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ANALYSIS OF DIELECTRIC BODY BY USING VOLUME INTEGRAL EQUATION COMBINED WITH MULTI-REGION ITERATIVE METHOD

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Abstract—In this research, a fast approach of method of moments (MoM) for analyzing 3-D dielectric body is proposed. The unknown polarization current in dielectric body is expanded into rectangular blocks with overlapping volume sinusoidal basis functions. To accelerate the matrix-solving CPU time in MoM, the multi-region iterative method, where the overlapping blocks are used as the iteration units, is applied to solving the matrix equation in the MoM. Some numerical results are given to show that the CPU time for solving unknown currents can be reduced effectively by multi-region iterative method.

1. INTRODUCTION

The method of moments (MoM) is one of the effective methods for electromagnetic analysis of radiation and scattering problems where both of conductors and dielectric bodies are involved. When the dielectric body is included in the analysis model of MoM, it costs much of the CPU time to fill the mutual impedance between the dielectric blocks because the expression of mutual impedance between the 3-D dielectric blocks includes multiple integral. The point matching technique can reduce the CPU time for integration [1], but it is difficult to obtain the accurate solution of self/mutual impedance. The Galerkin-MoM analysis for dielectric scatters by using sinusoidal reaction technique can give accurate solution, but a double volume integral is required. Many researches have been done to reduce double volume integral to lower dimensional integral. The double volume integral was reduced to a three-dimensional integral [2]. Recently, the double volume integral was further transformed into one-dimensional integral to reduce the CPU time [3, 4].

Although the CPU time of filling the self/mutual impedance can be reduced greatly, solving solution of matrix equation appearing in the MoM costs much more CPU time than matrix-filling when the dielectric body becomes larger. Many researches have been carried out in order to obtain the fast approach for a large-scale problem. The fast multipole method (FMM) reduces CPU time by accelerating the matrix-vector multiplication in an iterative solver [5]. Some other techniques are also given for the fast solving [6]. The multi-region iterative method is another method to reduce the CPU time in solving the matrix equation, which have been studied in two-dimensional or three-dimensional surface radiation and scattering problems of conducting bodies [7–12]. The iterative multi-region algorithm is also used by hybrid method [13].

In this research, the volume integral method combined with multi-region iterative method for 3-D volume radiation and scattering problems of dielectric bodies is studied. In the MoM, where 3-D dielectric body is involved, the unknown polarization current in dielectric body is expanded into rectangular blocks with overlapping volume sinusoidal functions. To accelerate the matrix-solving CPU time, the whole dielectric body is divided into many smaller overlapping volume sub-regions. The iterative algorithm, where the overlapping volume sub-region is regarded as the iteration unit, is applied to solving the matrix equation in the MoM. It is described that how the multi-region iterative method is implemented on the dielectric body problem. Some numerical results are given to show that the

CPU time for solving unknown currents can be reduced effectively by multi-region iterative method.

2. FORMULATION

In the MoM, the dielectric body is divided into dielectric blocks and the mutual impedance between the blocks can be expressed by a double volume integral

$$Z_{mn}^{dd} = j\omega\mu_0 \iiint\limits_{V_m} \iiint\limits_{V_n} \vec{g}_m(\vec{r},\vec{r}\,') \cdot \bar{\bar{G}}_0(\vec{r},\vec{r}\,') \cdot \vec{f}_n(\vec{r}) dV dV' + \delta_{m,n} \iiint\limits_{V_m} \left[\frac{1}{\sigma + j\omega(\varepsilon_r\varepsilon_0 - \varepsilon_0)}\right] \vec{g}_m(\vec{r}) \cdot \vec{f}_n(\vec{r}) dV \quad (1)$$

where $\bar{\bar{G}}_0(\vec{r},\vec{r}\,')$ is the free space dyadic Green's function, and $\vec{f}_n$ and $\vec{g}_m$ denote the sinusoidal basis functions and the sinusoidal test functions for the polarization current inside the dielectric scatters, respectively. The detailed calculation of self and mutual impedance by equation (1) can be found in [4]. The unknown polarization current is expanded into overlapping volume sinusoidal functions in x, y and z directions.

