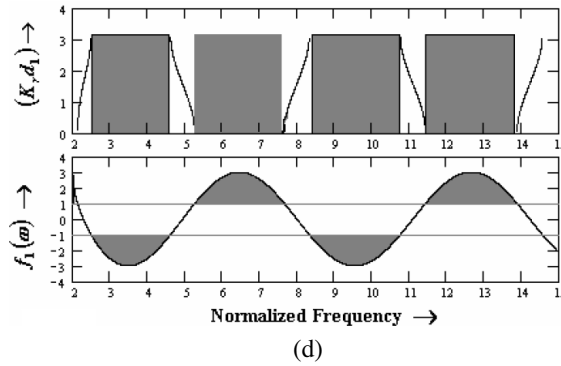
(a)



(b)



(c)



**Figure 2.** Dispersion curve and forbidden bands of the metallic SWG structure for (a)  $N' = 1$ , (b)  $N' = 2$ , (c)  $N' = 3$  and (d)  $N' = 4$ , and other parameters are  $\omega_p = 1.5 \times 10^{16}$  Hz,  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $\gamma = 0$ .

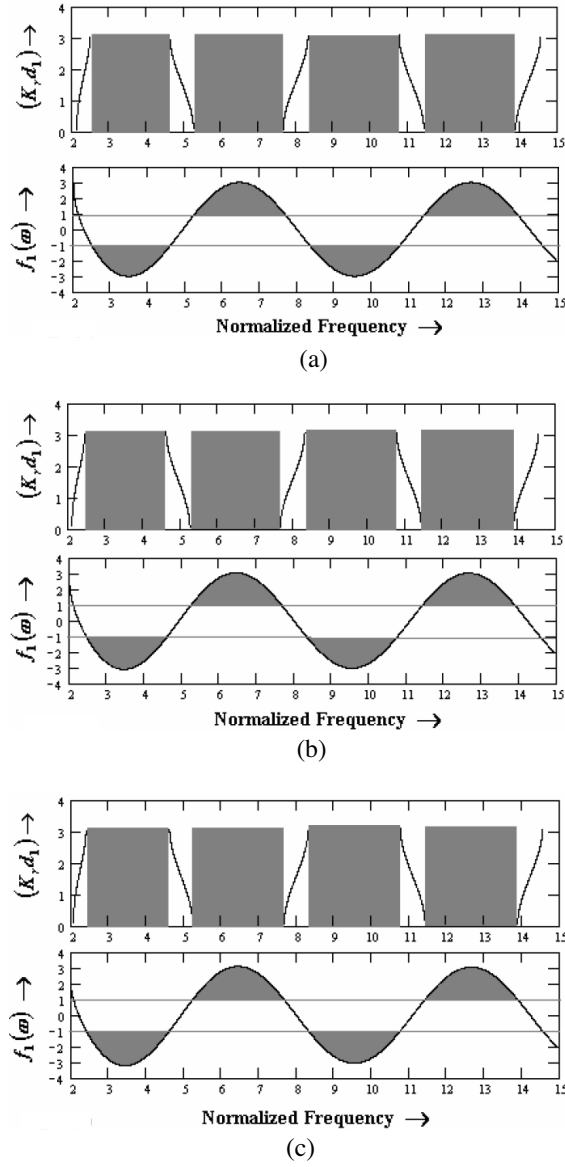
forbidden bands also depends on the number of grafted branches of the SWG structure. The forbidden range of frequency can be seen from the dispersion curves plotted between  $(K_r(\omega)d_1)$  Vs normalized frequency  $(\omega d_1/c)$  as well as with the cosine curve plotted between  $f_1(\omega)$  Vs normalized frequency  $(\omega d_1/c)$ . Since the maximum and minimum value of cosine is  $+1$  and  $-1$ , therefore the portion of the curves which lies within this limit gives allowed bands and which lies beyond this limit gives forbidden or reflection bands. Further, the study of Figs. 2(a)–(d) show that when the number of grafted side branches increases the width of the reflection bands becomes wider and wider even the total number of bands remains the same. For  $N' = 1$  the width of the 1st band is found to be  $1.473(\omega d_1/c)$  while for  $N' = 2, 3$  and  $4$  the width is found to be  $1.824, 2.008$  and  $2.212(\omega d_1/c)$  respectively. Similarly, the width of the 2nd, 3rd and 4th bands also get enhanced as the number of grafted branches in the structure increases, which are reported in the Table 1. The enlargement of forbidden bands by increasing the number of grafted branches arises due to multiple reflection of electromagnetic wave within the increased number of branches. Hence, it is emphasized that the width and the range of reflection bands of SWG structure can be tuned by varying the number of grafted branches without changing the other parameters of the structure. The effect of absorption of the constituent material on the metallic SWG structure has been shown in the Figs. 2(a) and 3(a)–(c) for different value of absorption *viz.*  $\gamma = 0.01\omega_p, 0.1\omega_p$  and  $0.2\omega_p$  respectively. The other parameters taken in this calculation are same as above except, here, we take  $N' = 4$ . From the analysis of

**Table 1.** Forbidden bands of the metallic SWG structure for  $\omega_p = 1.5 \times 10^{16}$  Hz,  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $\gamma = 0$  and  $N' = 1, 2, 3$ , and 4.

