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DEASTIGMATISM AND CIRCULARIZATION OF AN ELLIPTICAL GAUSSIAN BEAM BY OFF-AXIS ELLIPSOID REFLECTOR BASED OFF-FOCUS CONFIGURATION

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Abstract—Off-axis ellipsoid reflector based off-focus configuration for deastigmatism and circularization of an elliptical Gaussian beam is proposed. Mostly used off-axis ellipsoid reflector based conventional configuration is constructed by aligning the incident direction directed to one focus of the ellipsoid, which reflect the output beam to the another focus of the ellipsoid. However, such configuration is unavailable to deastigmatize and circularize an elliptical Gaussian beam. Therefore, the coupling efficiency between the reflected beam and an essentially circular beam is not well satisfied. In this case, an off-axis ellipsoid reflector based off-focus configuration is proposed to obtain better coupling efficiency. Different from the conventional configuration, in the proposed off-focus configuration, the incident beam direction is diverged from one focus of the ellipsoid. As a result, the coupling efficiency of no less than 99.9% (as compared with coupling efficiency of about 94.2% based on conventional configuration) can be obtained, which is verified with numerical calculations.

1. INTRODUCTION

In the applications of quasi-optical system, a confocal and circular Gaussian beam is always required. However, the reflected beam will induce two different phase centers (i.e., astigmatic) and two different beam radii in two transverse directions, as shown in Fig. 1, which is a common problem encountered in many applications. For example, beam from smooth-walled circular feed horn and rectangular feed horn [1], or many GaAs-based lasers [2] operating at wavelengths such as 650, 780, 810 and 850 nm all emit elliptical beam. Also, an elliptical beam may evolve from a circular beam via one or more quasi-optical element. Because Most of the reflectors or filters used in quasi-optical system have circular profile, an astigmatic and elliptical beam will suffer much power lost. Therefore, a Circular Gaussian beam, which has the same phase center (i.e., confocal) and the same beam waist in two transverse directions, is desired for higher-energy efficiency and easier beam manipulation.

Off-axis ellipsoid reflector based conventional configuration, in which Gaussian beam radiated from one focus is redirected and refocused to the other focus of the elliptical reflector, is impossible to deastigmatize and circularize an elliptical Gaussian beam. Therefore, the alternative configurations are developed. One of the most efficient configurations is cylindrical lenses having different refractive powers in two transverse directions [2]. Another configuration uses an anamorphic prism pair [3], in which the prism pair serves to change the beam width in one transverse direction to make the beam width equal in both transverse directions, thus, circularizes the beam. Another efficient configuration worthy to notice is astigmatic off-axis reflector based configuration, which require an astigmatic off-axis reflector to be designed by choosing the correct mirror curvature in two transverse directions [4].

In this work, a different approach to obtain the circular and confocal beam using the off-axis ellipsoid reflector based off-focus configuration is presented by diverging incident direction from one focus of the ellipsoid to find a correct mirror curvature and an incident angle.

Such an approach for deastigmatism and circularization of an elliptical Gaussian beam based on off-focus configuration is verified by numerical calculation in this paper and compared with the previous results by astigmatic off-axis reflector approach. The reflected field has a high coupling coefficient with circular Gaussian beam of no less than 99.9%, which indicates the high efficiency of this configuration.

2. CALCULATION OF THE REFLECTED GAUSSIAN BEAM

Based on the Gaussian beam property, the evolutions of the Gaussian beam in two transverse directions are entirely independent, thus the field distributions of incident Gaussian beam and reflected Gaussian beam can be calculated by the product of the field distributions in two orthogonal planes, which can be expressed [1] as:

$$E^i = E_x^i E_y^i \qquad E^r = E_x^r E_y^r \tag{1}$$

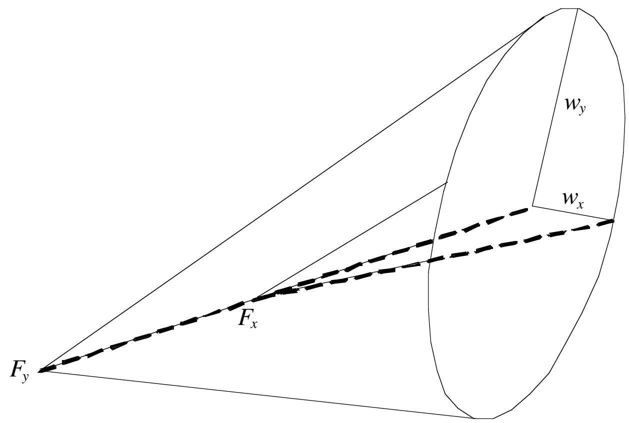

Figure 1. Illustration of elliptical and astigmatic Gaussian beam. w_x and w_y are beam radii in two transverse directions. F_x and F_y are two phase centers in two transverse directions.

