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DIFFRACTION BY A SEMI-INFINITE PARALLEL-PLATE WAVEGUIDE WITH SINUSOIDAL WALL CORRUGATION: COMBINED PERTURBATION AND WIENER-HOPF ANALYSIS

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Abstract—The diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation is analyzed for the E-polarized plane wave incidence using the Wiener-Hopf technique together with the perturbation method. The problem is formulated in terms of the simultaneous Wiener-Hopf equations by introducing the Fourier transform for the unknown scattered field and applying approximate boundary conditions in the transform domain. Employing the factorization and decomposition procedure together with a perturbation series expansion, the zero- and first-order solutions of the Wiener-Hopf equations are obtained. Explicit expressions of the scattered field inside and outside the waveguide are derived analytically by taking the inverse Fourier transform and applying the saddle point method. Far field scattering characteristics of the waveguide are discussed in detail via representative numerical examples.

1. INTRODUCTION

In microwave and optical engineering, there are many devices with periodic structures including resonators, filters, and couplers composed of gratings as well as reflector antennas. Therefore the analysis of the scattering by periodic structures is one of the important subjects in electromagnetic theory and optics. Various analytical and numerical methods have been developed thus far and the diffraction phenomena have been investigated for a number of periodic structures. The

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Riemann-Hilbert problem technique [1–3], the analytical regularization methods [3–5], the Yasuura method [6–8], the integral and differential method [9], the point matching method [10], and the Fourier series expansion method [11, 12] are efficient for the analysis of the diffraction by periodic structures. The authors of these papers have analyzed diffraction problems involving various gratings and periodic structures, and obtained important results. The Wiener-Hopf technique [13–17] is known as a powerful tool for analyzing electromagnetic wave problems related to canonical geometries rigorously, and can be applied efficiently to problems of the diffraction by specific periodic structures such as gratings. There are significant contributions to the analysis of the diffraction by gratings based on the Wiener-Hopf technique [18–22]. In the previous papers, we have analyzed the diffraction problems involving transmission-type gratings with the aid of the Wiener-Hopf technique [23–26], where rigorous solutions valid over a broad frequency range have been obtained.

It is noted that the analysis in most of the above-mentioned papers are restricted to periodic structures of infinite extent and plane boundaries. Therefore, it is important to investigate scattering problems involving periodic structures without these restrictions. As an example of infinite periodic structures with non-plane boundaries, Das Gupta [27] analyzed the plane wave diffraction by a half-plane with sinusoidal corrugation by means of the Wiener-Hopf technique together with the perturbation method. The method developed in [27] has been generalized thereafter by Chakrabarti and Dowerah [28] for the Wiener-Hopf analysis of the H-polarized plane wave diffraction by two parallel sinusoidal half-planes. We have also considered a finite sinusoidal grating as another important generalization and analyzed the plane wave diffraction by means of the Wiener-Hopf technique [29–31].

In this paper, we shall analyze the E-polarized plane wave diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation by the Wiener-Hopf technique together with the use of the perturbation method. As mentioned above, this problem was solved in the past for the H-polarized plane wave incidence by Chakrabarti and Dowerah [28] following a similar method, but their analysis was incomplete since important contributions to the scattered field were not taken into account. In addition, no numerical results were presented in their paper. We shall derive, in this paper, various new expressions of the scattered field inside and outside the waveguide via a more rigorous approach.

Assuming that the corrugation amplitude of the waveguide walls is small compared with the wavelength, we replace the original problem by the problem of diffraction by a flat, semi-infinite parallel-plate waveguide with impedance-type boundary conditions. Taking the Fourier transform of the two-dimensional Helmloltz equation and applying approximate boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener-Hopf equations satisfied by unknown spectral functions. The Wiener-Hopf equations are then solved via the factorization and decomposition procedure together with the perturbation scheme leading to the zero- and first-order perturbation solutions. Taking the Fourier inverse of the solution in the transform domain and applying the saddle point method, explicit expressions of the scattered field in the real space are derived analytically. Illustrative numerical examples on the scattered far field are presented, and the effect of sinusoidal corrugation of the waveguide walls is investigated in detail.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. FORMULATION OF THE PROBLEM

We consider the diffraction of an E-polarized plane wave by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation as shown in Fig. 1, where the E-polarization implies that the incident electric field is parallel to the y-axis. The surface of the two half-planes is assumed to be infinitely thin, perfectly conducting, and uniform in the y-direction, which is defined by

$$x = \pm b + h \sin mz, \quad z < 0, \tag{1}$$

where m and h are some positive constants. In view of the waveguide geometry and the characteristics of the incident field, this problem reduces to a two-dimensional problem.

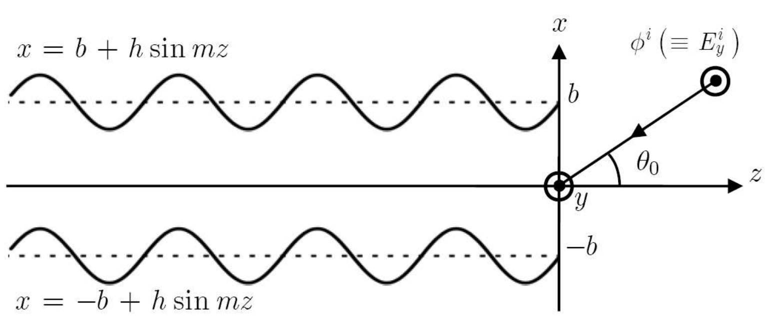

Figure 1. Geometry of the problem.

Let us define the total electric field $\phi^t(x,z)[\equiv E_y^t(x,z)]$ by

$$\phi^t(x,z) = \phi^i(x,z) + \phi(x,z), \tag{2}$$

where $\phi^i(x,z)$ is the incident field given by

$$\phi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)} \tag{3}$$

for $0 < \theta_0 < \pi/2$ with $k\left[\equiv \omega(\varepsilon_0\mu_0)^{1/2}\right]$ being the free-space wavenumber. The scattered field $\phi(x,z)$ satisfies the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2\right)\phi(x,z) = 0. \tag{4}$$

Nonzero components of the scattered electromagnetic fields are derived from the following relations:

$$(E_y,\, H_x,\, H_z) = \left[\phi,\, \frac{i}{\omega\mu_0}\frac{\partial\phi}{\partial z},\, \frac{1}{i\omega\mu_0}\frac{\partial\phi}{\partial x}\right]. \tag{5}$$

The total electric field satisfies the perfect conductor condition

$$\phi^t(\pm b + h\sin mz, z) = 0, \quad z < 0 \tag{6}$$

on the waveguide walls. Assuming that the corrugation amplitude $2h$ of the waveguide walls is small compared with the wavelength, we approximate the boundary condition (6) by ignoring the $O\left(h^2\right)$ terms with the aid of Taylor's theorem. Then we deduce that

$$\phi^t(\pm b, z) + h\sin mz\frac{\partial\phi^t(\pm b, z)}{\partial x} + O\left(h^2\right) = 0 \tag{7}$$

for $z < 0$. We note that, by letting $h \to 0$ in (7), the problem reduces to the classical diffraction problem involving a flat, semi-infinite parallel-plate waveguide [13–16].

For convenience of analysis, we assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real k is obtained by letting $k_2 \to 0$ at the end of analysis. In view of the radiation condition, it follows that, for any fixed x, the scattered field $\phi(x,z)$ in (2) shows the asymptotic behavior

$$\begin{aligned} \phi(x,z) &= O(e^{k_2 z\cos\theta_0}), \;\; z \to -\infty \\ &= O(e^{-k_2 z}), \;\; z \to \infty. \end{aligned} \tag{8}$$

Let us introduce the Fourier transform of the scattered field as in

$$\Phi(x,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi(x,z)e^{i\alpha z}dz \tag{9}$$

for $\alpha = \operatorname{Re}\alpha + i\operatorname{Im}\alpha (\equiv \sigma + i\tau)$. Using (8), we find from (9) that $\Phi(x,\alpha)$ is regular in the strip $-k_2 < \tau < k_2 \cos\theta_0$ of the complex α-plane. We further introduce the Fourier integrals as

$$\Phi_{\pm}(x,\alpha) = \pm(2\pi)^{-1/2} \int_0^{\pm\infty} \phi(x,z) e^{i\alpha z} dz. \tag{10}$$

Then it is seen from (8) that $\Phi_+(x,\alpha)$ and $\Phi_-(x,\alpha)$ are regular in $\tau > -k_2$ and $\tau < k_2 \cos\theta_0$, respectively. It follows from (9) and (10) that

$$\Phi(x,\alpha) = \Phi_+(x,\alpha) + \Phi_-(x,\alpha). \tag{11}$$

We also see with the aid of (8) that $\Phi(x,\alpha)$ is bounded as $|x| \to \infty$. Taking the Fourier transform of (4) and making use of (8), we derive that

$$\left[d^2/dx^2 - \gamma^2(\alpha)\right] \Phi(x,\alpha) = 0, \tag{12}$$

where $\gamma(\alpha) = (\alpha^2 - k^2)^{1/2}$ with $\operatorname{Re}\gamma(\alpha) > 0$. Equation (12) is the transformed wave equation and holds for any α in the strip $-k_2 < \tau < k_2 \cos\theta_0$.

Taking into account the radiation condition, the solution of (12) is expressed as

$$\begin{aligned} \Phi(x,\alpha) &= A(\alpha) e^{-\gamma(\alpha)x}, \ x > b, \\ &= B(\alpha) e^{-\gamma(\alpha)x} + C(\alpha) e^{\gamma(\alpha)x}, \ |x| < b, \\ &= D(\alpha) e^{\gamma(\alpha)x}, \ x < -b, \end{aligned} \tag{13}$$

where $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$ are unknown functions. For convenience of analysis, we introduce the Fourier integrals as in

$$P_+(\alpha) = \int_0^{\infty} \left[\phi(b+0,z) + h \sin mz \frac{\partial}{\partial x} \phi(b+0,z)\right] e^{i\alpha z} dz, \tag{14}$$

$$Q_+(\alpha) = \int_0^{\infty} \left[\phi(-b-0,z) + h \sin mz \frac{\partial}{\partial x} \phi(-b-0,z)\right] e^{i\alpha z} dz, \tag{15}$$

$$M_-(\alpha) = \int_{-\infty}^{0} \left[\frac{\partial}{\partial x} \phi(b+0,z) - \frac{\partial}{\partial x} \phi(b-0,z)\right] e^{i\alpha z} dz, \tag{16}$$

$$N_-(\alpha) = \int_{-\infty}^{0} \left[\frac{\partial}{\partial x} \phi(-b+0,z) - \frac{\partial}{\partial x} \phi(-b-0,z)\right] e^{i\alpha z} dz. \tag{17}$$

In Appendix A, we have investigated some important relations among the functions $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$ in (13) and the functions