After filling the self/mutual impedance, generally the following matrix equation can be obtained by

$$[Z][I] = [V] \quad (2)$$

By using the multi-region iterative method for solving above Eq. (2), the whole object is divided into smaller sub-regions. Considering the iterative convergence problem, the adjoining regions are overlapped generally [7, 8], in which the detailed discussions have been given. In this research, the method was further applied to 3-D volume problem. The volume multi-region model is shown in Fig. 1. There are many methods to divide a large object into many sub-regions. In order to simply and clearly explain the idea of sub-regions, we only use the simpler one-dimensional divided model as used in [7, 8] to illustrate the application of sub-regions iterative method, which can show the validity of our method. About the selection rule of overlapping regions can be detailed found in [7–9]. For example, in [7], the overlapping region size equal to 1/4–1/5 of the sub-region size is good enough to reach the full MoM solution.

Firstly, if the interactions from the other regions are neglected, the relationship between unknown current and excitation voltage in

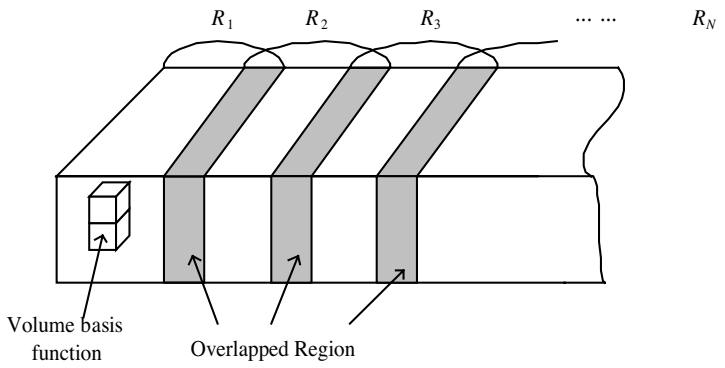

Figure 1. Many overlapped sub-regions for large object.

the first volume sub-region (R_1) can be expressed by

$$[Z_{11}]\,[I_1] = [V_1] \tag{3}$$

where $[Z_{11}]$ denotes the self impedance in volume sub-region1. $[I_1]$ is the unknown current coefficient vector in volume sub-region1. If the unknown current vector $[I_1']$ in non-overlapping part of sub-region1 is assumed to be unchanged, unknown current vector $[I_2]$ in volume sub-region2 can be solved by

$$[Z_{22}]\,[I_2] = [V_2] - [Z_{21}']\,[I_1'] \tag{4}$$

where $[Z_{21}']$ is the modified mutual impedance matrix, which represents the interaction between volume sub-region2 (R_2) and the non-overlapping volume part of sub-region1.

Repeating the above procedures, we can obtain the iterative solution of the lth volume sub-region currents by the following iterative matrix equation which can also be called forward and backward iterative method:
For the forward iterative equation:

$$\left\{\sum_{i=1}^{l-1}[Z_{li}'][I_i']^{(m)}\right\} + [Z_{ll}][I_l]^{(m)} + \left\{\sum_{i=l+1}^{n}[Z_{li}'][I_i']^{(m-1)}\right\} = [V_l] \tag{5}$$

For the backward iterative equation:

$$\left\{\sum_{i=1}^{l-1}[Z_{li}'][I_i']^{(m)}\right\} + [Z_{ll}][I_l]^{(m+1)} + \left\{\sum_{i=l+1}^{n}[Z_{li}'][I_i']^{(m+1)}\right\} = [V_l] \tag{6}$$

where l represents sub-region number. For (5), $l = 1, 2, \ldots, n$ and for (6), $l = n, \ldots, 2, 1$. m represents iterative step.

In above two equations, $[Z_{ll}]$ denotes the self impedance matrix of sub-region $l\,(R_l)$. $[Z'_{li}]$ denotes the modified mutual impedance matrix, which represents the interaction between sub-region l and the corresponding non-overlapping part of sub-region i. $[I'_i]$ denotes the unknown current matrix in non-overlapping part of sub-region i. $[V_l]$ denotes the exciting voltage matrix in sub-region l. The superscript m denotes the mth iteration. The initialized current $[I_l]^{(0)} = 0$. The Detailed discussions with regards to saving the calculation count are given and an iterative process is suggested for the high-order solutions in [7, 8].