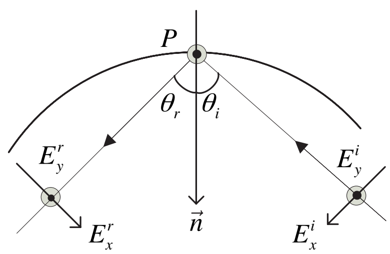

Figure 2. Illustration of reflection. $E_{x,y}^i$ and $E_{x,y}^r$ are the incident and reflected Gaussian beams in two transverse directions respectively, and θ_i and θ_r are the incident and reflected angles respectively, with n representing the normal of the reflector on the incident point P.

Take a reflector for example, shown in Fig. 2.

The beam radii of the reflected Gaussian beam on incident point in two transverse directions follow the regulations as:

$$\begin{aligned} \omega_x^r &= \omega_x^i \\ \omega_y^r &= \omega_y^i \end{aligned} \tag{2}$$

where, $\omega_{x,y}^i$ are the beam radii of the incident Gaussian beam on incident point P in two transverse directions.

The curvature matrixes of the equiphase surface of the incident and reflected beam on the incident point P defined as , and the curvature matrix of the reflector on the incident point P defined as $Q^{i,r}$, can be expressed as [5]:

$$Q^{i,r} = \begin{bmatrix} Q_{11}^{i,r} & Q_{12}^{i,r} \\ Q_{21}^{i,r} & Q_{22}^{i,r} \end{bmatrix} = \begin{bmatrix} 1/R_x^{i,r} & 0 \\ 0 & 1/R_y^{i,r} \end{bmatrix} \tag{3}$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 1/R_x^f & 0 \\ 0 & 1/R_y^f \end{bmatrix} \tag{4}$$

Where, $R_{x,y}^{i,r}$ are the curvature radii of the incident and reflected Gaussian beam on the incident point P in two transverse directions, respectively, and $R_{x,y}^f$ is the curvature radii of reflector on the incident point P.

The can be calculated [5] as:

$$Q^r = \begin{bmatrix} 2C_{11}\cos\theta_i + Q_{11}^i & 2C_{12} - Q_{12}^i \\ 2C_{12} - Q_{12}^i & 2C_{22}\sec\theta_i + Q_{22}^i \end{bmatrix} \tag{5}$$

Consequently, the $R_{x,y}^r$ are calculated as:

$$R_x^r = 1/\left(2\sec\upsilon_i/R_x^f + 1/R_x^i\right) \qquad R_y^r = 1/\left(2\cos\upsilon_i/R_y^f + 1/R_y^i\right) \tag{6}$$

Subsequently, $E_{x,y}^r$ can be derived from $R_{x,y}^r$ and $\omega_{x,y}^r$ [1].

Therefore, if the reflected field is to be circular and confocal, these following conditions have to be satisfied:

$$\omega_x^i = \omega_y^i \qquad 2\sec\upsilon_i/R_x^f + 1/R_x^i = 2\cos\upsilon_i/R_y^f + 1/R_y^i \tag{7}$$

3. OFF-AXIS ELLIPSOID REFLECTOR BASED CONVENTIONAL CONFIGURATION

The conventional configuration for Gaussian beam reflection and redirection is constructed by aligning the incident beam waist passing through one focus (e.g., F_1 in Figure 3) of the ellipsoid in order to have reflected beam collimated on the other focus.

In this case, the curvature radius of the reflector always satisfies:

$$\sec\upsilon_i/R_x^f = \cos\upsilon_i/R_y^f \tag{8}$$

Referring to equation (7), it is concluded that only when the incident beam is confocal and circular, the reflected beam can be

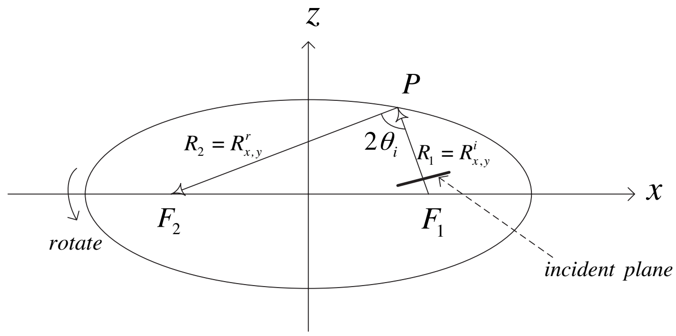

Figure 3. Off-axis reflector constructed by rotating a specific part of the x axis. F_1 and F_2 are two focuses of the ellipsoid.