$P_+(\alpha)$, $Q_+(\alpha)$, $M_-(\alpha)$, and $N_-(\alpha)$ defined by (14)–(17). Referring to (A13)–(A18) in Appendix A, it follows that

$$\begin{aligned} A(\alpha) &= -\frac{e^{\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha{+}m) - M_-(\alpha{-}m)\right]\right\} \\ &\quad -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha{+}m) - N_-(\alpha{-}m)\right]\right\}, \quad (18) \\ B(\alpha) &= -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha{+}m) - N_-(\alpha{-}m)\right]\right\}, \quad (19) \\ C(\alpha) &= -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha{-}m) - M_-(\alpha{+}m)\right]\right\}, \quad (20) \\ D(\alpha) &= -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha{-}m) - M_-(\alpha{+}m)\right]\right\} \\ &\quad -\frac{e^{\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha{-}m) - N_-(\alpha{+}m)\right]\right\} \quad (21) \end{aligned}$$

and

$$\begin{aligned} F_1(\alpha)+P_+(\alpha) &= e^{-2\gamma(\alpha)b}\left\{\frac{ih}{4}\left[N_-(\alpha+m) - N_-(\alpha-m)\right] - \frac{N_-(\alpha)}{2\gamma(\alpha)}\right\} \\ &\quad -\frac{M_-(\alpha)}{2\gamma(\alpha)} + \frac{ih\gamma(\alpha+m)}{2} \\ &\quad \cdot\left(e^{-2\gamma(\alpha+m)b}\left\{\frac{ih}{4}\left[N_-(\alpha{+}2m) - N_-(\alpha)\right] - \frac{N_-(\alpha{+}m)}{2\gamma(\alpha{+}m)}\right\}\right. \\ &\quad \left. +\frac{ih}{4}\left[M_-(\alpha) - M_-(\alpha+2m)\right]\right) - \frac{ih\gamma(\alpha-m)}{2} \\ &\quad \cdot\left(e^{-2\gamma(\alpha-m)b}\left\{\frac{ih}{4}\left[N_-(\alpha) - N_-(\alpha-2m)\right] - \frac{N_-(\alpha{-}m)}{2\gamma(\alpha-m)}\right\}\right. \\ &\quad \left. +\frac{ih}{4}\left[M_-(\alpha) - M_-(\alpha{-}2m)\right]\right), \quad (22) \\ F_2(\alpha)+Q_+(\alpha) &= e^{-2\gamma(\alpha)b}\left\{\frac{ih}{4}\left[M_-(\alpha{-}m) - M_-(\alpha+m)\right] - \frac{M_-(\alpha)}{2\gamma(\alpha)}\right\} \\ &\quad -\frac{N_-(\alpha)}{2\gamma(\alpha)} + \frac{ih\gamma(\alpha{+}m)}{2} \\ &\quad \cdot\left(e^{-2\gamma(\alpha+m)b}\left\{\frac{ih}{4}\left[M_-(\alpha{+}2m) - M_-(\alpha)\right] + \frac{M_-(\alpha{+}m)}{2\gamma(\alpha{+}m)}\right\}\right. \end{aligned}$$

$$+\frac{ih}{4}\left[N_-(\alpha+2m)-N_-(\alpha)\right]\Big)+\frac{ih\gamma(\alpha-m)}{2}$$
$$\cdot\left(e^{-2\gamma(\alpha-m)b}\left\{\frac{ih}{4}\left[M_-(\alpha-2m)-M_-(\alpha)\right]-\frac{M_-(\alpha-m)}{2\gamma(\alpha-m)}\right\}\right.$$
$$\left.+\frac{ih}{4}\left[N_-(\alpha-2m)-N_-(\alpha)\right]\right), \tag{23}$$

where

$$F_j(\alpha)=\int_{-\infty}^{0}f_j(z)e^{i\alpha z}dz, \qquad j=1,2 \tag{24}$$

with

$$f_{1,2}(z)=-\left[\phi^i(\pm b,z)+h\sin mz\frac{\partial}{\partial x}\phi^i(\pm b,z)\right]. \tag{25}$$

Using the boundary conditions and carrying out some manipulations, we derive that

$$\begin{aligned}S_+(\alpha)+G_1(\alpha) &= -K(\alpha)U_-(\alpha)\\ &\quad+\frac{ih}{4}\left\{V_-(\alpha-m)\left[e^{-2\gamma(\alpha)b}-e^{-2\gamma(\alpha-m)b}\right]\right.\\ &\quad\left.+V_-(\alpha+m)\left[e^{-2\gamma(\alpha+m)b}-e^{-2\gamma(\alpha)b}\right]\right\},\end{aligned} \tag{26}$$

$$\begin{aligned}D_+(\alpha)+G_2(\alpha) &= -L(\alpha)V_-(\alpha)\\ &\quad+\frac{ih}{4}\left\{U_-(\alpha+m)\left[e^{-2\gamma(\alpha)b}-e^{-2\gamma(\alpha+m)b}\right]\right.\\ &\quad\left.+U_-(\alpha-m)\left[e^{-2\gamma(\alpha-m)b}-e^{-2\gamma(\alpha)b}\right]\right\},\end{aligned} \tag{27}$$

where

$$S_+(\alpha) = P_+(\alpha)+Q_+(\alpha), \tag{28}$$
$$D_+(\alpha) = P_+(\alpha)-Q_+(\alpha), \tag{29}$$
$$U_-(\alpha) = M_-(\alpha)+N_-(\alpha), \tag{30}$$
$$V_-(\alpha) = M_-(\alpha)-N_-(\alpha), \tag{31}$$

$$\begin{aligned}G_1(\alpha)&=F_1(\alpha)+F_2(\alpha)\\ &=2i\cos(kb\sin\theta_0)\left[\frac{1}{\alpha-k\cos\theta_0}-\frac{kh\sin\theta_0}{m^2-(\alpha-k\cos\theta_0)^2}\right],\end{aligned} \tag{32}$$

$$\begin{aligned}G_2(\alpha)&=F_1(\alpha)-F_2(\alpha)\\ &=2\sin(kb\sin\theta_0)\left[\frac{1}{\alpha-k\cos\theta_0}-\frac{kh\sin\theta_0}{m^2-(\alpha-k\cos\theta_0)^2}\right],\end{aligned} \tag{33}$$

$$K(\alpha) = e^{-\gamma(\alpha)b}\frac{\cosh[\gamma(\alpha)b]}{\gamma(\alpha)}, \tag{34}$$

$$L(\alpha) = e^{-\gamma(\alpha)b}\frac{\sinh[\gamma(\alpha)b]}{\gamma(\alpha)}. \tag{35}$$

Equations (26) and (27) are the simultaneous Wiener-Hopf equations satisfied by $S_+(\alpha)$, $D_+(\alpha)$, $U_-(\alpha)$, and $V_-(\alpha)$, which hold for any α in the strip $-k_2 < \tau < k_2 \cos\theta_0$. In the above, $K(\alpha)$ and $L(\alpha)$ defined by (34) and (35) are kernel functions. It is noted that, by taking the limit $h \to 0$, (26) and (27) reduce to the Wiener-Hopf equations arising in the problem of diffraction by a flat, semi-infinite parallel-plate waveguide [13–16].

3. PERTURBATION SERIES SOLUTIONS OF THE WIENER-HOPF EQUATIONS

In order to solve the Wiener-Hopf equations (26) and (27), we express the unknown functions $S_+(\alpha)$, $D_+(\alpha)$, $U_-(\alpha)$, and $V_-(\alpha)$ in terms of a perturbation series expansion in h as

$$S_+(\alpha) = S_+^0(\alpha) + hS_+^1(\alpha) + O(h^2), \tag{36}$$

$$D_+(\alpha) = D_+^0(\alpha) + hD_+^1(\alpha) + O(h^2), \tag{37}$$

$$U_-(\alpha) = U_-^0(\alpha) + hU_-^1(\alpha) + O(h^2), \tag{38}$$

$$V_-(\alpha) = V_-^0(\alpha) + hV_-^1(\alpha) + O(h^2). \tag{39}$$

We can also express the known functions $G_1(\alpha)$ and $G_2(\alpha)$ defined by (32) and (33) in the form of a perturbation series in h as in

$$G_1(\alpha) = G_1^0(\alpha) + hG_1^1(\alpha) + O(h^2), \tag{40}$$

$$G_2(\alpha) = G_2^0(\alpha) + hG_2^1(\alpha) + O(h^2). \tag{41}$$

Substituting (36)–(41) into (26) and (27), the original Wiener-Hopf equations can be separated into the zero-order equations

$$S_+^0(\alpha) = -K(\alpha)U_-^0(\alpha) - \frac{2i\cos(kb\sin\theta_0)}{\alpha - k\cos\theta_0}, \tag{42}$$

$$D_+^0(\alpha) = -L(\alpha)V_-^0(\alpha) - \frac{2\sin(kb\sin\theta_0)}{\alpha - k\cos\theta_0} \tag{43}$$

and the first-order equations

$$\begin{aligned}
S_+^1(\alpha) \;=\; & -K(\alpha)U_-^1(\alpha) + \frac{i}{4}\left[e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha-m)b}\right]V_-^0(\alpha-m) \\
& +\frac{i}{4}\left[e^{-2\gamma(\alpha+m)b} - e^{-2\gamma(\alpha)b}\right]V_-^0(\alpha+m) \\
& +\frac{2ik\sin\theta_0\cos(kb\sin\theta_0)}{m^2-(\alpha-k\cos\theta_0)^2}, \qquad (44)
\end{aligned}$$

$$\begin{aligned}
D_+^1(\alpha) \;=\; & -L(\alpha)V_-^1(\alpha) + \frac{i}{4}\left[e^{-2\gamma(\alpha-m)b} - e^{-2\gamma(\alpha)b}\right]U_-^0(\alpha-m) \\
& +\frac{i}{4}\left[e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha+m)b}\right]U_-^0(\alpha+m) \\
& +\frac{2k\sin\theta_0\sin(kb\sin\theta_0)}{m^2-(\alpha-k\cos\theta_0)^2}. \qquad (45)
\end{aligned}$$

The kernel functions $K(\alpha)$ and $L(\alpha)$ defined by (34) and (35) are factorized as [13–16]

$$K(\alpha) \;=\; K_+(\alpha)K_-(\alpha) = K_+(\alpha)K_+(-\alpha), \qquad (46)$$

$$L(\alpha) \;=\; L_+(\alpha)L_-(\alpha) = L_+(\alpha)L_+(-\alpha), \qquad (47)$$

where $K_\pm(\alpha)$ and $L_\pm(\alpha)$ are the split functions given by

$$\begin{aligned}
K_+(\alpha) = & (\cos kb)^{1/2}e^{i\pi/4}(k+\alpha)^{-1/2}\exp\left[\frac{i\gamma(\alpha)b}{\pi}\ln\frac{\alpha-\gamma(\alpha)}{k}\right] \\
& \cdot\exp\left[\frac{i\alpha b}{\pi}\left(1-C+\ln\frac{\pi}{2kb}+i\frac{\pi}{2}\right)\right]\prod_{n=1,\mathrm{odd}}^{\infty}\left(1+\frac{\alpha}{i\gamma_n}\right)e^{2i\alpha b/n\pi}, \qquad (48)
\end{aligned}$$

$$\begin{aligned}
L_+(\alpha) = & \left(\frac{\sin kb}{k}\right)^{1/2}\exp\left[\frac{i\gamma(\alpha)b}{\pi}\ln\frac{\alpha-\gamma(\alpha)}{k}\right] \\
& \cdot\exp\left[\frac{i\alpha b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right]\prod_{n=2,\mathrm{even}}^{\infty}\left(1+\frac{\alpha}{i\gamma_n}\right)e^{2i\alpha b/n\pi} \qquad (49)
\end{aligned}$$

with $C\,(=\,0.57721566\cdots)$ being Euler's constant and

$$\gamma_n = \left[(n\pi/2b)^2 - k^2\right]^{1/2}. \qquad (50)$$

It is seem from (48) and (49) that $K_\pm(\alpha)$ and $L_\pm(\alpha)$ are regular and nonzero in $\tau \gtrless \mp k_2$. We can also verify that

$$K_\pm(\alpha) \sim (\mp 2i\alpha)^{-1/2}, \quad L_\pm(\alpha) \sim (\mp 2i\alpha)^{-1/2} \qquad (51)$$

as $\alpha \to \infty$ with $\tau \gtrless \mp k_2$.

