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DUAL BEAM SYNTHESIS USING ELEMENT POSITION PERTURBATIONS AND THE G3-GA ALGORITHM

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Abstract—The position perturbations of linear antenna elements are used for designing non-uniformly spaced reconfigurable antennas radiating with multiple pattern such that the same amplitude distribution and perturbed positions produces either a pencil or a flat topped beam, the difference being dependent upon phase distribution of the array. The perturbation method consists of inducing small perturbations in the element positions of a linear array to obtain the desired patterns and offer the flexibility of simple design and is similar to other adaptive techniques like phase only or phase/amplitude synthesis. The problem of finding the element position perturbations is treated as a non linear problem and has been solved using a the generalized generation gap steady state genetic algorithm (G3-GA) using parent centric crossover. In the G3-GA approach the population diversity versus selection pressure problem considers both the parent selection and the replacement plans of GA. The position-phase synthesis method using the G3-GA approach is compared with the G3-GA phase-only synthesis technique. It is seen that, an optimal set of element-perturbed positions in a constrained position range with uniform amplitude, unequally spaced elements with unequal phases has the potential to overcome the design challenge of phase only syntheses that uses a larger number of elements to get the same desired side lobe level. Further when the main beam is scanned it is found that the proposed method can maintain a sidelobe level .without distortion during beam steering for the angular positions studied.

1. INTRODUCTION

Shaped beam radiation pattern synthesis of antenna arrays using adaptive techniques have received increased attention in recent times [1]. The adaptive techniques suppresses interference and noise dynamically and in many cases have been preferred over traditional classical synthesis procedures such as Dolph-Chebyshev, Fourier inversion the numerical optimization for producing the radiation pattern that is closest to the desired pattern [2]. In all of the above, the desired sidelobe level is usually achieved by optimizing the amplitudes and/or phases with uniform spacing. However, this synthesis method degrades the total radiation power efficiency due to the amplitude tapering, which requires a complicated feed system. Therefore, it is often desirable to use a non-uniformly spaced array to improve the sidelobe level, while maintaining the total radiation efficiency [3].

Non-uniformly spaced arrays have been previously dealt with by Unz [4], who developed a matrix formulation to obtain the current distribution necessary to generate a prescribed radiation pattern from an unequally spaced array with prescribed geometry. Recent design techniques focus on two categories of non-uniform arrays: arrays with randomly spaced elements and thinned arrays, which are derived by selectively zeroing some elements of an initial equally, spaced array. In the random class, the methods are by Harrington [5] for reducing sidelobe levels of uniform excited elements, by Miller and Goldman [6] who applied Prony's method to estimate the parameters of a sum of exponentials and by Lo and Lee [7] who studied the probabilistic properties of a planar antenna array with its elements placed over an aperture. However, more recently the focus has been on the methods based on evolutionary algorithm concepts. The statistical thinning of arrays with quantized element weights [8], the genetic algorithm [9–16], the tabu search algorithm [17] the simulated annealing [18–20] and the ant touring optimization [21] all come under this class.

In the last few years there has been a growing interest in the design and application of multple antenna arrays for communication and radar. For example, different beams for simultaneous aircraft tracking and environmental surveillance with the same radar unit have been considered. To define the three dimensional structure of storms, weather radars employ narrow pencil beams that are scanned in azimuth and elevation. Aircraft surveillance radars use broad beams that are fan shaped in elevation. The beam is scanned in azimuth at a high rate to provide the rapid sequence of target echoes to track the fast moving aircraft. The design problem to obtain the dual beam is to find element excitations that will result in a flat top pattern beam with low side lobes with the additional requirement that the same excitation amplitudes applied to the array with zero phase should result in a high directivity, low side lobe and pencil shaped main beam.

The generation of multiple radiation patterns by a single antenna array greatly simplifies the design and implementation of the feed network. Solutions to the design problem of multiple pattern arrays have been considered using amplitude and or phase control in several ways. The synthesis of phase only multiple radiation patterns with pre-fixed amplitude distributions using a modified Woodward Lawson technique has been reported by Ares et al. [22]. The method of projection to synthesize reconfigurable array antennas with asymmetrical and flat top beams using common amplitude and varying phase distribution has been proposed by Bucci [23]. The design of a phase differentiated reconfigurable array has been described [24] using particle swarm optimization in the theta domain. Phase only beam shaping with pre-fixed amplitude distributions has been reported in [25] using an analytical technique. Design of phase differentiated multiple antenna arrays has been reported based on simulated annealing optimization technique [26]. Synthesized reconfigurable array antennas with phase only control of a 6-bit discrete phase shifter and continuous amplitude distribution using generalized generation gap and a parent centric crossover (PCX) model genetic algorithm and better synthesis results has been obtained by Basker et al. [27].