Number of grafted branches ( $N'$ )	Range and width of the forbidden bands in $(\omega d_1/c)$
$N' = 1$	$2.768 - 4.241 = 1.473$ $5.629 - 7.266 = 1.637$ $8.701 - 10.363 = 1.662$ $11.811 - 13.481 = 1.670$
$N' = 2$	$2.610 - 4.434 = 1.824$ $5.430 - 7.468 = 2.038$ $8.498 - 10.567 = 2.069$ $11.607 - 13.687 = 2.080$
$N' = 3$	$2.530 - 4.538 = 2.008$ $5.322 - 7.577 = 2.255$ $8.388 - 10.679 = 2.291$ $11.495 - 13.799 = 2.304$
$N' = 4$	$2.482 - 4.606 = 2.124$ $5.254 - 7.649 = 2.395$ $8.317 - 10.751 = 2.434$ $11.424 - 13.869 = 2.445$

the Figs. 2(d) and 3(a)–(c), it has been observed that for small value of absorption ( $\gamma = 0.01\omega_p$ ) the width of the forbidden bands remains almost unchanged, i.e., the width of the forbidden bands for  $\gamma = 0$  and  $\gamma = 0.01\omega_p$  remains approximately same which can be seen from the Table 2. But when the absorption of the structure increases the band width increases slightly. For  $\gamma = 0.1\omega_p$  the width of the four bands are 2.170, 2.408, 2.440 and 2.451( $\omega d_1/c$ ), if we compare this result with the bandwidth of structure having zero absorption then it is observed that the increment in the band width is very small. Further, for  $\gamma = 0.2\omega_p$  the band width does not change appreciably though the first two bands have slightly larger width in comparison with zero absorption. Thus, from these observations it is inferred that absorption of the constituent materials does not have much influence on the reflection





**Figure 3.** Dispersion curve and forbidden bands of the metallic SWG structure for (a)  $\gamma = 0.01\omega_p$ , (b)  $\gamma = 0.1\omega_p$  and (c)  $\gamma = 0.2\omega_p$  respectively and other parameters are  $\omega_p = 1.5 \times 10^{16}$  Hz,  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $N' = 4$ .

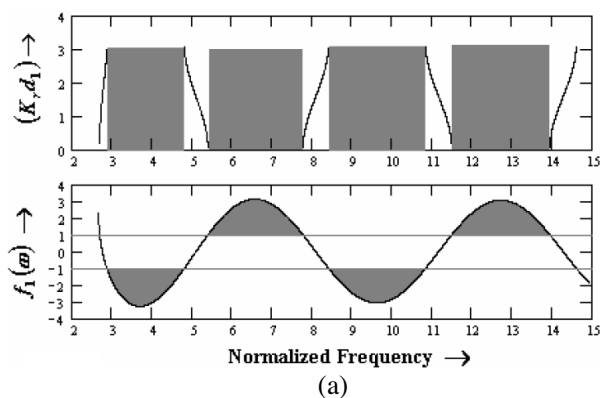
**Table 2.** Forbidden bands of the metallic SWG structure for  $\omega_p = 1.5 \times 10^{16}$  Hz,  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $N' = 4$  and  $\gamma = 0.0\omega_p$ ,  $0.01\omega_p$ ,  $0.1\omega_p$  and  $0.2\omega_p$  respectively.

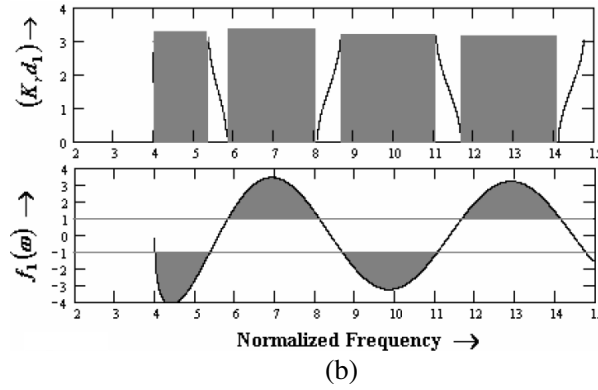
Absorption of materials ( $\gamma$ )	Range and width of the forbidden bands in $\left(\omega d_1/c\right)$
$\gamma = 0$	$2.482 - 4.606 = 2.124$ $5.254 - 7.649 = 2.395$ $8.317 - 10.751 = 2.434$ $11.424 - 13.869 = 2.445$
$\gamma = 0.01\omega_p$	$2.478 - 4.608 = 2.130$ $5.253 - 7.648 = 2.395$ $8.316 - 10.751 = 2.435$ $11.424 - 13.869 = 2.445$
$\gamma = 0.1\omega_p$	$2.438 - 4.608 = 2.170$ $5.242 - 7.650 = 2.408$ $8.312 - 10.752 = 2.440$ $11.421 - 13.872 = 2.451$
$\gamma = 0.2\omega_p$	$2.305 - 4.609 = 2.304$ $5.232 - 7.653 = 2.421$ $8.305 - 10.753 = 2.448$ $11.417 - 13.873 = 2.456$