First let us consider the solution of the zero-order Wiener-Hopf equations (42) and (43). These are the Wiener-Hopf equations arising in the classical diffraction problem for a flat, semi-infinite parallel-plate waveguide, and have the following exact solution [13–16]:

$$U_-^0(\alpha) = \frac{-2i\cos(kb\sin\theta_0)}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)K_-(\alpha)}, \tag{52}$$

$$V_-^0(\alpha) = -\frac{2\sin(kb\sin\theta_0)}{L_+(k\cos\theta_0)(\alpha - k\cos\theta_0)L_-(\alpha)}. \tag{53}$$

Next we shall solve the first order Wiener-Hopf equations as given by (44) and (45). Multiplying both sides of (44) and (45) by $1/K_+(\alpha)$ and $1/L_+(\alpha)$, respectively and rearranging the results, it follows that

$$\begin{aligned} &\frac{S_+^1(\alpha)}{K_+(\alpha)} - \frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{mK_+(\alpha)}\left[\frac{1}{(\alpha - k\cos\theta_1)} - \frac{1}{(\alpha - k\cos\theta_2)}\right] \\ &= -K_-(\alpha)U_-^1(\alpha) + \frac{i}{4}\frac{V_-^0(\alpha+m)\left[e^{-2\gamma(\alpha+m)b} - e^{-2\gamma(\alpha)b}\right]}{K_+(\alpha)} \\ &\quad + \frac{i}{4}\frac{V_-^0(\alpha-m)\left[e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha-m)b}\right]}{K_+(\alpha)}, \end{aligned} \tag{54}$$

$$\begin{aligned} &\frac{D_+^1(\alpha)}{L_+(\alpha)} - \frac{k\sin\theta_0\sin(kb\sin\theta_0)}{mL_+(\alpha)}\left[\frac{1}{(\alpha - k\cos\theta_1)} - \frac{1}{(\alpha - k\cos\theta_2)}\right] \\ &= -L_-(\alpha)V_-^1(\alpha) + \frac{i}{4}\frac{U_-^0(\alpha+m)\left[e^{-2\gamma(\alpha)b} - e^{-2\gamma(\alpha+m)b}\right]}{L_+(\alpha)} \\ &\quad + \frac{i}{4}\frac{U_-^0(\alpha-m)\left[e^{-2\gamma(\alpha-m)b} - e^{-2\gamma(\alpha)b}\right]}{L_+(\alpha)}. \end{aligned} \tag{55}$$

Applying the decomposition theorem [13, 15–17] to (54) and (55), we obtain that

$$\begin{aligned} &\frac{S_+^1(\alpha)}{K_+(\alpha)} - \frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{m(\alpha - k\cos\theta_1)}\left[\frac{1}{K_+(\alpha)} - \frac{1}{K_+(k\cos\theta_1)}\right] \\ &+ \frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{m(\alpha - k\cos\theta_2)}\left[\frac{1}{K_+(\alpha)} - \frac{1}{K_+(k\cos\theta_2)}\right] \\ &+ \frac{1}{8\pi}\int_C \frac{V_-^0(u+m)\left[e^{-2\gamma(u)b} - e^{-2\gamma(u+m)b}\right]}{K_+(\alpha)(u-\alpha)}du \\ &+ \frac{1}{8\pi}\int_C \frac{V_-^0(u-m)\left[e^{-2\gamma(u-m)b} - e^{-2\gamma(u)b}\right]}{K_+(\alpha)(u-\alpha)}du \end{aligned}$$

$$= -K_-(\alpha)U_-^1(\alpha) + \frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{m(\alpha - k\cos\theta_1)K_+(k\cos\theta_1)}$$
$$-\frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{m(\alpha - k\cos\theta_2)K_+(k\cos\theta_2)} + H_1(\alpha) + H_2(\alpha), \qquad (56)$$

$$\frac{D_+^1(\alpha)}{L_+(\alpha)} - \frac{k\sin\theta_0\sin(kb\sin\theta_0)}{m(\alpha - k\cos\theta_1)}\left[\frac{1}{L_+(\alpha)} - \frac{1}{L_+(k\cos\theta_1)}\right]$$
$$+\frac{k\sin\theta_0\sin(kb\sin\theta_0)}{m(\alpha - k\cos\theta_2)}\left[\frac{1}{L_+(\alpha)} - \frac{1}{L_+(k\cos\theta_2)}\right]$$
$$-\frac{1}{8\pi}\int_C \frac{U_-^0(u+m)\left[e^{-2\gamma(u)b} - e^{-2\gamma(u+m)b}\right]}{L_+(\alpha)(u-\alpha)}du$$
$$-\frac{1}{8\pi}\int_C \frac{U_-^0(u-m)\left[e^{-2\gamma(u-m)b} - e^{-2\gamma(u)b}\right]}{L_+(\alpha)(u-\alpha)}du$$
$$= -L_-(\alpha)V_-^1(\alpha) + \frac{k\sin\theta_0\sin(kb\sin\theta_0)}{m(\alpha - k\cos\theta_1)L_+(k\cos\theta_1)}$$
$$-\frac{k\sin\theta_0\sin(kb\sin\theta_0)}{m(\alpha - k\cos\theta_2)L_+(k\cos\theta_2)} - J_1(\alpha) - J_2(\alpha), \qquad (57)$$

where

$$H_{1,2}(\alpha) = \pm\frac{1}{8\pi}\int_D \frac{V_-^0(u\pm m)[e^{-2\gamma(u)b} - e^{-2\gamma(u\pm m)b}]}{K_+(u)(u-\alpha)}du, \qquad (58)$$
$$J_{1,2}(\alpha) = \pm\frac{1}{8\pi}\int_D \frac{U_-^0(u\pm m)[e^{-2\gamma(u)b} - e^{-2\gamma(u\pm m)b}]}{L_+(u)(u-\alpha)}du. \qquad (59)$$

In (56)–(59), C and D are infinite integration paths running parallel to the real axis in the u-plane, as shown in Fig. 2, where c and d are some constants such that $-k_2 < c < \tau < d < k_2\cos\theta_0$ with $\tau = \mathrm{Im}\,\alpha$.

It is seen that the left-hand and right-hand sides of (56) are regular in the upper ($\tau > -k_2$) and lower ($\tau < k_2\cos\theta_0$) half-planes, respectively, and both sides have a common strip of regularity $-k_2 < \tau < k_2\cos\theta_0$. Hence by analytic continuation, we can show that both sides of (56) must be equal to an entire function, which we denote by $P(\alpha)$. Taking into account the edge condition, we deduce that

$$S_+^1(\alpha) = o(\alpha^{-1/2}), \quad \tau > -k_2, \qquad (60)$$
$$U_-^1(\alpha) = o(\alpha^{1/2}), \quad \tau < k_2\cos\theta_0 \qquad (61)$$

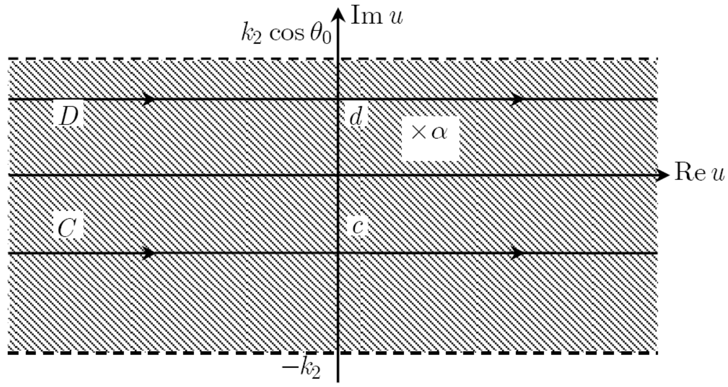

Figure 2. Integration paths C and $D(-k_2 < c < \tau < d < k_2 \cos\theta_0)$.

as $\alpha \to \infty$. We also see from (51) that the split functions $K_\pm(\alpha)$ are $O(\alpha^{-1/2})$ as $\alpha \to \infty$ with $\tau \gtrless \mp k_2$. These considerations show that

$$P(\alpha) = o(1), \quad \alpha \to \infty. \tag{62}$$

Thus, we can conclude with the aid of Liouville's theorem that $P(\alpha)$ must be identically zero. The same argument can also be applied to (57) and its both sides are identically equal to zero. Therefore equating the right-hand sides of (56) and (57) to zero, it follows that

$$\begin{aligned} U_-^1(\alpha) &= \frac{1}{K_-(\alpha)}\left[H_1(\alpha) + H_2(\alpha)\right] \\ &\quad + \frac{A_1}{K_-(\alpha)\,(\alpha - k\cos\theta_1)} + \frac{A_2}{K_-(\alpha)\,(\alpha - k\cos\theta_2)}, \end{aligned} \tag{63}$$

$$\begin{aligned} V_-^1(\alpha) &= -\frac{1}{L_-(\alpha)}\left[J_1(\alpha) + J_2(\alpha)\right] \\ &\quad + \frac{B_1}{L_-(\alpha)(\alpha - k\cos\theta_1)} - \frac{B_2}{L_-(\alpha)(\alpha - k\cos\theta_2)}, \end{aligned} \tag{64}$$

where

$$\cos\theta_{1,2} = \cos\theta_0 \pm m/k, \tag{65}$$

$$A_{1,2} = \frac{ik\sin\theta_0\cos(kb\sin\theta_0)}{mK_+(k\cos\theta_{1,2})}, \tag{66}$$

$$B_{1,2} = \frac{k\sin\theta_0\sin(kb\sin\theta_0)}{mL_+(k\cos\theta_{1,2})}. \tag{67}$$

Equations (63) and (64) contain the infinite integrals $H_{1,2}(\alpha)$ and $J_{1,2}(\alpha)$ defined by (58) and (59), respectively. These integrals can be evaluated in closed form by using the residue theorem. Taking into account (52) and (53) and carrying out some manipulations, we arrive at

$$\begin{aligned} H_{1,2}(\alpha) \;=\; & \mp\frac{B_0}{4}\left(\frac{k}{\sin kb}\right)^{1/2}\sum_{q=2,\text{even}}^{\infty} i\gamma_q \prod_{\substack{n=2,\text{even}\\ n\neq q}}^{\infty}\frac{\gamma_n\exp(-2\gamma_q b/n\pi)}{\gamma_n-\gamma_q} \\ & \cdot\exp\left[-\frac{2\gamma_q b}{q\pi}-\frac{\gamma_q b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right] \\ & \cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_q-\gamma(\alpha)}{k}\right] \\ & \cdot\left[\frac{e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}}{(u\pm k\cos\theta_{1,2})K_\mp(u)(u\pm\alpha)}\right]_{u=\mp i\gamma_q+m}, \end{aligned} \tag{68}$$

$$\begin{aligned} J_{1,2}(\alpha) \;=\; & \frac{\mp iC_0}{4\exp(i\pi/4)(\cos kb)^{1/2}} \\ & \cdot\sum_{p=1,\text{odd}}^{\infty} i\gamma_p\,(k-i\gamma_p)^{1/2}\prod_{\substack{n=1,\text{odd}\\ n\neq p}}^{\infty}\frac{\gamma_n\exp(-2\gamma_p b/n\pi)}{\gamma_n-\gamma_p} \\ & \cdot\exp\left[-\frac{2\gamma_p b}{p\pi}-\frac{\gamma_p b}{\pi}\left(1-C+\ln\frac{\pi}{2kb}+i\frac{\pi}{2}\right)\right] \\ & \cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_p-\gamma(\alpha)}{k}\right] \\ & \cdot\left[\frac{e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}}{(u\pm k\cos\theta_{1,2})L_\mp(u)(u\pm\alpha)}\right]_{u=\mp i\gamma_p+m}. \end{aligned} \tag{69}$$

where

$$B_0 \;=\; -\frac{2\sin(kb\sin\theta_0)}{L_+(k\cos\theta_0)}, \tag{70}$$

$$C_0 \;=\; -\frac{2i\cos(kb\sin\theta_0)}{K_+(k\cos\theta_0)}. \tag{71}$$

Substituting (68) and (69) into (63) and (64), respectively, we obtain that

$$U_-^1(\alpha) = \frac{B_0}{4K_-(\alpha)}\left(\frac{k}{\sin kb}\right)^{1/2}\left\{\sum_{q=2,\text{even}}^{\infty} i\gamma_q \prod_{\substack{n=2,\text{even}\\ n\neq q}}^{\infty}\frac{\gamma_n\exp(-2\gamma_q b/n\pi)}{\gamma_n-\gamma_q}\right.$$
$$\cdot\exp\left[-\frac{2\gamma_q b}{q\pi}-\frac{\gamma_q b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right]$$
$$\cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_q-\gamma(\alpha)}{k}\right]$$
$$\cdot\left[\frac{e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}}{(u+k\cos\theta_1)K_-(u)(u+\alpha)}\right]_{u=-i\gamma_q+m}$$
$$-\sum_{q=2,\text{even}}^{\infty} i\gamma_q \prod_{\substack{n=2,\text{even}\\ n\neq q}}^{\infty}\frac{\gamma_n\exp(-2\gamma_q b/n\pi)}{\gamma_n-\gamma_q}$$
$$\cdot\exp\left[-\frac{2\gamma_q b}{q\pi}-\frac{\gamma_q b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right]$$
$$\cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_q-\gamma(\alpha)}{k}\right]$$
$$\left.\cdot\left[\frac{e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}}{(u-k\cos\theta_2)K_+(u)(u-\alpha)}\right]_{u=i\gamma_q+m}\right\}$$
$$+\frac{A_1}{K_-(\alpha)\,(\alpha-k\cos\theta_1)}+\frac{A_2}{K_-(\alpha)\,(\alpha-k\cos\theta_2)}, \tag{72}$$