In general, the pattern synthesis techniques require complex weights, phase shifters, or attenuators, which are variable at the elements of the array [28-33]. The most versatile technique is the control of both amplitudes and phases of the array elements [16]. A second technique is achieved through controlling only the phases of the array elements [31–33], which is an attractive solution, since in a phased array the required controls are available at no extra cost. The pattern synthesis is also possible by controlling only the amplitudes [15, 35], which overcome the limitations of phase only method while simplifying the adaptive system. However, there are many practical difficulties in the design of non-uniform amplitude arrays, in particular the amplitude tapered antenna arrays, which are a popular choice among antenna designers. Although, the amplitude tapered array obtains the required beam characteristics such as the minimum sidelobe level, the array elements are not operated at the maximum attainable power. Therefore, if we consider the radiated power for a given number of elements as the design criterion, then the amplitude taper in the elements is not the optimum way of the design. In this case, the phase-only synthesis enables us to operate the elements at maximum uniform amplitudes, maximizing the radiated power in the broadside direction. The drawback of the phase-only synthesis using equal element spacing is the requirement of a large number of elements compared to the amplitude tapered arrays for achieving lowered sidelobe levels.

In this paper, the reconfigurable dual pattern is obtained for the first time using an optimal set of element-perturbed positions in a constrained position range using the G3-GA algorithm. The range of the antenna for the optimised array structure is examined to see if the desired low level side lobe is maintained during the scanning process.

Further, it is the aim of this paper to show that an optimal set of element-perturbed positions in a constrained position range with uniform amplitude, unequally spaced elements and with unequal phases have the potential to overcome the above design challenges. The perturbation method consists of inducing small perturbations in the element positions of a linear uniform array to obtain the desired patterns [34]. Towards this a parent-centric re-combinational operator and a steady-state, elite preserving model genetic algorithm [35] is presented. The recombination operator is the main search operator in the GA. GA directly manipulates two or more parents to generate one or more offspring. The recombination operator to maintain adequate diversity in the population must increase the population variance. In mean-centric recombination, offspring are produced near the centroid of the participating parents (male and female). In parent-centric recombination, offspring are created near the parents by assigning each parent an equal probability of creating offspring in its neighbourhood. Hitherto in the conventional GA used with phased array antenna synthesis, a population of GAs is altered by the four user-defined plans in the evolution process. These are selection, generation, replacement and update plans. No strategy for male and female identification is proposed In the G3-GA approach the population diversity versus selection pressure problem considers both the parent selection and the replacement plans of GA. Each female member is considered as a niche in the population and the species formation takes place around these niches. Species formation is based on Euclidean distance between female and male members. Each species contains one female member and zero or more male members. All species gets equal chances to produce offspring The sexual selection strategy selects female and required number of male members, from species to perform recombination operation. After a certain generation the performance of the species is evaluated. If species is not performing well, then the merging to the nearby species takes place. The parent centric self-adaptive multi-parent recombination operators are used to explore the search space leading to faster convergence. To our best knowledge, it is the first attempt to use the G3-GA method with above emphasis (perturbation) in synthesizing the dual beam pattern problem.

2. FORMULATION FOR DUAL BEAM OPTIMIZATION

As already mentioned multiple beam optimisation has been obtained by using different techniques. These are described in brief in the following sub sections.