The iterative error is estimated by the residual norm, which is defined by

$$\Pi = \|[Z][I] - [V]\| \,/\, \|[V]\| \tag{7}$$

3. NUMERICAL RESULTS

The model for a numerical analysis is shown in Fig. 2. A wire dipole antenna is located in the vicinity of a dielectric rectangular box. The dielectric body is divided into dielectric blocks. The sinusoidal basis and test functions are also used for the dipole antenna.

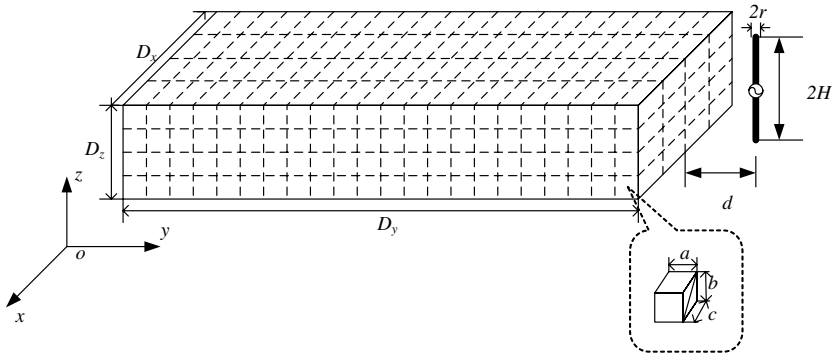

Figure 2. Simulation model of dipole antenna located in the vicinity of dielectric box.

In order to show the validity of the multi-region iterative method on based of volume integral method, the numerical example is given. $D_x = 10.5\,\text{mm}$, $D_y = 112\,\text{mm}$, $D_z = 10.5\,\text{mm}$, $d = 1\,\text{mm}$, $2H = 10.5\,\text{mm}$, $r = 0.1\,\text{mm}$, $\varepsilon_r = 5$, $\sigma = 0$. The size of volume monopole basis function is $3.5\,\text{mm} \times 3.5\,\text{mm} \times 3.5\,\text{mm}$. The multi-region model

is shown in Fig. 3. The dielectric scatter is divided into n overlapped volume sub-regions, which is shown in the following figure.

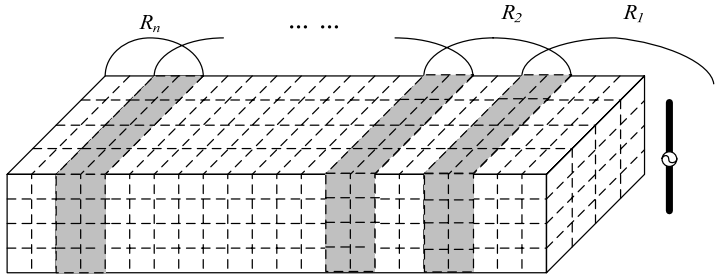

Figure 3. Multi-region model for antenna and dielectric box.

Gauss-Jordan method ($n = 1$) and multi-region iterative method ($n > 1$) are used for the same size body. In the sub-region iterative method, the residual norm is smaller than 1×10^{-6}.

The CPU-times with different number of sub-regions when operation frequency is 4.5 GHz are shown in the Table 1 where a Pentium-III 2.8 GHz PC with 1.5 GB of memory is used.

Table 1. CPU time results for different number of sub-regions.

Number of sub-regions	Time for inversing matrix T_1 [sec.]	Iterative steps	Time for iteration T_2 [sec.]	Whole time T [sec.]
1	444.218			444.218
3	203.078	4	0.828	203.906
5	50.296	6	1.218	51.514
7	23.500	9	1.781	25.281
9	6.0625	10	2.203	8.265

From the Table 1 we can see that compared with the conventional solution ($K = 1$) without overlapping regions, the sub-region iterative method ($K > 1$) effectively reduces the whole CPU time. We also note that when number of sub-regions increases, the time T_1 becomes smaller, however the iterative steps become larger and the time T_2 also becomes larger. In general, when the number of sub-region increases, the T_1 will becomes smaller and T_2 will become larger. Thus, we will try

not to increase T_2 greatly when we reduce T_1 by sub-regions method. In order to reach the optimum selection, some basic relations or rules should be considered, for example, in [7], the overlapping region size equal to 1/4–1/5 of the sub-region size is good enough to reach the full MoM solution; When number of sub-regions is constant, overlapping regions becomes larger, T_1 will become larger and iterative steps will become smaller. The detailed discussions about that can be found in [7–9].