$$V_-^1(\alpha) = \frac{iC_0}{4\exp(i\pi/4)(\cos kb)^{1/2}L_-(\alpha)}$$
$$\cdot\left\{\sum_{p=1,\text{odd}}^{\infty} i\gamma_p\,(k-i\gamma_p)^{1/2}\prod_{\substack{n=1,\text{odd}\\ n\neq p}}^{\infty}\frac{\gamma_n\exp(-2\gamma_p b/n\pi)}{\gamma_n-\gamma_p}\right.$$
$$\cdot\exp\left[-\frac{2\gamma_p b}{p\pi}-\frac{\gamma_p b}{\pi}\left(1-C+\ln\frac{\pi}{2kb}+i\frac{\pi}{2}\right)\right]$$
$$\cdot\exp\left[\frac{ib\gamma(\alpha)}{\pi}\ln\frac{-i\gamma_p-\gamma(\alpha)}{k}\right]$$
$$\cdot\left[\frac{e^{-2\gamma(u)b}-e^{-2\gamma(u-m)b}}{(u+k\cos\theta_1)L_-(u)(u+\alpha)}\right]_{u=-i\gamma_p+m}$$

$$
\begin{aligned}
&- \sum_{p=1,\text{odd}}^{\infty} i\gamma_p \left(k - i\gamma_p\right)^{1/2} \prod_{\substack{n=1,\text{odd}\\ n\neq p}}^{\infty} \frac{\gamma_n \exp(-2\gamma_p b/n\pi)}{\gamma_n - \gamma_p} \\
&\cdot \exp\left[-\frac{2\gamma_p b}{p\pi} - \frac{\gamma_p b}{\pi}\left(1 - C + \ln\frac{\pi}{2kb} + i\frac{\pi}{2}\right)\right] \\
&\cdot \exp\left[\frac{ib\gamma(\alpha)}{\pi} \ln \frac{-i\gamma_p - \gamma(\alpha)}{k}\right] \\
&\cdot \left.\left[\frac{e^{-2\gamma(u)b} - e^{-2\gamma(u-m)b}}{(u - k\cos\theta_2)L_+(u)(u-\alpha)}\right]_{u=i\gamma_p+m}\right\} \\
&+ \frac{B_1}{L_-(\alpha)(\alpha - k\cos\theta_1)} - \frac{B_2}{L_-(\alpha)(\alpha - k\cos\theta_2)}.
\end{aligned}
\tag{73}
$$

Equations (72) and (73) provide exact solutions of the first order Wiener-Hopf equations (44) and (45), respectively.

4. SCATTERED FIELD

In this section, we shall derive a scattered field expression inside and outside the waveguide explicitly by using the results obtained in Section 3. The scattered field in the real space can be derived by taking the inverse Fourier transform according to the formula

$$
\phi(x,z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \Phi(x,\alpha) e^{-i\alpha z} d\alpha, \tag{74}
$$

where c is a constant satisfying $-k_2 < c < k_2 \cos\theta_0$.

First we consider the scattered field inside the waveguide. Substituting the field representation for $|x| < b$ in (13) into (74) and taking into account (19), (20), (30), (31), (38), and (39), the scattered field is expressed using the zero- and first-order terms as follows:

$$
\begin{aligned}
\phi(x,z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} &\left\{ -\frac{U_-^0(\alpha)\cosh\gamma(\alpha)x + V_-^0(\alpha)\sinh\gamma(\alpha)x}{2\gamma(\alpha)} \right. \\
&+ h\left[\frac{U_-^0(\alpha+m)\sinh\gamma(\alpha)x + V_-^0(\alpha+m)\cosh\gamma(\alpha)x}{4i}\right. \\
&- \frac{U_-^0(\alpha-m)\sinh\gamma(\alpha)x + V_-^0(\alpha-m)\cosh\gamma(\alpha)x}{4i} \\
&\left.\left. - \frac{U_-^1(\alpha)\cosh\gamma(\alpha)x + V_-^1(\alpha)\sinh\gamma(\alpha)x}{2\gamma(\alpha)}\right]\right\} e^{-\gamma(\alpha)b - i\alpha z} d\alpha.
\end{aligned}
\tag{75}
$$

Substituting (52), (53), (72), and (73) into (75) and evaluating the resultant integral, we derive, after some manipulations, that

$$\phi(x,z) = \phi^0(x,z) + h\phi^1(x,z), \tag{76}$$

where $\phi^0(x,z)$ and $\phi^1(x,z)$ are the zero- and first-order scattered fields, respectively, and take the form

$$\phi^0(x,z) = -\phi^i(x,z) + \sum_{n=1}^{\infty} T_n^0 \sin\frac{n\pi}{2b}(x+b)e^{\gamma_n z}, \tag{77}$$

$$\phi^1(x,z) = \sum_{n=1}^{\infty} T_n^1 \cos\frac{n\pi}{2b}\,(x+b)\,e^{\gamma_n z}, \tag{78}$$

where

$$\begin{aligned} T_n^0 &= -\frac{n\pi}{2b^2\gamma_n}U_-^0(i\gamma_n)L(i\gamma_n) \quad \text{for odd } n, \\ &= \frac{n\pi}{2b^2\gamma_n}V_-^0(i\gamma_n)N(i\gamma_n) \quad \text{for even } n, \end{aligned} \tag{79}$$

$$\begin{aligned} T_n^1 &= \frac{iC_0}{2}\left[\frac{\sin(kb\sin\theta_1)}{K_-(k\cos\theta_1)}K_n^1(i\gamma_n - m)\right. \\ &\quad -\frac{\sin(kb\sin\theta_2)}{K_-(k\cos\theta_2)}K_n^1(i\gamma_n + m) \\ &\quad \left.+\frac{n\pi}{2^{1/2}be^{i\pi/4}}K_n^1(i\gamma_n)\right] \quad \text{for odd } n, \\ &= \frac{B_0}{2}\left[\frac{\cos(kb\sin\theta_1)}{L_-(k\cos\theta_1)}L_n^1(i\gamma_n - m)\right. \\ &\quad +\frac{\cos(kb\sin\theta_2)}{L_-(k\cos\theta_2)}L_n^1(i\gamma_n + m) \\ &\quad \left.-\frac{n\pi}{2^{1/2}b}L_n^1(i\gamma_n)\right] \quad \text{for even } n \end{aligned} \tag{80}$$

with

$$\begin{aligned} K_n^1(i\gamma_n \pm m) &= \frac{\exp\left[\pm mi - \gamma_n - ikb\sin(i\gamma_n \pm m)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(\pi/2kb) + i\pi/2\right]\right\}} \\ &\quad \cdot\frac{\sin\left[kb\sin(i\gamma_n \pm m)\right]}{\exp\left(\left[ib\gamma(i\gamma_n \pm m)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n \pm m)\right]/k\right\}\right)} \\ &\quad \cdot\frac{k - i\gamma_n \pm m}{\prod\limits_{\substack{p=1,\text{odd} \\ p\neq n}}^{\infty}\left[(\gamma_p - \gamma_n)/\gamma_p\right]\exp(2\gamma_p b/p\pi)}, \end{aligned} \tag{81}$$

$$L_n^1(i\gamma_n \pm m) = \left(\frac{k}{\sin kb}\right)^{1/2} \frac{\exp\left[\pm mi - \gamma_n - ikb\sin(i\gamma_n \pm m)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(2\pi/kb) + i\pi/2\right]\right\}} \cdot \frac{\cos\left[kb\sin(i\gamma_n \pm m)\right]}{\exp\left(\left[ib\gamma(i\gamma_n \pm m)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n \pm m)\right]/k\right\}\right)} \cdot \frac{k - i\gamma_n \pm m}{\prod\limits_{\substack{q=2,\text{even}\\ q\neq n}}^{\infty}\left[\left(\gamma_q - \gamma_n\right)/\gamma_q\right]\exp(2\gamma_q b/q\pi)}, \tag{82}$$

$$K_n^1(i\gamma_n) = \frac{\exp\left[-\gamma_n - ikb\sin(i\gamma_n)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(\pi/2kb) + i\pi/2\right]\right\}} \cdot \frac{\sin\left[kb\sin(i\gamma_n \pm m)\right]}{\exp\left(\left[ib\gamma(i\gamma_n)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n)\right]/k\right\}\right)} \cdot \frac{k - i\gamma_n}{\prod\limits_{\substack{p=1,\text{odd}\\ p\neq n}}^{\infty}\left[(\gamma_p - \gamma_n)/\gamma_p\right]\exp(2\gamma_p b/p\pi)}, \tag{83}$$

$$L_n^1(i\gamma_n) = \left(\frac{k}{\sin kb}\right)^{1/2} \frac{\exp\left[-\gamma_n - ikb\sin(i\gamma_n)\right]}{\exp\left\{(\gamma_n b/\pi)\left[1 - C + \ln(2\pi/kb) + i\pi/2\right]\right\}} \cdot \frac{\cos\left[kb\sin(i\gamma_n)\right]}{\exp\left(\left[ib\gamma(i\gamma_n)/\pi\right]\ln\left\{\left[-i\gamma_n - \gamma(i\gamma_n)\right]/k\right\}\right)} \cdot \frac{k - i\gamma_n}{\prod\limits_{\substack{q=2,\text{even}\\ q\neq n}}^{\infty}\left[\left(\gamma_q - \gamma_n\right)/\gamma_q\right]\exp(2\gamma_q b/q\pi)}. \tag{84}$$

Equations (77) and (78) are explicit expressions of the zero- and first-order scattered fields inside the waveguide, respectively. It is seen that by letting $h \to 0$, (76) is reduced to the scattered field expression for the diffraction problem involving a flat, semi-infinite parallel-plate waveguide [13–16].

Next we shall consider the field outside the waveguide and derive the scattered far field. The region outside the waveguide actually includes $z > 0$ with $|x| < b$, but contributions from this region are negligibly small at large distances from the origin. Therefore, only the scattered far field for $|x| > b$ will be discussed in the following. Substituting (18) and (21) into (13) and using (30) and (31) together with (38) and (39), an integral representation of the scattered field for

$x \gtrless \pm b$ is given by

$$\phi(x,z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{\infty+ic} \left\{ -\frac{U_-^0(\alpha)\cosh\gamma(\alpha)b \pm V_-^0(\alpha)\sinh\gamma(\alpha)b}{2\gamma(\alpha)} \right.$$
$$+h\left[\mp\frac{U_-^0(\alpha+m)\cosh\gamma(\alpha)b \mp V_-^0(\alpha+m)\sinh\gamma(\alpha)b}{4i}\right.$$
$$\pm\frac{U_-^0(\alpha-m)\cosh\gamma(\alpha)b \pm V_-^0(\alpha-m)\sinh\gamma(\alpha)b}{4i}$$
$$\left.\left.-\frac{U_-^1(\alpha)\cosh\gamma(\alpha)b \pm V_-^1(\alpha)\sinh\gamma(\alpha)b}{2\gamma(\alpha)}\right]\right\} e^{\mp\gamma(\alpha)x - i\alpha z} d\alpha, \quad (85)$$

where $-k_2 < c < k_2 \cos\theta_0$. Let us introduce the cylindrical coordinates $(\rho_\pm, \theta_\pm)$ centered at the waveguide edges $(x,z) = (\pm b, 0)$ as follows:

$$x - b = \rho_+ \sin\theta_+, \; z = \rho_+ \cos\theta_+ \;\; \text{for} \;\; 0 < \theta_+ < \pi, \quad (86)$$
$$x + b = \rho_- \sin\theta_-, \; z = \rho_- \cos\theta_- \;\; \text{for} \;\; -\pi < \theta_- < 0. \quad (87)$$

Applying the saddle point method together with the use of the formulas presented in Appendix B, we can derive a far field asymptotic expression of the scattered field for $x \gtrless \pm b$. Omitting the details, we obtain the results presented below.