2.1. Amplitude/Phase Synthesis and Phase only Synthesis

Consider a linear array of N equispaced isotropic antenna elements (see Figure 1). The array pattern is given by

$$F(u) = \sum_{n=1}^{N} a_n \exp(jd_n u + \delta_n) \tag{1}$$

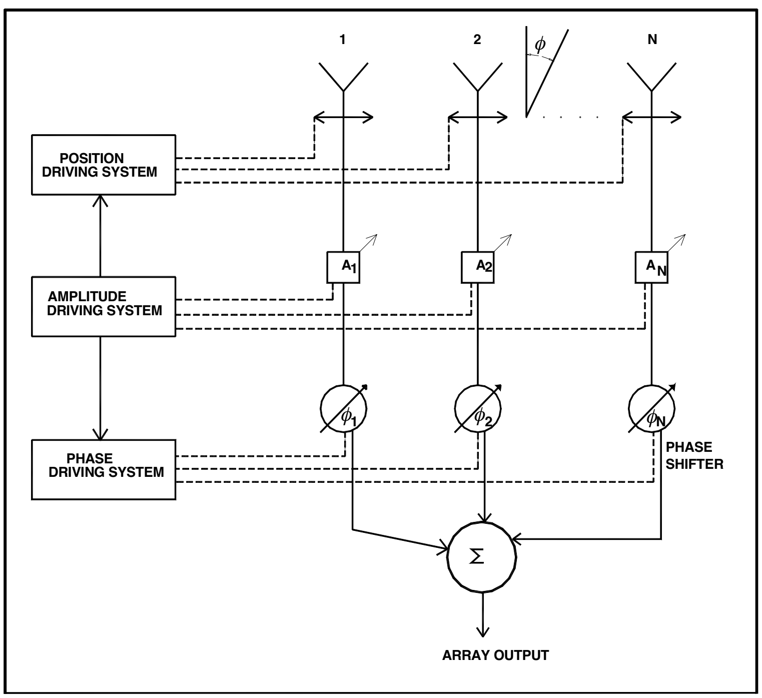

Figure 1. An adaptive linear array geometry.

where a_n is the excitation amplitude of the nth element and $u = k\sin(\theta)$, θ is the angle from broadside, the wavenumber $k = 2\pi/\lambda$, λ is wavelength, d_n is the distance between the nth element position and the array centre, δ_n is the excitation phase of the nth element.

All the excitation phases are set at 0° to generate a pencil pattern and are varied in the range $-180° \leq \phi \leq 180°$ to form a sector beam pattern. In the phase only synthesis a common amplitude distribution generates either a pencil or a sector beam power pattern, when the phase distribution of the array is modified appropriately. For example, with a pre fixed Gaussian amplitude distribution, the excitation phases in the range $-180° \leq \phi \leq 180°$ are used to generate the pencil as well as the flat top patterns.

2.2. Perturbed Position Synthesis

The purpose here is find an optimal set of element-perturbed positions $\{\Delta_1, \Delta_2, \ldots, \Delta_n\}$ in the constrained perturbation position range that achieves the desired dual beam pattern and minimizes the element position perturbations simultaneously for the computed common amplitude distribution. The method starts from a given original pattern with the desired main beam and sidelobe envelope corresponding to a set of optimized coefficients and initial inter-element equal spacing d_0. These element positions are then perturbed to obtain the second beam simplifying the feed network for the adaptive array.

For the nth element with perturbed position Δ_n only, the resultant distance from the array centre is $d_n^p = d_n + \Delta_n$. The designed perturbed pattern corresponding to the element position perturbations can be written as

$$F(u) = \sum_{n=1}^{N} a_n \exp(jd_n^p u) = \sum_{n=1}^{N} a_n \exp(j\Delta_n u)\exp(jd_n u) \qquad (2)$$

The above result can be written as

$$F(u) = \sum_{n=1}^{N} W_n \exp(jd_n u) \qquad (3)$$

where

$$W_n = a_n \exp(j\Delta_n u). \qquad (4)$$

The problem of finding the element position perturbations is a non linear problem and has been solved using the generalized generation gap steady state genetic algorithm (G3-GA) using parent centric crossover described below [35].

3. OPTIMIZATION USING THE GENETIC ALGORITHM

A genetic algorithm (GA) offers an alternative to traditional local search algorithms. It is an optimization algorithm inspired by well known biological process of genetics and evolution. A combination of genetics and evolution is analogous to numerical optimization in that they both seek to find a good result within constraints on the variables. Input to the objective function is a chromosome. The output of the objective function is known as the cost when minimizing. Each chromosome consists of genes or individual variables. A group of chromosomes is known as a population. GA begins its search with a population of guess solutions. Thereafter, in all iterations the population is updated by using a population-update algorithm. At the end of each iteration, this set is updated to a new by using user-defined plans. A baseline genetic algorithm has the following steps:

In first step, Choose μ solutions (the set P) from B using *selection plan* (SP). The selection plan (SP) for choosing μ solutions must emphasize the better solutions of B. A set of μ solutions can be chosen either by directly emphasizing the better solutions in B or by de-emphasizing the worst solutions of B.