In the following, the convergences for the different number of sub-regions are also shown in the following Fig. 4.

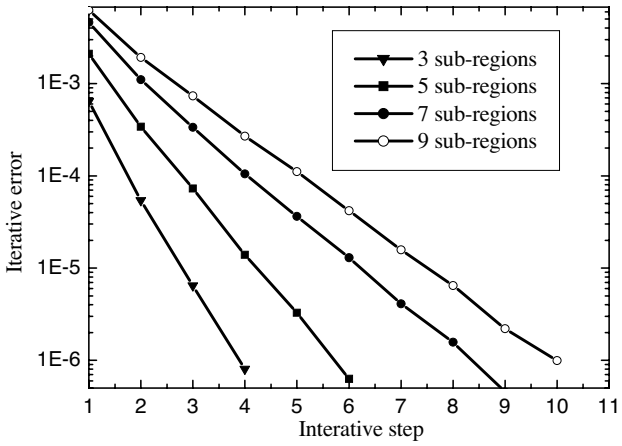

Figure 4. The convergence for the different number of sub-regions.

From above table, we can see that the whole CPU time of solving unknowns can be effectively reduced by multi-region iterative method compared with conventional Gauss-Jordan method.

The number of iteration steps versus the total number of the unknowns is shown in Fig. 5. The computational model is same as one in Fig. 3. When unknowns increase, it means the distance along y direction of rectangular dielectric body in Fig. 3 will increase. When the K is constant, overlapping region is nearly equal to 1/3 of sub-region size.

The total computational time T can be approximatively expressed by $T \approx T_1 + T_2 = \alpha \sum_1^K M_i^3 + \beta L \sum_1^K M_i^2$, where, K is number of sub-regions, N is the whole number of unknowns, and M_i denotes the number of unknowns in sub-region i. Because each sub-regions have overlapping regions, the sum of all unknowns $\sum_1^K M_i$ will be larger than N. The first term is for evaluating $[Z_{ll}]^{-1}$, the second term is for iterating process, and the α, β are constants depending on

the computer performance, L denotes the number of iterative steps. From Fig. 5, we can see the iterative steps L tend to be stable when the whole unknowns N become larger in the case that the number of sub-regions (K) is constant. At the same time, we can also find the L gives some increase when K (the number of all sub-regions) becomes larger in the Fig. 5.

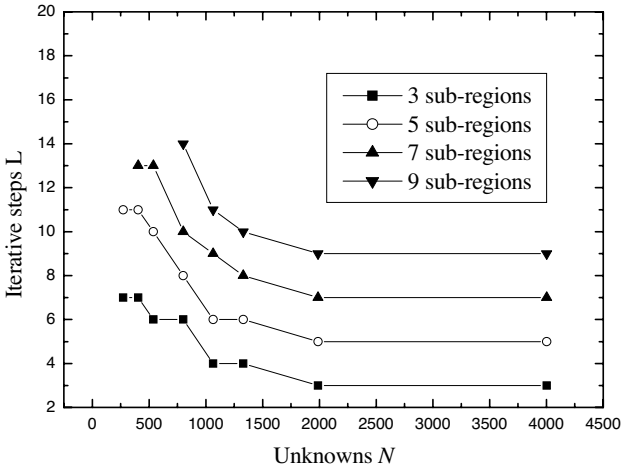

Figure 5. Iterative steps versus the number of unknowns.

4. CONCLUSION

In this research, multi-region iterative method has been effectively applied to dielectric body problem on basis of volume integral equation. The numerical results show that the computational CPU time can be reduced significantly if the multi-region iterative method is applied.

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