(i) Scattered far field for $x > b$

The scattered far field for region $x > b$ (i.e., $0 < \theta_+ < \pi$) is derived as

$$\phi(\rho_+, \theta_+) \sim \phi_r^+(\rho_+, \theta_+) + \phi_d^+(\rho_+, \theta_+) \quad (88)$$

for $k\rho_+ \to \infty$, where

$$\phi_r^+(\rho_+, \theta_+) = \sum_{j=0}^{2} \phi_{rj}(\theta_0, \theta_j) \cdot \left\{ e^{-ik\rho_+ \cos(\theta_+ - \theta_j)} F\left[(2k\rho_+)^{1/2} \cos\frac{\theta_+ - \theta_j}{2}\right] + e^{-ik\rho_+ \cos(\theta_+ + \theta_j)} F\left[(2k\rho_+)^{1/2} \cos\frac{\theta_+ + \theta_j}{2}\right] \right\}, \quad (89)$$

$$\phi_d^+(\rho_+, \theta_+) = \sum_{j=0}^{2} \phi_{dj}(\theta_+, \theta_0, \theta_j) \frac{e^{i(k\rho_+ - \pi/4)}}{(k\rho_+)^{1/2}} \quad (90)$$

with

$$\phi_{r0}(\theta_0, \theta_0) = -e^{-ikb\sin\theta_0}, \quad (91)$$

$$\begin{aligned}\phi_{r1}(\theta_0,\theta_1) &= \frac{hk\sin\theta_0}{2}\left[e^{-ikb\sin\theta_0}-e^{ikb(\sin\theta_0-2\sin\theta_1)}\right]\\ &\quad+\frac{h}{8}\left(e^{-2ikb\sin\theta_1}-e^{-2ikb\sin\theta_0}\right)\\ &\quad\cdot\left[\frac{C_0}{K_-(k\cos\theta_0)}-\frac{B_0}{L_-(k\cos\theta_0)}\right],\end{aligned} \tag{92}$$

$$\begin{aligned}\phi_{r2}(\theta_0,\theta_2) &= \frac{hk\sin\theta_0}{2}e^{ikb\sin\theta_0}\left(1+e^{-2ikb\sin\theta_2}\right)\\ &\quad+\frac{h}{8}\left(e^{-2ikb\sin\theta_0}-e^{-2ikb\sin\theta_2}\right)\\ &\quad\cdot\left[\frac{C_0}{K_-(k\cos\theta_0)}-\frac{B_0}{L_-(k\cos\theta_0)}\right],\end{aligned} \tag{93}$$

$$\begin{aligned}\phi_{d1,2}(\theta_+,\theta_0,\theta_{1,2}) &= \pm\frac{ih\sin\theta_+}{2^{3/2}\pi^{1/2}(\cos\theta_++\cos\theta_{1,2})}\\ &\quad\cdot\left[U_-^0(\pm m-k\cos\theta_+)\cos(kb\sin\theta_+)\right.\\ &\quad\left.+V_-^0(\pm m-k\cos\theta_+)\sin(kb\sin\theta_+)\right]\\ &\quad\mp\frac{ih}{2^{3/2}\pi^{1/2}k(\cos\theta_++\cos\theta_{1,2})}\\ &\quad\cdot\left[\frac{A_{1,2}\cos(kb\sin\theta_+)}{K_+(k\cos\theta_+)}+\frac{B_{1,2}\sin(kb\sin\theta_+)}{L_+(k\cos\theta_+)}\right],\end{aligned} \tag{94}$$

$$\begin{aligned}\phi_{d0}(\theta_+,\theta_0,\theta_0) &= \frac{1}{2k(\cos\theta_++\cos\theta_0)}\\ &\quad\cdot\left[U_-^0(-k\cos\theta_+)\cos(kb\sin\theta_+)\right.\\ &\quad\left.-V_-^0(-k\cos\theta_+)\sin(kb\sin\theta_+)\right]\\ &\quad+\frac{iC_0h}{8}\sum_{n=1,\mathrm{odd}}^{\infty}\left\{\frac{L_-(k\cos\theta_+)\left[e^{-2\gamma(i\gamma_n)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{p_n^-(\theta_+)(m-k\cos\theta_+-i\gamma_n)}\right.\\ &\quad\left.+\frac{L_-(k\cos\theta_+)\left[e^{-2\gamma(i\gamma_n+m)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{p_n^+(\theta_+)(-m-k\cos\theta_+-i\gamma_n)}\right\}\\ &\quad+\frac{iB_0h}{8}\sum_{n=2,\mathrm{even}}^{\infty}\left\{\frac{K_-(k\cos\theta_+)\left[e^{-2\gamma(i\gamma_n)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{q_n^-(\theta_+)(m-k\cos\theta_+-i\gamma_n)}\right.\\ &\quad\left.+\frac{K_-(k\cos\theta_+)\left[e^{-2\gamma(i\gamma_n+m)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{q_n^+(\theta_+)(-m-k\cos\theta_+-i\gamma_n)}\right\},\end{aligned} \tag{95}$$

$$p_n^{\pm}(\Theta) = \frac{L_-(k\cos\Theta)}{(i\gamma_n - k\cos\theta_0)\, L_+(i\gamma_n \pm m)\,\mathrm{Res}\,[K_-(i\gamma_n)]}, \tag{96}$$

$$q_n^{\pm}(\Theta) = \frac{K_-(k\cos\Theta)}{(i\gamma_n - k\cos\theta_0)\, K_+(i\gamma_n \pm m)\,\mathrm{Res}\,[L_-(i\gamma_n)]}, \tag{97}$$

$$\mathrm{Res}\,[K_-(i\gamma_n)] = \frac{e^{i\pi/4}(\cos kb)^{1/2} e^{2\gamma_n b/n\pi}}{(k - i\gamma_n)^{1/2}\exp\left([ib\gamma(i\gamma_n)/\pi]\ln\{[-i\gamma_n - \gamma(i\gamma_n)]/k\}\right)} \cdot \frac{\exp\{(\gamma_n b/\pi)[1 - C + \ln(\pi/2kb) + i\pi/2]\}}{\prod\limits_{\substack{p=1,\mathrm{odd}\\ p\neq n}}^{\infty} \dfrac{\gamma_p \exp(-2\gamma_p b/p\pi)}{\gamma_p - \gamma_n}}, \tag{98}$$

$$\mathrm{Res}\,[L_-(i\gamma_n)] = \frac{(\sin kb/k)^{1/2}\exp(2\gamma_n b/n\pi)}{\exp\{(\gamma_n b/\pi)[1 - C + \ln(2\pi/kb) + i\pi/2]\}} \cdot \frac{\exp\left([ib\gamma(i\gamma_n)/\pi]\ln\{[-i\gamma_n - \gamma(i\gamma_n)]/k\}\right)}{\prod\limits_{\substack{q=2,\mathrm{even}\\ q\neq n}}^{\infty} \dfrac{\gamma_q \exp(-2\gamma_q b/q\pi)}{\gamma_q - \gamma_n}}, \tag{99}$$

In (89), $F(\cdot)$ is the Fresnel integral defined by (B4) in Appendix B. It is noted that (88) is a uniform asymptotic expression of the scattered far field for region $x > b$, which holds for any incidence and observation angles including the shadow boundaries.

(ii) Scattered far field for $x < -b$

The scattered far field for region $x < -b$ (i.e., $-\pi < \theta_- < 0$) is derived as

$$\phi(\rho_-, \theta_-) \sim \phi_r^-(\rho_-, \theta_-) + \phi_d^-(\rho_-, \theta_-) \tag{100}$$

for $k\rho_- \to \infty$, where

$$\begin{aligned}\phi_r^-(\rho_-, \theta_-) = {} & \phi_{r0}(\theta_0, \theta_0) \\ & \cdot \Big\{\exp[-ik\rho_- \cos(\theta_- - \theta_0)] F\Big[(2k\rho_-)^{1/2}\cos\frac{\theta_- - \theta_0}{2}\Big] \\ & + \exp[-ik\rho_- \cos(\theta_- + \theta_0)] F\Big[(2k\rho_-)^{1/2}\cos\frac{\theta_- + \theta_0}{2}\Big]\Big\},\end{aligned} \tag{101}$$

$$\phi_d^-(\rho_-, \theta_-) = \sum_{j=0}^{2} \phi_{dj}(\theta_-, \theta_0, \theta_j) \frac{e^{i(k\rho_- - \pi/4)}}{(k\rho_-)^{1/2}} \tag{102}$$

with

$$\phi_{r0}(\theta_0, \theta_0) = -e^{ikb\sin\theta_0}, \tag{103}$$

$$\begin{aligned}\phi_{d1,2}(\theta_-,\theta_0,\theta_{1,2}) = &\mp\frac{ih\sin\theta_-}{2^{5/2}\pi^{1/2}(\cos\theta_-+\cos\theta_{1,2})}\\ &\cdot\left[U_-^0(\pm m-k\cos\theta_-)\cos(kb\sin\theta_-)\right.\\ &\left.-V_-^0(\pm m-k\cos\theta_-)\sin(kb\sin\theta_-)\right]\\ &\pm\frac{ih}{2^{3/2}\pi^{1/2}k(\cos\theta_-+\cos\theta_{1,2})}\\ &\cdot\left[\frac{A_{1,2}\cos(kb\sin\theta_-)}{K_+(k\cos\theta_-)}-\frac{B_{1,2}\sin(kb\sin\theta_-)}{L_+(k\cos\theta_-)}\right],\end{aligned}\tag{104}$$

$$\begin{aligned}\phi_{d0}(\theta_-,\theta_0,\theta_0) = &-\frac{1}{2k(\cos\theta_-+\cos\theta_0)}\\ &\cdot\left[U_-^0(-k\cos\theta_-)\cos(kb\sin\theta_-)\right.\\ &\left.-V_-^0(-k\cos\theta_-)\sin(kb\sin\theta_-)\right]\\ &+\frac{iC_0h}{8}\sum_{n=1,\text{odd}}^{\infty}\left\{\frac{L_-(k\cos\theta_-)\left[e^{-2\gamma(i\gamma_n)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{p_n^-(\theta_-)(m-k\cos\theta_--i\gamma_n)}\right.\\ &\left.+\frac{L_-(k\cos\theta_-)\left[e^{-2\gamma(i\gamma_n+m)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{p_n^+(\theta_-)(-m-k\cos\theta_--i\gamma_n)}\right\}\\ &+\frac{iB_0h}{8}\sum_{n=2,\text{even}}^{\infty}\left\{\frac{K_-(k\cos\theta_-)\left[e^{-2\gamma(i\gamma_n)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{q_n^-(\theta_-)(m-k\cos\theta_--i\gamma_n)}\right.\\ &\left.+\frac{K_-(k\cos\theta_-)\left[e^{-2\gamma(i\gamma_n+m)b}-e^{-2\gamma(i\gamma_n-m)b}\right]}{q_n^+(\theta_-)(-m-k\cos\theta_--i\gamma_n)}\right\}.\end{aligned}\tag{105}$$

It is noted that (100) is a uniform asymptotic expression of the scattered far field for region $x < -b$ and holds for any incident and observation angles.

(iii) Physical interpretation of the results

As mentioned earlier, (88) and (100) are uniform asymptotic expressions of the scattered far field for $x > b$ and $x < -b$, respectively, which are valid for any incidence and observation angles including the shadow boundaries. In the following, we shall make physical interpretation of the results obtained in (i) and (ii) above.

In (88) and (100), $\phi_r^+(\rho_+,\theta_+)$ and $\phi_r^-(\rho_-,\theta_-)$ comprise contributions due to the geometrical optics fields and the singly diffracted fields. In particular, we can verify by using the asymptotic expansion of the Fresnel integral that $\phi_r^+(\rho_+,\theta_+)$ in (88) contain the reflected waves propagating along the directions at $\pi-\theta_0$, $\pi-\theta_1$, and

$\pi - \theta_2$ and the singly diffracted fields emanating from the waveguide edges at $x = \pm b$. It is important to note that $\pi - \theta_0$, $\pi - \theta_1$, and $\pi - \theta_2$ are, respectively, propagation directions of the (0), (−1), and (+1) order diffracted waves involved in the Floquet space harmonic modes arising in periodic structures of infinite extent. Similarly it can be shown that $\phi_r^-(\rho_-, \theta_-)$ in (100) contain the geometrical optics term canceling exactly the incident field and the singly diffracted fields from the waveguide edges at $x = \pm b$. In addition, $\phi_d^+(\rho_+, \theta_+)$ and $\phi_d^-(\rho_-, \theta_-)$ denote the multiply diffracted fields for regions $x > b$ and $x < -b$, respectively, which account for the higher-order diffraction between the edges of the two sinusoidal half-planes.