In the second step, λ new solutions are created from the chosen set P by using generation plan GP.

In the third step, r solutions are chosen from the solution bank B for replacement. Here, different replacement plans (RP) are possible. The RP can simply choose r solutions at random or include some or all members of P to ensure diversity preservation. The RP can also pick a set of bad solutions from B to constitute a faster search.

In the fourth step, the r chosen members are updated by r members chosen from R, P and C by using update plan (UP). To really ensure elite-preservation, the RP should choose the best solutions of B and a combined P and C sets needs to be considered in Step 4.

4. THE STEADY STATE GENETIC ALGORITHM BASED ON PARENT CENTRIC RECOMBINATION

Generational GA and steady state GA are two strategies for reproducing the population members in GA. In generational GA, in each iteration, a complete set of new solutions is created. In steady state GA, for preserving elite solutions both the parent and the child populations are compared and the best solutions are retained. The term generation gap is used to describe the size of the population overlap in steady state. The selection pressure is more in steady-state

GA but its memory requirement is less as compared to generational GA.

For a steady-state GA the four plans are as follows

SP: Choose μ solutions (the set P) from solution bank B.

GP: Create the offspring set C from P. Solutions in set C, may or may not be created iteratively, from set P using operators.

RP: Choose r solutions (the set R) from B.

UP: Update these r members by r solutions chosen from a comparison set of R, P and C.

5. ALGORITHM DETAILS

Step 1: Form a population of N individuals; select the best parent and $(\mu - 1)$ other parents randomly.

Step 2: Generate λ offspring from the chosen μ parents using the PCX operator as follows: firstly, the mean vector $\vec{g}$ of the μ parents is computed. Thereafter, for each offspring, a parent $x^{(p)}$ is chosen from μ parents with equal probability and the direction vector $\vec{d}^{(p)} = x^{(p)} - \vec{g}(p)$ is calculated. Then, from each of the other $(\mu - 1)$ parents, perpendicular distances D_i to the line $\vec{d}^{(p)}$ are computed and their average D is found. The offspring y is created using the following equation:

$$y = x^{(p)} + \omega_\alpha \vec{d}^{(p)} + \sum_{\substack{i=1 \\ i \neq p}}^{\mu} \omega_\beta D \vec{e}^{(i)} \tag{5}$$

where $\vec{e}^{(i)}$ are $(\mu - 1)$ orthonormal bases that span the subspace perpendicular to $\vec{d}^{(p)}$, and ω_α and ω_β are zero mean normally distributed random variables with variance σ_α^2 and σ_β^2, respectively.

Step 3: Choose two parents at random from the population.

Step 4: From the combined subpopulation of two chosen parents and the created λ offspring solutions, choose the best two solutions and replace the chosen two parents with these solutions.

Step 5: Repeat steps 2–4 until a stopping criterion, such as a sufficiently good best solution being discovered or a maximum number of iterations/function evaluations being completed, is satisfied.

6. FITNESS FUNCTION

The optimal choice of the perturbations determined by the genetic algorithm requires only the fitness values of diverse perturbation choices, guiding potential solutions and gradually evolving towards better solutions through repetitive application of the genetic operations above.

Table 1. Design specifications for the dual beam synthesis.

Design Parameters	Pencil Pattern	Flat Top Pattern
Side-lobe level (SLL)	−30 dB	−25 dB
Half power bandwidth (HPBW)	7.0 degrees	24 degrees
Ripple	N/A	0.5 dB

For the dual beam array optimization, the objective function must quantify the entire array radiation pattern. The fitness function to be minimized for the dual pattern optimization is expressed as follows:

$$\text{Fitness} = \sum_{i-1}^{2}\left(P_{i,d}^{(p)} - P_i^{(p)}\right)^2 + \sum_{i-1}^{3}\left(P_{i,d}^{(s)} - P_i^{(s)}\right)^2 \qquad (6)$$

where the superscript p specifies fitness factors for the pencil pattern and superscript s specifies fitness factors for the sector pattern. The subscript d represents the desired values for each fitness factor. Finally, P represents the applicable fitness factors specified in Table 1. The first summation is performed over the first column of Table 1 and the second summation is performed over the second column.