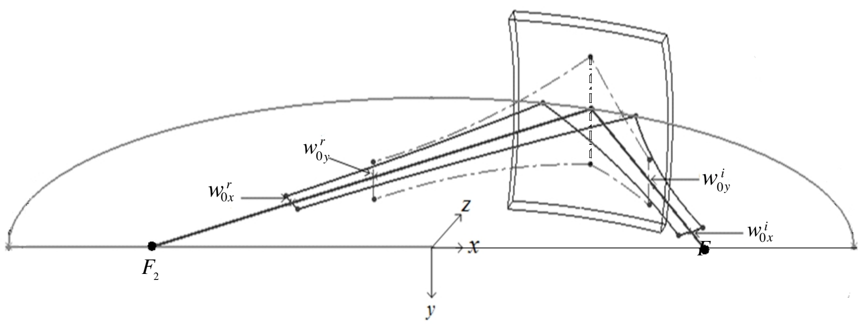

Figure 4. An astigmatic and elliptical Gaussian beam transformed to another astigmatic and elliptical Gaussian beam based on conventional configuration. F_1 and F_2 are the two focuses of the ellipsoid.

confocal and circular in above conventional configuration. In other words, if the incident beam is astigmatic and elliptical, the reflected beam is astigmatic and elliptical either, illustrated in Fig. 4.

Fig. 4 indicates that the reflected beam has two different focuses located near the focus of elliptical reflector.

Efforts have been made to correct the above problem, such as astigmatic ellipsoid reflector based off-focus configuration as mentioned in the introduction. We proposed a method of off-axis ellipsoid reflector based off-focus configuration, obtaining the same efficient as astigmatic ellipsoid reflector based off-focus configuration without designing an astigmatic ellipsoid reflector, thus avoiding fabrication difficulty.

4. OFF-AXIS ELLIPSOID REFLECTOR BASED OFF-FOCUS CONFIGURATION FOR DEASTIGMATISM AND CIRCULARIZATION OF AN ASTIGMATIC AND ELLIPTICAL GAUSSIAN BEAM

Our proposed off-focus configuration is constructed by diverging the incident direction from one focus of the ellipsoid. Following the Gaussian beam property, it is possible to modulate the radii and locations of the incident Gaussian beam waists in two transverse directions, thus two different Gaussian beam waist radii in two transverse directions can be adjusted the same, and the two separated reflected Gaussian beam waist locations in two transverse directions can be adjusted together.

Based on the above analysis, there can be three implementation methods to derive off-focus configuration, as described below.

4.1. Method 1: Change the Incident Point, with the Same Incident Angle

In the condition of the same incident angle v_i, find an incident plane and an incident point Q, with which the beam radii and the curvature radii satisfy Equation (7). It would induce the result that the actual intersections (i.e., F_1' and F_2') depart from focuses of the ellipsoid (i.e., F_1 and F_2), which is illustrated in Figure 5.

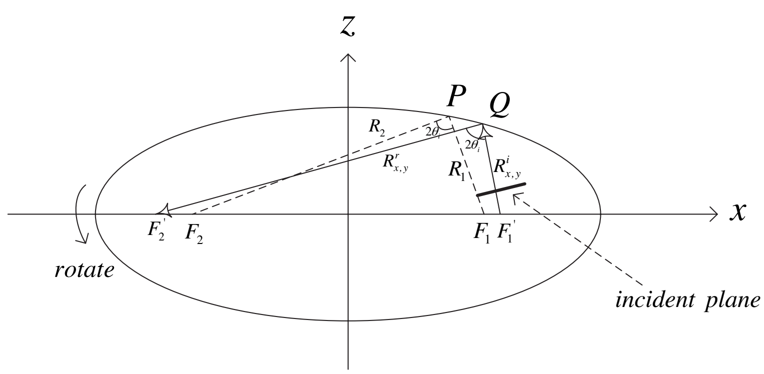

Figure 5. Off-axis reflector constructed by rotating a specific part of the x axis. F_1 and F_2 are the two focuses of the ellipsoid. F_1' and F_2' are the actual intersections of the incident and reflected directions with the x axis, respectively.