bands of the metallic SWG structure, though the effect of absorption may be appreciable for very high value of absorption. Finally, the effect of variation of plasma frequency on the reflection bands of metallic SWG structure has been investigated and the corresponding graphs for different values of plasma frequencies are illustrated in the Figs. 3(b) and 4(a)–(b) respectively. The calculation is made here for plasma frequencies having values  $\omega_p = 1.5 \times 10^{16}$ ,  $2.0 \times 10^{16}$  and  $3.0 \times 10^{16}$  Hz, respectively,  $\gamma = 0.1\omega_p$ , and  $N' = 4$  while the values of  $d_1$  and  $d_2$  remains the same as previous. From the study of Figs. 3(b) and 4(a)–(b), it is obvious that when the plasma frequency of the metallic SWG structure increases the width of the forbidden bands decreases and at the same time it is shifted towards the higher frequency side of the EM spectrum which are reported in Table 3. The reason for this change in the band width of the metallic SWG structure is due the increase in the absolute value of the dielectric constant with increase in the plasma frequency. Since, the existence of the band gaps in SWG structures

**Table 3.** Forbidden bands of the metallic SWG structure for  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $\gamma = 0.1\omega_p$ ,  $N' = 4$  and  $\omega_p = 1.5 \times 10^{16}$ ,  $2.0 \times 10^{16}$  and  $3.0 \times 10^{16}$  Hz respectively.

Plasma Frequency ( $\omega_p$ ) in Hz	Range and width of the forbidden bands in ( $\omega d_1/c$ )
$\omega_p = 1.5 \times 10^{16}$	$2.438 - 4.608 = 2.170$ $5.242 - 7.650 = 2.408$ $8.312 - 10.752 = 2.440$ $11.421 - 13.872 = 2.451$
$\omega_p = 2.0 \times 10^{16}$	$2.859 - 4.801 = 1.942$ $5.389 - 7.761 = 2.372$ $8.398 - 10.829 = 2.431$ $11.484 - 13.932 = 2.448$
$\omega_p = 3.0 \times 10^{16}$	$3.985 - 5.365 = 1.380$ $5.814 - 8.081 = 2.267$ $8.652 - 11.055 = 2.403$ $11.664 - 14.104 = 2.440$





**Figure 4.** Dispersion curve and forbidden bands of the metallic SWG structure for (a)  $\omega_p = 2.0 \times 10^{16}$  Hz and (b)  $\omega_p = 3.0 \times 10^{16}$  Hz respectively, and other parameters are  $d_1 = d_2 = 4 \times 10^{-8}$  m,  $N' = 4$ .

or CWG structures do not require the contrast in the refractive index of the constituent materials and have comparatively larger forbidden bands for the smaller value of the refractive index of the materials, as mentioned earlier. Therefore, the metallic SWG structures having larger values of plasma frequency have comparatively larger value of refractive index and hence the smaller forbidden bands.

#### 4. CONCLUSION

In conclusion, we have investigated theoretically a different type of photonic structure called metallic star waveguide structure. The analysis of dispersion relation of the proposed structure shows that the band gaps do not depend on the refractive index contrast of the constituent materials whereas it is must in the case of usual photonic band gap structure. The width of band gaps of the metallic SWG structure can be increased by increasing the number of side branches grafted periodically and at the same time the range of forbidden bands can be tuned to different value without changing the other parameters of the structure. Further, we have studied the effect absorption of the constituent materials on the band structure and it is found that for small absorption of the material the forbidden bands remains almost the same and for large absorption the forbidden bands get enhanced slightly. Thus the absorption of materials has not much impact on the reflection bands of metallic (SWG) structure. Finally, we have shown how the variation plasma frequency affects the band gaps of

the metallic (SWG) structure. From the analysis it observed that the structure having larger value of plasma frequency have smaller forbidden bands and the range of forbidden bands shifted towards the higher frequency side of the spectrum. Therefore, the range and width of the forbidden bands can also be tuned to different value of varying the plasma frequency of the structure without changing other parameters. This type of metallic SWG structures can be used to design the optical filters, optical reflectors, and wavelength multiplexing etc. in any region of the electromagnetic spectrum by the proper selection of material and geometrical parameters and have potential applications in the field of optoelectronic devices due to the easier manufacturing technique (lithography) and geometrical construction.

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