5. NUMERICAL RESULTS AND DISCUSSION

In this section, we shall present illustrative numerical examples of the scattered far field for various physical parameters and investigate the scattering characteristics of the waveguide in detail. For convenience, we introduce the cylindrical coordinate

$$x = \rho \sin\theta, \quad z = \rho \cos\theta \quad \text{for} -\pi < \theta < \pi \tag{106}$$

and define the scattered far field intensity as

$$|\phi(\rho, \theta)| \, [\text{dB}] = 20 \log_{10} \left[\frac{\lim\limits_{\rho \to \infty} \left| (k\rho)^{1/2} \phi(\rho, \theta) \right|}{\max\limits_{|\theta| < \pi} \lim\limits_{\rho \to \infty} \left| (k\rho)^{1/2} \phi(\rho, \theta) \right|} \right], \tag{107}$$

where

$$\begin{aligned} \phi(\rho, \theta) &= \phi_r^+(\rho_+, \theta_+) + \phi_d^+(\rho_+, \theta_+) \quad \text{for} \quad 0 < \theta < \pi, \\ &= \phi_r^-(\rho_-, \theta_-) + \phi_d^-(\rho_-, \theta_-) \quad \text{for} \quad -\pi < \theta < 0. \end{aligned} \tag{108}$$

Comparing (106) with (86) and (87), we see that, in the far field, $\rho_\pm$ and $\theta_\pm$ can be approximately replaced by ρ and θ, respectively for the amplitude terms involved in (108). By careful numerical experimentation, we have found that, when the corrugation depth $2h$ and the corrugation period $2\pi/m$ satisfy $kh \leq 1.0$ and $mh/kh \leq 0.3$, the approximate boundary condition given by (7) can be employed to simulate a perfectly conducting sinusoidal surface with sufficient accuracy.

Figures 3–6 show numerical examples of the scattered far field intensity as a function of observation angle θ for various values of kh, mh and kb, where the incidence angle θ_0 is fixed as 60°. In order

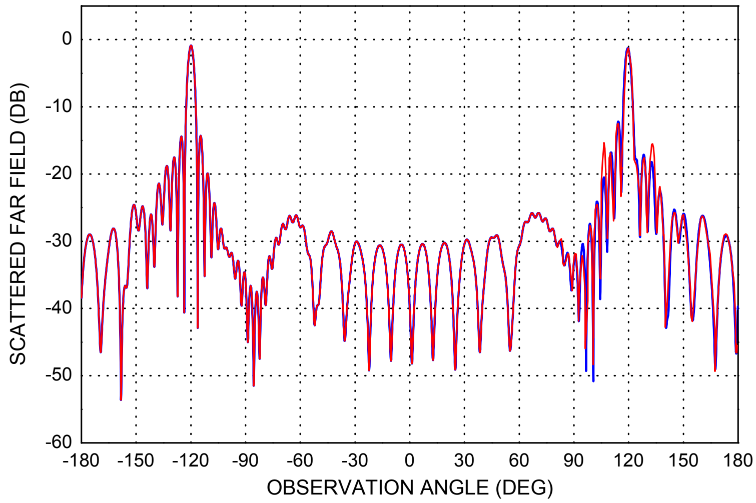

Figure 3(a). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 0.1$. ▬: corrugated waveguide. ▬: flat waveguide.

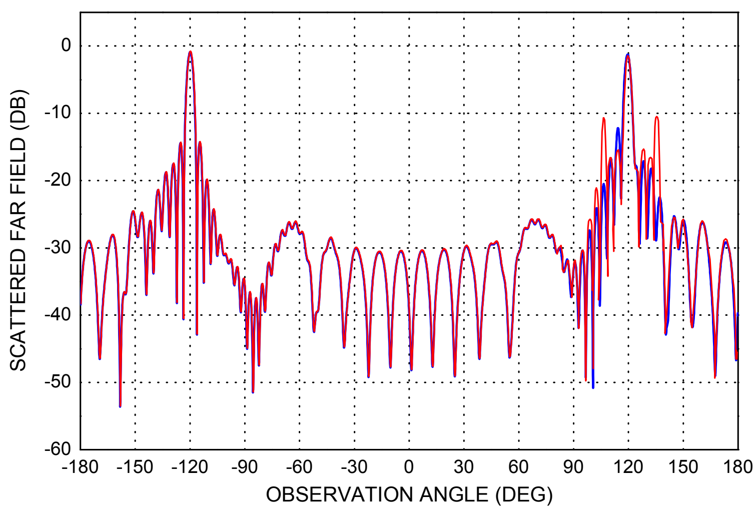

Figure 3(b). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 0.5$. ▬: corrugated waveguide. ▬: flat waveguide.

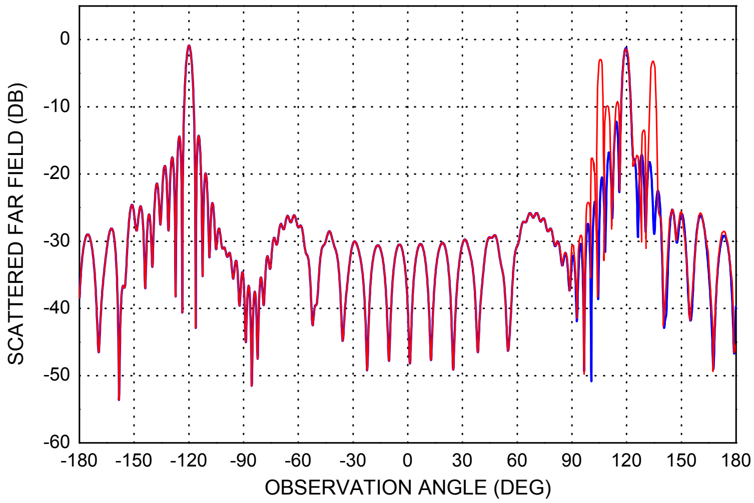

Figure 3(c). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 15.7$, $kh = 1.0$. ▬: corrugated waveguide. ▬: flat waveguide.

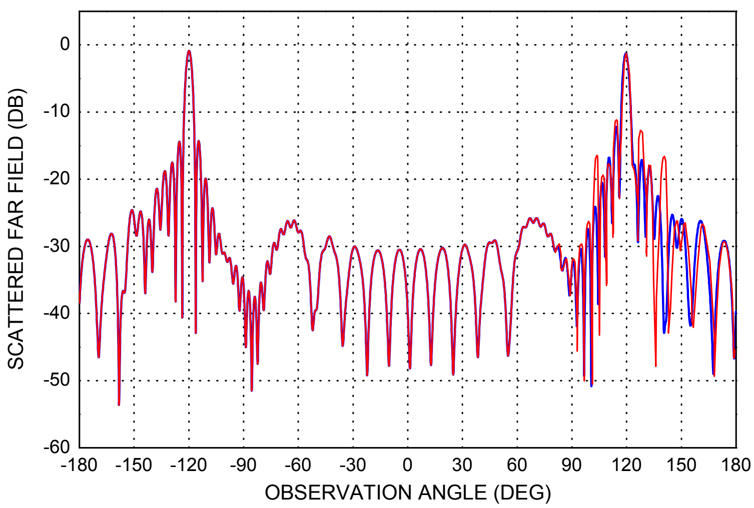

Figure 4(a). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 0.1$. ▬: corrugated waveguide. ▬: flat waveguide.

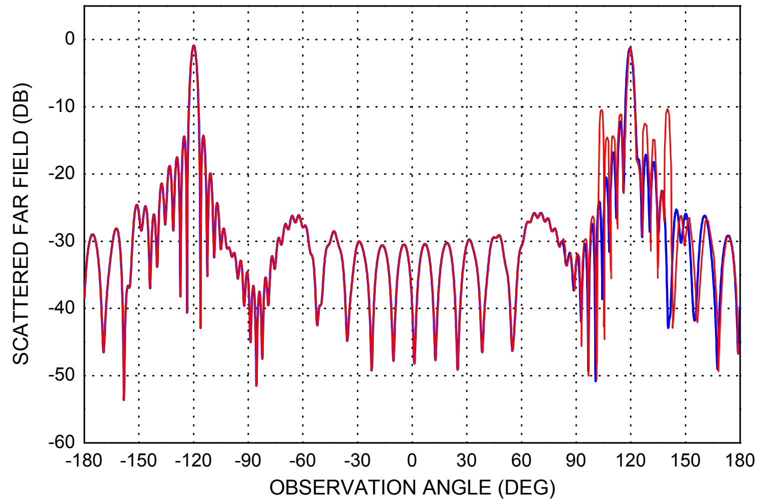

Figure 4(b). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 0.5$. ▬ : corrugated waveguide. ▬ : flat waveguide.

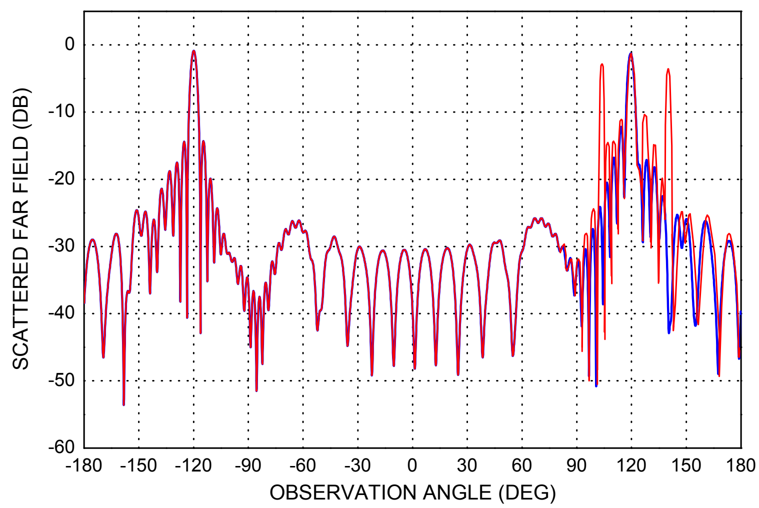

Figure 4(c). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 15.7$, $kh = 1.0$. ▬ : corrugated waveguide. ▬ : flat waveguide.

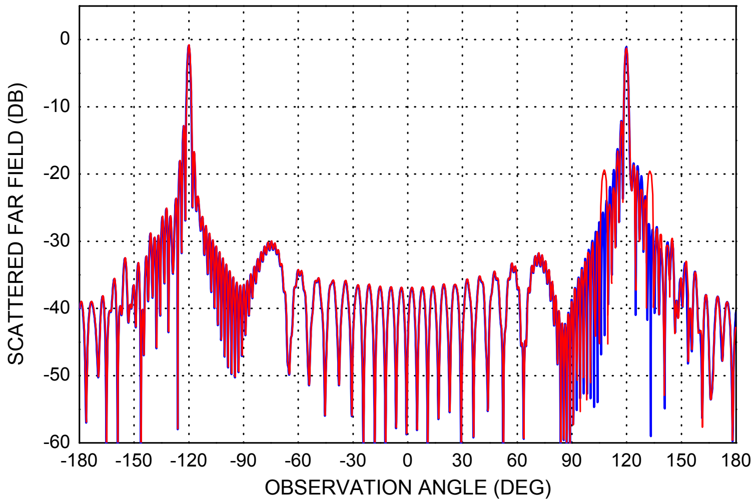

Figure 5(a). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 0.1$. ▬: corrugated waveguide. ▬: flat waveguide.

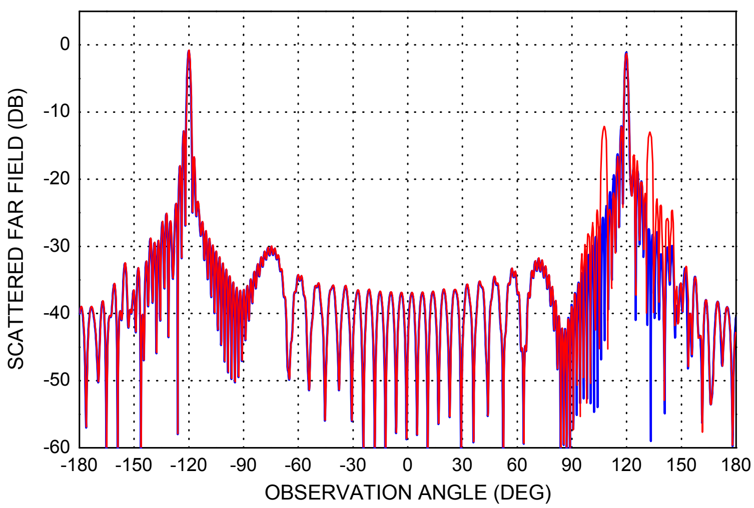

Figure 5(b). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 0.5$. ▬: corrugated waveguide. ▬: flat waveguide.