4.2. Method 2: Change the Incident Angle, with the Same Incident Point

In the condition of the same incident point P, find an incident plane and an incident angle θ_i', with which the beam radii and the curvature radii satisfy equation (7). It would also induce the result that the actual intersections (i.e., F_1' and F_2') depart from focuses of the ellipsoid (i.e., F_1 and F_2),which is illustrated in Fig. 6.

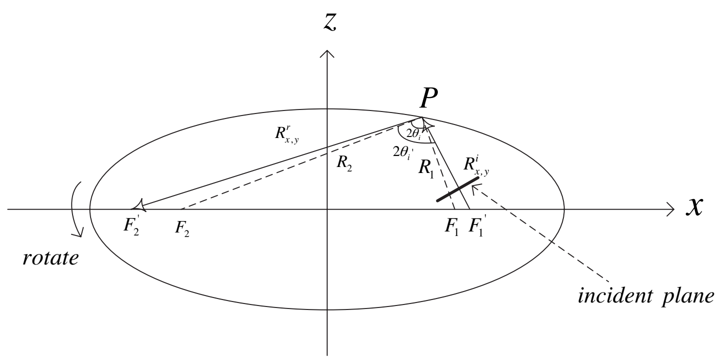

Figure 6. Off-axis reflector constructed by rotating a specific part of the x axis. F_1 and F_2 are the two focuses of the ellipsoid. F_1' and F_2' re the actual intersections of the incident and reflected directions with the x axis, respectively.

4.3. Method 3: Change the Incident Point and the Incident Angle

The two methods can also be combined to find an incident point Q and incident angle θ_i', with which the beam radii and the curvature radii satisfy Equation (7). The configuration is illustrated in Fig. 7.

With the proposed off-focus configuration, astigmatic and ellipsoid Gaussian beam can be transformed to a confocal and circular Gaussian beam, illustrated in Fig. 8.

5. NUMERICAL SIMULATIONS

An incident Gaussian beam with $\omega_{0x} = 0.05\,\text{m}$ and $\omega_{0y} = 0.1\,\text{m}$ operating at 54 GHz, is shown in Fig. 9 below. Referring to Figures 10–14, the reflected beams at a spacing of 2m departing from the incident point, are calculated by Diffracted Gaussian Beam Analysis (DGBA)

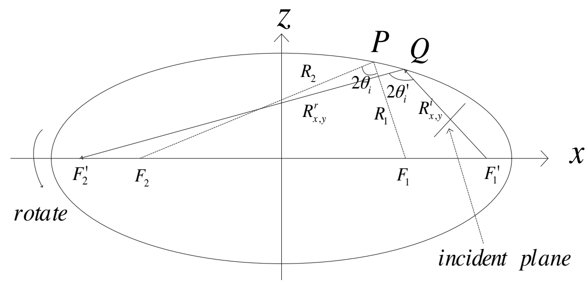

Figure 7. Off-axis reflector constructed by rotating a specific part of the x axis. F_1 and F_2 are the two focuses of the ellipsoid. F_1' and F_2' are the actual intersections of the incident and reflected directions with the x axis, respectively.

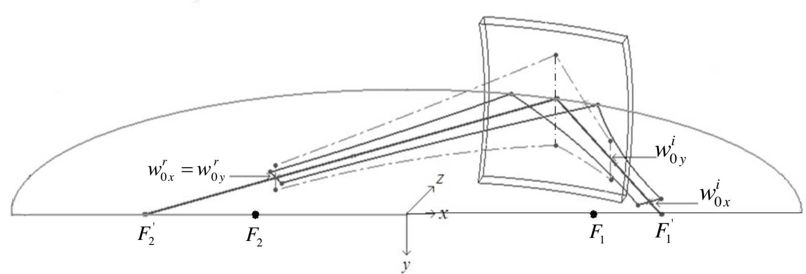

Figure 8. An astigmatic and elliptical Gaussian beam transformed to a confocal and circle beam based on off-focus configuration. F_1 and F_2 are the two focuses of the ellipsoid. F_1' and F_2' are the actual intersections of the incident and reflected directions with the x axis, individually.

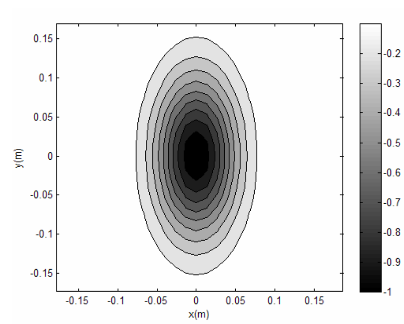

Figure 9. Incident Gaussian beam.