7. NUMERICAL EXPERIMENTS

Experiments are first carried out to verify the optimization and fitness evaluation routines. Both of the desired radiation patterns, the pencil beam and the fan (flat top) beams are synthesized independently using the G3-GA code developed. An array of 20 equispaced elements with incremental spacing $0.5\,\lambda$ is considered. Because of symmetry only 10 phases, 10 perturbed positions (assuming symmetry around centre) and 10 amplitudes are considered for optimisation. The genetic algorithm is run with the an initial population of 60, number of offspring 9, PCX operation $\mu = 3$, $\sigma_\beta = \sigma_\beta = 0.1$.

All phases are restricted to lie between $-180°$ and $180°$; and the amplitudes between 0 and 1. The fitness is computed using either the $P^{(p)}$ or the $P^{(s)}$ terms, depending on which pattern is desired. Figure 2 shows the resulting radiation patterns for the optimized 20 element arrays with the excitation amplitude, phase and position perturbations given in Table 2. It is seen that the results agree and meet the desired design specifications (HPBW $= 7°$ for the pencil beam with SLL of -30 dB; $24°$ for the flat top beam with SLL of -25 dB) indicating the feasibility of applying the proposed technique for the synthesis of linear arrays considered.

Table 2. Computed amplitude and element positions for independent optimization of the pencil and fan (flat top) beams.

Pencil Beam: SLL= -30.2775 dB; HPBW=7^0

Element Number	Amplitude	Phase in degrees	Element Spacing d_k/λ
1/20	0.1681	14.8038	0.5082
2/19	0.0362	157.8460	0.5877
3/18	0.2764	65.2568	0.5363
4/17	0.0192	111.3832	0.5619
5/16	0.5113	144.7039	0.5804
6/15	0.7078	54.4629	0.5303
7/14	0.6687	-41.4931	0.4769
8/13	0.4083	54.8899	0.5305
9/12	0.9121	55.7749	0.5310
10/11	0.7603	-76.5591	0.4575

Flat Top Beam: SLL=-25.0058 dB; HPBW=24.6^0

Element Number	Amplitude	Phase in degrees	Element Spacing d_k/λ
1/20	0.0971	-100.6019	0.4441
2/19	0.8441	-95.1329	0.4471
3/18	0.0193	16.6441	0.5092
4/17	0.2619	-154.84556	0.4140
5/16	1.0000	-82.4927	0.4542
6/15	0.0764	54.4047	0.5302
7/14	0.9096	-58.1295	0.4677
8/13	0.6529	-55.2363	0.4693
9/12	0.8159	-11.8465	0.4934
10/11	0.9427	-11.3464	0.4937

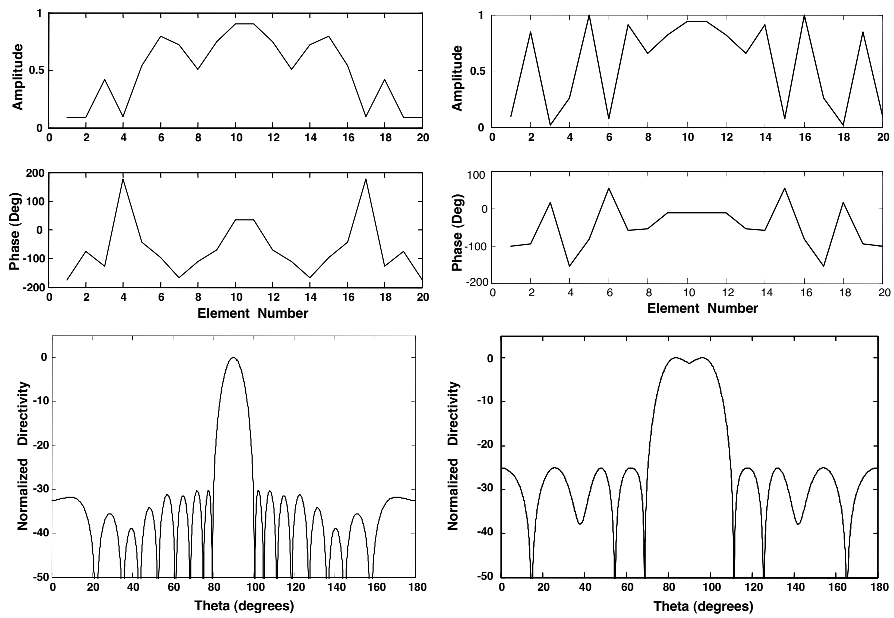

Figure 2. Optimization of the individual pencil and flat top beams for the parameters given in Table 1.