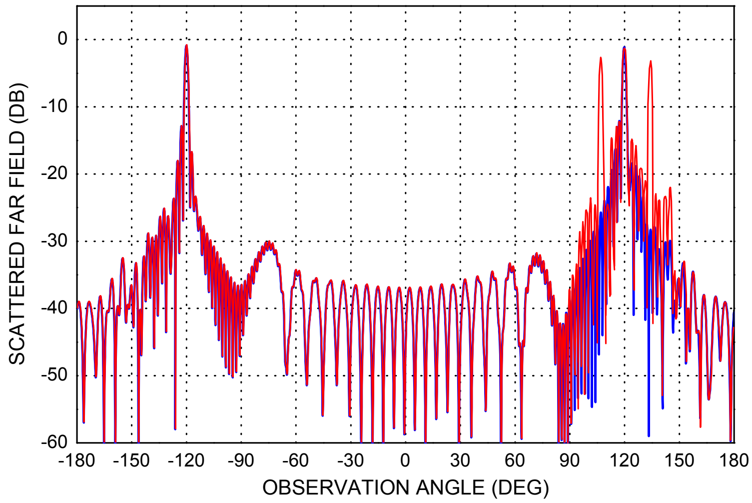

Figure 5(c). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.2$, $kb = 31.4$, $kh = 1.0$. ▬: corrugated waveguide. ▬: flat waveguide.

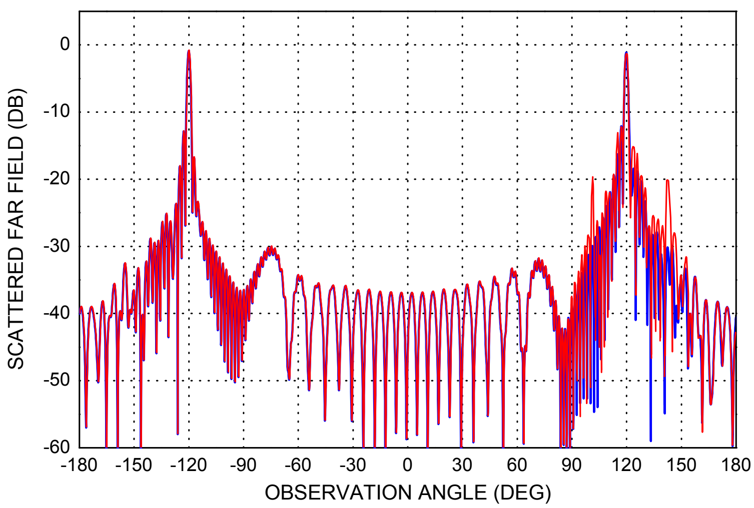

Figure 6(a). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 0.1$. ▬: corrugated waveguide. ▬: flat waveguide.

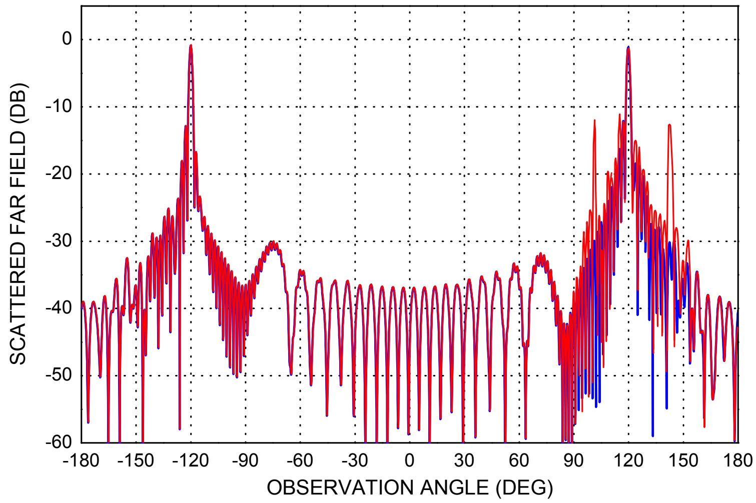

Figure 6(b). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 0.5$. red line: corrugated waveguide. blue line: flat waveguide.

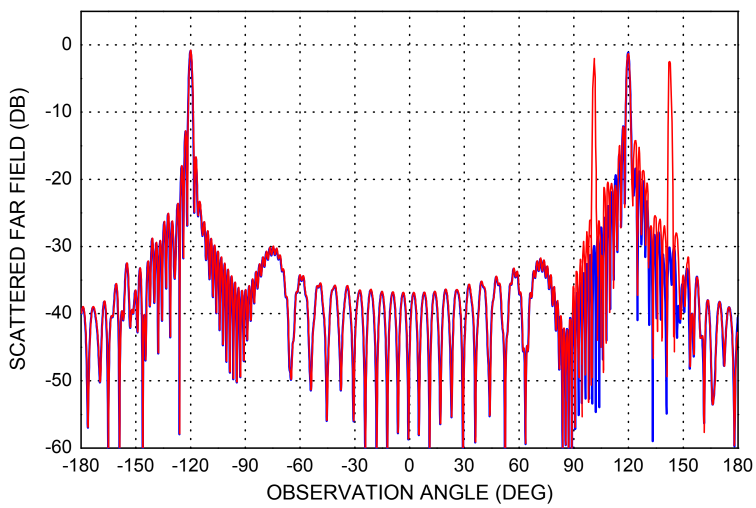

Figure 6(c). Scattered far field for $\theta_0 = 60°$, $mh/kh = 0.3$, $kb = 31.4$, $kh = 1.0$. red line: corrugated waveguide. blue line: flat waveguide.

to investigate the effect of sinusoidal corrugation of the waveguide walls in detail, we have chosen the normalized corrugation depth kh and the periodicity parameter mh/kh as $kh = 0.1, 0.5, 1.0$ and $mh/kh = 0.2, 0.3$. In addition, the normalized waveguide spacing kb has been taken as $kb = 15.7, 31.4$. The results for a flat, semi-infinite parallel-plate waveguide have also been added for comparison.

It is seen from all the figures that the scattered far field has maximum peaks at $\theta = -120°$ as this direction corresponds to the incident shadow boundary. Comparing the results for the corrugated waveguide with those for the flat waveguide, we observe that the effect of sinusoidal corrugation of the waveguide walls is noticeable for the range $90° < \theta < 180°$, and the scattered far field intensity has sharp peaks at two particular observation angles around the specularly reflected direction at $\theta = 120°$. Consideration on the structure of an infinite sinusoidal surface may offer a physical understanding of the scattering mechanism at these particular observation angles. Referring to (65), it is seen that $\pi - \theta_1$ and $\pi - \theta_2$ are, respectively, propagation directions of the (-1) and $(+1)$ order diffracted waves involved in the Floquet space harmonic modes arising in the periodic structures of infinite extent. These angles are 107.5°, 134.4° for the parameters chosen in Figs. 3 and 5, and 101.5°, 143.1° for those in Figs. 4 and 6, at which somewhat large reflection is expected. In fact, we see that observation angles associated with the two peaks around $\pi - \theta_0$ in each figure are very close to $\pi - \theta_1$ and $\pi - \theta_2$. On the other hand, the peaks along the specular reflection are also expected from the grating theory since they exactly correspond to the propagation direction of the zero-order Floquet mode. Therefore it is confirmed that the three peaks at $\pi - \theta_0$, $\pi - \theta_1$, and $\pi - \theta_2$ in numerical examples are due to the effect of periodicity of the sinusoidal surface of the waveguide. It can also be observed from the figures for fixed kb and mh/kh that the peaks occurring in the $\pi - \theta_1$ and $\pi - \theta_2$ directions become sharper with an increase of kh. This is because the waves in propagation directions of the particular Floquet modes are strongly excited for larger kh. We also find by comparing the results for $kb = 15.7$ in Figs. 3 and 4 with those for $kb = 31.4$ in Figs. 5 and 6 that sharp oscillation is observed for the case $kb = 31.4$ since the waveguide dimension then moves towards the high-frequency range. In addition, the effect of sinusoidal corrugation of the waveguide walls is seen more clearly for larger kb.

6. CONCLUDING REMARKS

In this paper, we have considered a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation and analyzed the E-polarized plane wave diffraction by means of the Wiener-Hopf technique combined with perturbation method. Explicit expressions of the scattered field inside and outside the waveguide have been obtained. In particular, the field outside the waveguide has been evaluated asymptotically using the saddle point method leading to the far field expressions, which are uniformly valid in arbitrary incidence and observation angles. We have carried out numerical computation of the scattered far field for various physical parameters, and investigated the effect of sinusoidal corrugation of the waveguide walls in detail. The results obtained in this paper are valid for the corrugation amplitude, small compared with the wavelength.

APPENDIX A. SOME IMPORTANT RELATIONS SATISFIED BY THE UNKNOWN FUNCTIONS IN THE WIENER-HOPF EQUATIONS

This appendix is concerned with the derivation of some important relations satisfied by the unknown functions arising in the Wiener-Hopf equations (26) and (27). Taking into account the approximate boundary condition on the waveguide surface as given by (7) and carrying out some manipulations, we find from (14), (15), (24), and (25) that

$$F_1(\alpha) + P_+(\alpha) = \Phi\left(b+0,\alpha\right) + \frac{h}{2i}\left[\Phi'\left(b+0,\alpha+m\right) - \Phi'\left(b+0,\alpha-m\right)\right], \tag{A1}$$

$$F_2(\alpha) + Q_+(\alpha) = \Phi\left(-b-0,\alpha\right) + \frac{h}{2i}\left[\Phi'\left(-b-0,\alpha+m\right) - \Phi'\left(-b-0,\alpha-m\right)\right], \tag{A2}$$

where the prime denotes differentiation with respect to x. Substituting the scattered field expression in (13) into (A1), (A2), (16), and (17), it follows that

$$F_1(\alpha)+P_+(\alpha) = A(\alpha)e^{-\gamma(\alpha)b}+\frac{ih}{2}\Big[\gamma(\alpha+m)A(\alpha+m)e^{-\gamma(\alpha+m)b} - \gamma(\alpha-m)A(\alpha-m)e^{-\gamma(\alpha-m)b}\Big], \tag{A3}$$

$$F_2(\alpha)+Q_+(\alpha) = D(\alpha)e^{-\gamma(\alpha)b}-\frac{ih}{2}\Big[\gamma(\alpha+m)D(\alpha+m)e^{-\gamma(\alpha+m)b} - \gamma(\alpha-m)D(\alpha-m)e^{-\gamma(\alpha-m)b}\Big]. \tag{A4}$$

$$M_-(\alpha) = -\gamma(\alpha)\Big[A(\alpha)e^{-\gamma(\alpha)b}-B(\alpha)e^{-\gamma(\alpha)b}+C(\alpha)e^{\gamma(\alpha)b}\Big], \tag{A5}$$

$$N_-(\alpha) = -\gamma(\alpha)\Big[B(\alpha)e^{\gamma(\alpha)b}-C(\alpha)e^{-\gamma(\alpha)b}+D(\alpha)e^{-\gamma(\alpha)b}\Big], \tag{A6}$$

$$\Phi''(b+0,\alpha)-\Phi''(b-0,\alpha) = \gamma^2(\alpha)\Big[A(\alpha)e^{-r(\alpha)b}-B(\alpha)e^{-r(\alpha)b}-C(\alpha)e^{r(\alpha)b}\Big], \tag{A7}$$

$$\Phi(-b+0,\alpha)-\Phi(-b-0,\alpha) = B(\alpha)e^{r(\alpha)b}+[C(\alpha)-D(\alpha)e]\,e^{-r(\alpha)b}. \tag{A8}$$

Making use of the continuity of tangential electric fields across $x=\pm b$, we deduce the following relations:

$$\Phi(-b+0,\alpha)-\Phi(-b-0,\alpha)=-\frac{h}{2i}\left[N_-(\alpha+m)-N_-(\alpha-m)\right], \tag{A9}$$

$$\Phi''(b+0,\alpha)-\Phi''(b-0,\alpha)=\gamma^2(\alpha)\left[\Phi(b+0,\alpha)-\Phi(b-0,\alpha)\right]. \tag{A10}$$