[6–14] algorithm based on the configurations described above, including off-axis reflector based conventional configuration, astigmatic off-axis reflector based configuration, and off-axis reflector based off-focus configuration. DGBA algorithm has nearly the same calculation efficiency with PO [6].

5.1. Result of Conventional Configuration

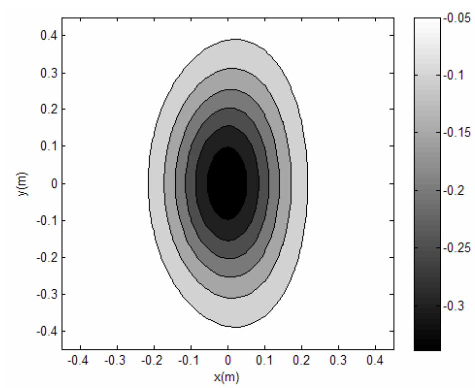

Figure 10. Reflected Gaussian beam.

5.2. Result of Astigmatic Off-axis Reflector Based Configuration

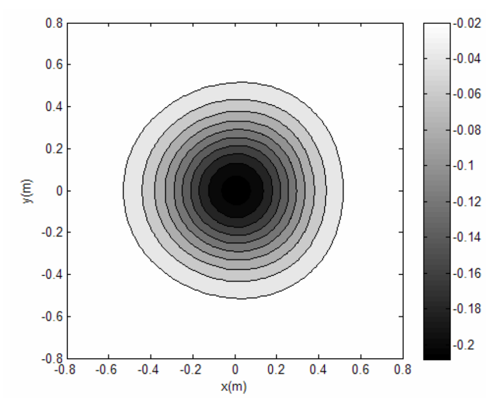

Figure 11. Reflected Gaussian beam.

5.3. Result of Off-axis Reflector Based Off-focus Configuration

Method 1:

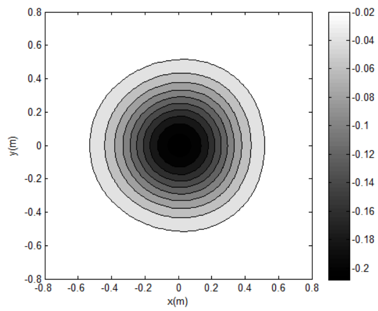

Figure 12. Reflected Gaussian beam.

Method 2:

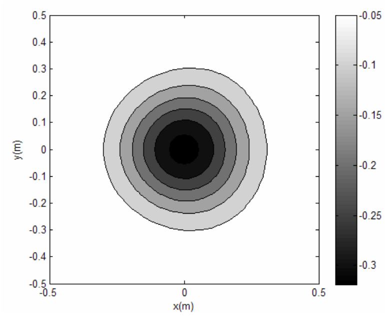

Figure 13. Reflected Gaussian beam.

Method 3:

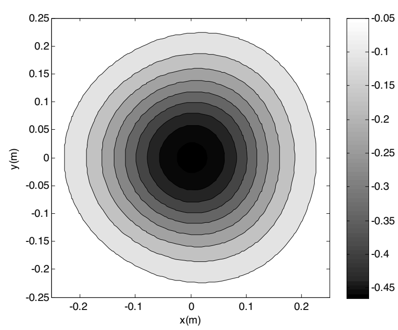

Figure 14. Reflected Gaussian beam.

The coupling coefficient between incident or reflected beam and a confocal and circular Gaussian beam can be defined as:

$$C = \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_1(x,y) E_2^*(x,y) dxdy \right| \tag{9}$$

where, $E_1(x,y)$ and $E_2(x,y)$ represent electromagnetic field distribution of incident or reflected beam and a confocal and circular Gaussian beam respectively, and the coupling coefficient between each incident or reflected beam derived by the configurations above and confocal and circular Gaussian beam is shown in Table 1.

Table 1. Coupling coefficient C.

Beam types		C
Incident beam		0.9439
Reflected beam based on off-axis reflector based conventional configuration		0.9419
Reflected beam based on astigmatic off-axis reflector based configuration		0.9990
Reflected beam based on off-axis reflector based off-focus configuration	Method 1	0.9991
	Method 2	0.9992
	Combination of method 1 and 2	0.9991

The phase center spacing of each reflected beam in two transverse directions can be defined as:

$$D = |z_{w0\_x} - z_{w0\_y}| \tag{10}$$

Where, z_{w0_x} and z_{w0_y} represent beam waist locations in two transverse directions respectively, and the phase center spacing of each incident or reflected beam is shown in Table 2.