Next, a linear array of 20 isotropic elements and initially spaced $0.5\,\lambda$ apart is considered. Because of symmetry, only 10 phases, 10 perturbed positions and 10 amplitudes are considered for optimisation. To speed up convergence, the optimised amplitude and perturbed positions of the independently optimised pencil beam pattern is used as an input to compute the initial population for the dual pattern optimisation. The element phases are randomly varied from their initial position in the range $-180°$ to $+180°$ to obtain the common amplitude, common perturbed position pencil and fan (flat top) beams, the difference being dependent upon the phase distribution of the arrays. The fitness is computed using the $P^{(p)}$ and the $P^{(s)}$ tterms For the design specifications as given in Table 1 the genetic algorithm is run with the an initial population of 60, number of offspring 9, maximum number of generations 1300, PCX operation $\mu = 3$.

Table 3 lists the optimised amplitude-phase distributions and element positions for the dual beam synthesis. Fig. 3 shows the optimised radiation patterns for the amplitude-phase distributions and for the perturbed element positions of $\pm 5\%$ and $\pm 10\%$. The corresponding HPBW and the maximum sidelobe levels for the

Table 3. Joint optimization of the pencil and fan (flat top) beams for two different perturbations.

(a) Pertubation =±5%

SLL(sector)= -26.2210 dB, HPBW(sector)=24^0, Ripple=0.3dB,
SLL(pencil)= -30.6213 dB, HPBW(sector)=7.6^0

Element Number	Amplitude	Phase in degree	Element Spacing d_k/λ
1/20	0.0749	-123.2240	0.4883
2/19	0.2058	-92.4829	0.4765
3/18	0.2021	-84.5468	0.4742
4/17	0.4843	-84.0763	0.4736
5/16	0.3388	92.0390	0.4504
6/15	0.4077	-93.5078	0.5066
7/14	0.7742	-114.7111	0.4842
8/13	0.6625	-98.4051	0.4801
9/12	0.8902	-179.9759	0.4977
10/11	0.8492	-134.9815	0.4872

(b) Pertubation =±10%

SLL(sector)= -25.003 dB, HPBW(sector)=23.9^0, Ripple=0.021dB,
SLL(pencil)= -30.013 dB, HPBW(sector)=7.2^0

Element Number	Amplitude	Phase in degree	Element Spacing d_k/λ
1/ 20	0.1914	-97.5297	0.4782
2/ 19	0.0892	-86.5294	0.4723
3/ 18	0.3699	-80.9005	0.4746
4/ 17	0.2881	-84.0600	0.4529
5/ 16	0.3218	96.2999	0.5051
6/ 15	0.5704	-99.5834	0.4791
7/ 14	0.5257	-128.1218	0.4835
8/ 13	0.7580	-141.2450	0.4861
9/ 12	0.7678	-147.4568	0.4890
10/11	0.7812	-177.4974	0.4829

optimised patterns are indicated in Table 3. The convergence curves are shown in Figure 4. There is, in general, a good agreement between the desired and the synthesized results using G3_GA method adopted. Next, the scanning properties of the optimised beam are examined to see if the proposed method can maintain the desired sidelobe levels without pattern distortion during beam steering. Figure 5 shows the radiation pattern when the main beam is scanned for different angular positions indicating the advantage of the method when compared to

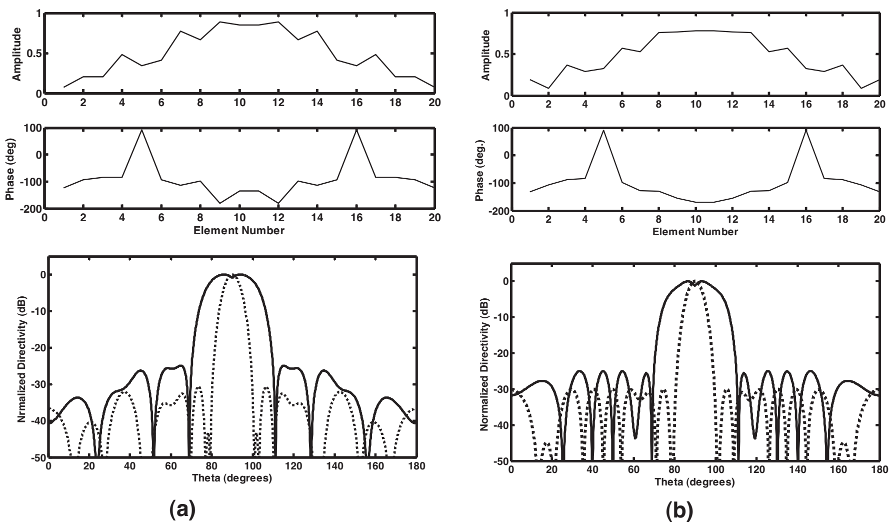

Figure 3. Dual beam optimised array patterns using element position perturbations of (a) $\pm 5\%$ and (b) $\pm 10\%$ for the corresponding excitation amplitudes-phase; - - - - - indicates the pencil beam and —— indicates the fan shaped beam.

Table 4. Position and phases of a 20 element pencil beam array derived using the G3-GA phase only synthesis and G3-GA position-ohase synthesis.

Element	Phase only	Phase-Position	
	δ_n	δ_n	d_k/λ
1/20	-120.8265	-91 .85	0.4883
2/19	-138.94	-155.32	0.4765
3/18	-177.92	-128.44	0.4742
4/17	-132.72	-134.70	0.4736
5/16	-151.45	-151.11	0.4504
6/15	-159.61	-145.77	0.5066
7/14	-149.35	-153.64	0.4842
8/13	-166.57	-163.64	0.4801
9/12	-146.43	-158.48	0.4977
10/11	-169.72	-165.20	0.4872

other linear gradient search algorithms.

Finally, the G3-GA-based position-phase synthesis and the G3-GA based phase-only synthesis are used for designing a pencil beam array. The number of elements in both the synthesis methods is

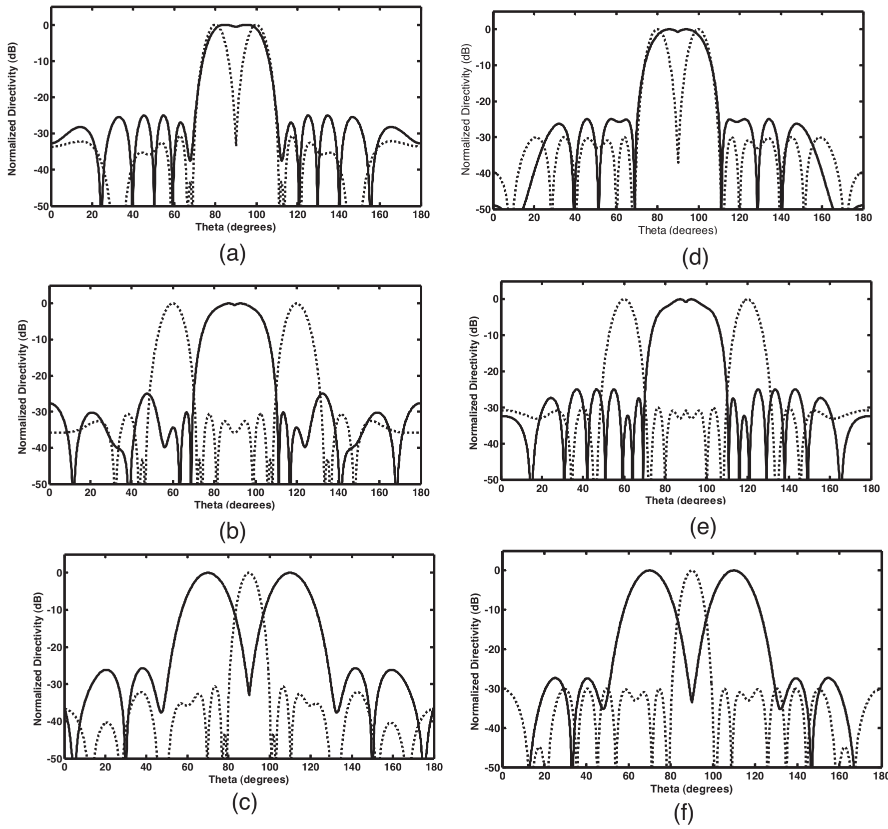

Figure 4. Scanning properties of optimised array patterns for element position perturbations of ±5% (a) pencil beam 10° (b) pencil beam 30° (c) sector beam 30° and 10% (d) pencil beam 10° (e) pencil beam 30° (f) sector beam 30° and ±10%.