5.4. Discussion

Referring to Table 1 and Table 2, both astigmatic off-axis reflector based configuration and off-axis reflector based off-focus configuration have transformed an astigmatic and ellipsoid Gaussian beam with 5.6% power degradation of a circular Gaussian beam and phase center spacing of 0.0399 m to a beam with less than 0.1% power degradation of a circular Gaussian beam and phase center spacing of zero, which is an

Table 2. Phase center spacing D.

Beam types		D
Incident beam		0.0399 m
Reflected beam based on off-axis reflector based conventional configuration		0.7691 m
Reflected beam based on astigmatic off-axis reflector based configuration		0
Reflected beam based on off-axis reflector based off-focus configuration	Method 1	0
	Method 2	0
	Combination of method 1 and 2	0

improvement of the beam with 5.8% degradation of a circular Gaussian beam and phase center spacing of 0.7691 m based on conventional configuration. In addition, off-focus configuration is easier to achieve because the conventional off-axis reflector is easier processing than astigmatic off-axis reflector.

6. CONCLUSION

A method using off-axis ellipsoid reflector based off-focus configuration to transform an astigmatic and ellipsoid Gaussian beam to a confocal and circle Gaussian beam is proposed. Numerical calculations have been implemented for verifying the off-focus configuration. Compared with other configurations such as off-axis reflector based conventional configuration and astigmatic off-axis reflector based configuration, the off-focus configuration can obtain high efficiency and easy to achieve, thus this configuration is promising in the applications of quasi-optical system for higher-energy efficiency and easier beam manipulation.

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REFERENCES

1. Goldsmith, P. F., *Quasioptical Systems: Gaussian Beam Quasioptical Propogation and Applications*, IEEE Press, January 1998.

2. Gaebe, C. E., S. Huang, K. A. Miller, T. Stanley, and G. T. Wiand, "Cruciform cylindrical lens for elliptical beam transformation," U.S. patent 5,973,853, October 26, 1999.
3. Hanada, H., "Beam shaping optical system," U.S. patent 4,318,594, March 9, 1982.
4. Wagner-Gentner, A., U. U. Graf, M. Philipp, and D. Rabanus, "A simple method to design astigmatic off-axis mirrors," *Infrared Physics & Technology*, Vol. 50, No. 11, 42–46, 2007.
5. James, G. L. and P. Peregrinus, *Geometrical Theory of Diffraction for Electromagnetic Waves*, 1976.
6. Rieckmann, C., "Novel modular approach based on Gaussian beam diffraction for analysing quasi-optical multi-reflector antennas," *IEE Proc.-Microw. Antennus Propag.*, Vol. 149. No. 3, June 2002.
7. McEwan, N. J. and P. F. Goldsmith, "Gaussian beam techniques for illuminating reflector antennas," *IEEE Transactions on Antennas and Propagation*, 2002.
8. Kildal, P.-S., "Synthesis of multireflector antennas by kinematic and dynamic ray tracing," *IEEE Transactions on Antennas and Propagation*, Vol. 38, 1587–1599, 1990.
9. Martin, R. J. and D. H. Martin, "Quasi-optical antennas for radiometric remote-sensing," *Electron. Commun. Eng. J.*, 37–48, 1996.
10. Chou, H.-T. and P. H. Pathak, "Uniform asymptotic solution for the EM reflection and diffraction of an arbitrary Gaussian beam by a smooth surface with an edge," *Radio Sci.*, Vol. 32, No. 4, 1319–1336, 1997.
11. Chou, H.-T., P. H. Pathak, and R. J. Burkholder, "Novel Gaussian beam method for the rapid analysis of large reflector antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 49, No. 6, 880–893, 2001.
12. Daubechies, I., "The wavelet transform, time-frequency localization and signal analysis," *IEEE Transactions on Information Theory*, Vol. 36, No. 5, 961–1005, 1990.
13. Einziger, P. D., S. Raz, and M. Shapira, "Gabor representation and aperture theory," *Journal of the Optical Society of America A*, Vol. 3, No. 4, 508–521, 1986.
14. Lugara, D. and C. Letrou, "Alternative to Gabor's representation of plane aperture radiation," *Electron. Lett.*, Vol. 34, No. 24, 2286–2287, 1998.