20. For the position-phase synthesis, the prior limits assumed in the maximum distance between the elements is 5%. For phase-only synthesis, the uniform distances between the elements are assumed to be $0.5\,\lambda$. Table 4 shows the phases and positions derived using the G3-GA-based phase-only synthesis and position-phase synthesis. Fig. 6 shows the corresponding array patterns. It is easily seen that for the position-phase synthesis, the sidelobe level has been lowered by about 2.5 to 3 dB when compared with phase only synthesis. In order to study the effects of the number of elements on the array performance the G3-GA synthesis procedure is repeated for synthesizing arrays upto fifty elements. For the phase only synthesis, the uniform distance

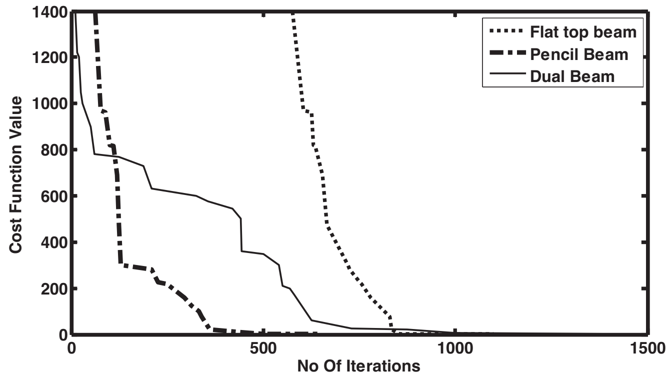

Figure 5. Convergence Curves for the individual pencil and flat top beams and for the dual beam.

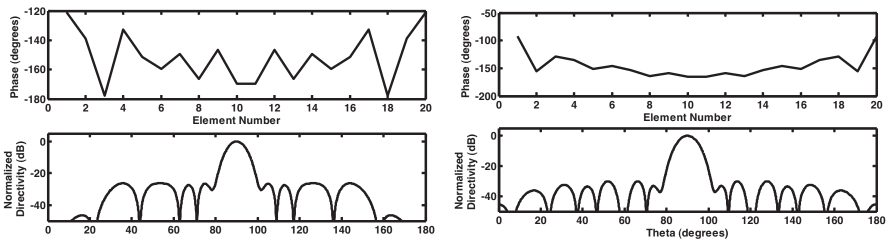

Figure 6. Comparison of the phase and array patterns for the G3-GA based phase only (left column) and the G3-GA phase-position synthesis for the 20 element unequally.

Table 5. Maximum SLL and beamwidth with number of elements for phase only and position-phase synthesis.

No of elements	Beamwidth (degrees)		SLL (dB)	
	Phase Only	Position-Phase	Phase only	Position -Phase
20	8.13	7.61	-27.5	-30.07
30	3.42	3.05	-30.12	-33.21
40	2.59	2.51	-33.67	-35.93
50	2.24	2.10	-35.08	-37.26

between the elements is $0.5\,\lambda$. Table 5 shows the achieved maximum sidelobe levels and the beamwidths for the different synthesis methods indicating the superiority of the phase-position method when compared to the phase only synthesis method.

8. CONCLUSION

In this paper, the reconfigurable dual pattern is obtained for the *first time* using an optimal set of element-perturbed positions in a constrained position range using the G3-GA algorithm. The range of the antenna for the optimised array structure maintains the desired low level side lobe during the scanning process for the scanning ranges studied. From the study of unequally spaced arrays derived using the G3-GA based phase only and phase-position synthesis, the position-phase synthesis showed that the position-phase synthesis results in reduced sidelobe when compared to the phase only synthesis. Although only linear antenna arrays have been considered here, the method can be easily extended for arrays with complex geometries as well as non isotropic-elements.

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