We now substitute (A7) and (A8) into (A10) and (A9), respectively, and making arrangements for the results. This leads to

$$[A(\alpha)-B(\alpha)]e^{-r(\alpha)b}-C(\alpha)e^{r(\alpha)b}=\frac{ih}{2}[M_-(\alpha+m)-M_-(\alpha-m)], \tag{A11}$$

$$B(\alpha)e^{r(\alpha)b}+[C(\alpha)-D(\alpha)]e^{-r(\alpha)b}=-\frac{ih}{2}[N_-(\alpha+m)-N_-(\alpha-m)]. \tag{A12}$$

Equations (A5), (A6), (A11), and (A12) constitute a system of simultaneous equations, which relates the unknown functions $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$ to the unknown functions $M(\alpha)$ and $N(\alpha)$. Solving these equations for $A(\alpha)$, $B(\alpha)$, $C(\alpha)$, and $D(\alpha)$, we derive that

$$A(\alpha) = -\frac{e^{\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha+m) - M_-(\alpha-m)\right]\right\}$$
$$-\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha+m) - N_-(\alpha-m)\right]\right\}, \quad \text{(A13)}$$

$$B(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha+m) - N_-(\alpha-m)\right]\right\}, \quad \text{(A14)}$$

$$C(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha-m) - M_-(\alpha+m)\right]\right\}, \quad \text{(A15)}$$

$$D(\alpha) = -\frac{e^{-\gamma(\alpha)b}}{2}\left\{\frac{M_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[M_-(\alpha-m) - M_-(\alpha+m)\right]\right\}$$
$$-\frac{e^{\gamma(\alpha)b}}{2}\left\{\frac{N_-(\alpha)}{\gamma(\alpha)} - \frac{ih}{2}\left[N_-(\alpha-m) - N_-(\alpha+m)\right]\right\}. \quad \text{(A16)}$$

Substituting (A13) and (A16) into (A1) and (A2), respectively and arranging the results, we deduce that

$$\begin{aligned}
&F_1(\alpha) + P_+(\alpha) \\
=\ & e^{-2\gamma(\alpha)b}\left\{\frac{ih}{4}\left[N_-(\alpha+m) - N_-(\alpha-m)\right] - \frac{N_-(\alpha)}{2\gamma(\alpha)}\right\} \\
&-\frac{M_-(\alpha)}{2\gamma(\alpha)} + \frac{ih\gamma(\alpha+m)}{2} \\
&\cdot\left(e^{-2\gamma(\alpha+m)b}\left\{\frac{ih}{4}\left[N_-(\alpha+2m) - N_-(\alpha)\right] - \frac{N_-(\alpha+m)}{2\gamma(\alpha+m)}\right\}\right. \\
&\left. + \frac{ih}{4}\left[M_-(\alpha) - M_-(\alpha+2m)\right]\right) - \frac{ih\gamma(\alpha-m)}{2} \\
&\cdot\left(e^{-2\gamma(\alpha-m)b}\left\{\frac{ih}{4}\left[N_-(\alpha) - N_-(\alpha-2m)\right] - \frac{N_-(\alpha-m)}{2\gamma(\alpha-m)}\right\}\right. \\
&\left. + \frac{ih}{4}\left[M_-(\alpha) - M_-(\alpha-2m)\right]\right), \qquad \text{(A17)}
\end{aligned}$$

$$
\begin{aligned}
&F_2(\alpha)+Q_+(\alpha)\\
=\ &e^{-2\gamma(\alpha)b}\left\{\frac{ih}{4}\left[M_-(\alpha-m)-M_-(\alpha+m)\right]-\frac{M_-(\alpha)}{2\gamma(\alpha)}\right\}\\
&-\frac{N_-(\alpha)}{2\gamma(\alpha)}+\frac{ih\gamma(\alpha+m)}{2}\\
&\cdot\left(e^{-2\gamma(\alpha+m)b}\left\{\frac{ih}{4}\left[M_-(\alpha+2m)-M_-(\alpha)\right]+\frac{M_-(\alpha+m)}{2\gamma(\alpha+m)}\right\}\right.\\
&\left.+\frac{ih}{4}\left[N_-(\alpha+2m)-N_-(\alpha)\right]\right)+\frac{ih\gamma(\alpha-m)}{2}\\
&\cdot\left(e^{-2\gamma(\alpha-m)b}\left\{\frac{ih}{4}\left[M_-(\alpha-2m)-M_-(\alpha)\right]-\frac{M_-(\alpha-m)}{2\gamma(\alpha-m)}\right\}\right.\\
&\left.+\frac{ih}{4}\left[N_-(\alpha-2m)-N_-(\alpha)\right]\right). \qquad \text{(A18)}
\end{aligned}
$$

It should be noted that, in the derivation of (A13)–(A18), the $O\left(h^2\right)$ terms occurring in the unknown functions $P_+(\alpha)$, $Q_+(\alpha)$, $M_-(\alpha)$, and $N_-(\alpha)$ have been ignored according to the boundary condition as given by (7). Equations (A13)–(A18) can be conveniently used in the derivation of the Wiener-Hopf equations.

APPENDIX B. EVALUATION OF SOME CANONICAL INTEGRALS IN TERMS OF THE FRESNEL INTEGRAL

This appendix is concerned with the evaluation of some canonical integrals in terms of the Fresnel integral. Let us define the integrals $I_\pm$ as

$$
I_\pm=\int_{-\infty+ic}^{\infty+ic}\frac{e^{-\gamma|x|-i\alpha z}}{(\alpha\pm k)^{1/2}(\alpha-k\cos\theta_0)}d\alpha \qquad \text{(B1)}
$$

for real x and z with γ and k being $\gamma=(\alpha^2-k^2)^{1/2}$ and $k=k_1+ik_2$, where $0<\theta_0<\pi/2$ and $-k_2<c<k_2\cos\theta_0$. We take a proper branch of the double-valued function γ as $\mathrm{Re}\,\gamma>0$. Introducing the cylindrical coordinate $x=\rho\sin\theta$, $z=\rho\cos\theta$ for $-\pi<\theta<\pi$, (B1) can be evaluated exactly as [16, 17]

$$I_+ = \left(\frac{2}{k}\right)^{1/2} \pi i \sec\frac{\theta_0}{2}\left\{e^{-ik\rho\cos(\theta-\theta_0)}F\left[(2k\rho)^{1/2}\cos\frac{\theta-\theta_0}{2}\right] + e^{-ik\rho\cos(\theta+\theta_0)}F\left[(2k\rho)^{1/2}\cos\frac{\theta+\theta_0}{2}\right]\right\}, \tag{B2}$$

$$I_- = \left(\frac{2}{k}\right)^{1/2} \pi \mathrm{cosec}\frac{\theta_0}{2}\mathrm{sgn}(\theta)\left\{e^{-ik\rho\cos(\theta-\theta_0)}F\left[(2k\rho)^{1/2}\cos\frac{\theta-\theta_0}{2}\right] - e^{-ik\rho\cos(\theta+\theta_0)}F\left[(2k\rho)^{1/2}\cos\frac{\theta+\theta_0}{2}\right]\right\}, \tag{B3}$$

where $F(\cdot)$ is the Fresnel integral defined by

$$F(x) = \frac{e^{-i\pi/4}}{\pi^{1/2}}\int_x^\infty e^{it^2}dt, \tag{B4}$$

and

$$\mathrm{sgn}(\xi) = 1 \quad \text{for } \xi > 0, \tag{B5}$$
$$= -1 \quad \text{for } \xi < 0. \tag{B6}$$

REFERENCES

1. Shestopalov, V. P., *The Riemann-Hilbert Method in the Theory of Diffraction and Propagation of Electromagnetic Waves*, Kharkov University Press, Kharkov, 1971 (in Russian).
2. Nosich, A. I., "Green's function-dual series approach in wave scattering by combined resonant scatterers," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Chap. 9, Science House, Tokyo, 1993.
3. Shestopalov, V. P., L. N. Litvinenko, S. A. Masalov, and V. G. Sologub, *Diffraction of Waves by Gratings*, Kharkov University Press, Kharkov, 1973 (in Russian).
4. Shestopalov, V. P., A. A. Kirilenko, and S. A. Masalov, *Convolution-Type Matrix Equations in the Theory of Diffraction*, Naukova Dumka Publishing, Kiev, 1984 (in Russian).
5. Nosich, A. I., "The method of analytical regularization in wave-scattering and eigenvalue problems: Foundations and review of solutions," *IEEE Antennas Propagat. Mag.*, Vol. 41, No. 3, 34–49, 1999.

6. Ikuno, H. and K. Yasuura, "Improved point-matching method with application to scattering from a periodic surface," *IEEE Trans. Antennas Propagat.*, Vol. 21, No. 5, 657–662, 1973.
7. Okuno, Y., "The mode-matching method," *Analysis Methods for Electromagnetic Wave Problems*, E. Yamashita (ed.), Chap. 4, Artech House, Boston, 1990.
8. Okuno, Y., "An introduction to the Yasuura Method," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Chap. 11, Science House, Tokyo, 1993.
9. Petit, R. (ed.), *Electromagnetic Theory of Gratings*, Springer-Verlag, Berlin, 1980.
10. Hinata, T. and T. Hosono, "On the scattering of electromagnetic wave by plane grating placed in homogeneous medium — Mathematical foundation of point-matching method and numerical analysis," *Trans. IECE Japan*, Vol. J59-B, No. 12, 571–578, 1976 (in Japanese).
11. Yamasaki, T., K. Isono, and T. Hinata, "Analysis of electromagnetic fields in inhomogeneous media by Fourier series expansion methods — The case of a dielectric constant mixed a positive and negative regions," *IEICE Trans. Electron.*, Vol. E88-C, No. 12, 2216–2222, 2005.
12. Ozaki, R., T. Yamasaki and T. Hinata, "Scattering of electromagnetic waves by multilayered inhomogeneous columnar dielectric gratings," *IEICE Trans. Electron.*, Vol. E90-C, No. 2, 295–303, 2007.
13. Noble, B., *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*, Pergamon, London, 1958.
14. Weinstein, L. A., *The Theory of Diffraction and the Factorization Method*, The Golem Press, Boulder, 1969.
15. Mittra, R. and S.-W. Lee, *Analytical Techniques in the Theory of Guided Waves*, Macmillan, New York, 1971.
16. Kobayashi, K., "The Wiener-Hopf technique with applications to scattering and diffraction problems," *A Course of Applied Mathematics*, K. Horiuchi (ed.), Chap. 9, Corona Publishing, Tokyo, 1989 (in Japanese).
17. Kobayashi, K., "Wiener-Hopf and modified residue calculus techniques," *Analysis Methods for Electromagnetic Wave Problems*, E. Yamashita (ed.), Chap. 8, Artech House, Boston, 1990.
18. Baldwin, G. L. and A. E. Heins, "On the diffraction of a plane

wave by an infinite plane grating," *Math. Scand.*, Vol. 2, 103–118, 1954.

19. Hills, N. L. and S. N. Karp, "Semi-infinite diffraction gratings — I," *Comm. Pure Appl. Math.*, Vol. 18, No. 1–2, 203–233, 1965.
20. Lüneburg, E. and K. Westpfahl, "Diffraction of plane waves by an infinite strip grating," *Ann. Phys.*, Vol. 27, No. 3, 257–288, 1971.
21. Lüneburg, E., "Diffraction by an infinite set of parallel half-planes and by an infinite strip grating," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Chap. 7, Science House, Tokyo, 1993.
22. Serbest, A. H., A. Kara, and E. Lüneburg, "Scattering of plane waves at the junction of two corrugated half-planes," *Electromagnetics*, Vol. 25, No. 1, 21–38, 2005.
23. Kobayashi, K., "Diffraction of a plane wave by the parallel plate grating with dielectric loading," *Trans. IECE Japan*, Vol. J64-B, No. 10, 1091–1098, 1981 (in Japanese).
24. Kobayashi, K., "Diffraction of a plane electromagnetic wave by a parallel plate grating with dielectric loading: the case of transverse magnetic incidence," *Can. J. Phys.*, Vol. 63, No. 4, 453–465, 1985.
25. Kobayashi, K. and T. Inoue, "Diffraction of a plane wave by an inclined parallel plate grating," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 10, 1424–1434, 1988.
26. Kobayashi, K. and K. Miura, "Diffraction of a plane wave by a thick strip grating," *IEEE Trans. Antennas Propagat.*, Vol. 37, No. 4, 459–470, 1989.
27. Das Gupta, S. P., "Diffraction by a corrugated half-plane," *Proc. Vib. Prob.*, Vol. 3, No. 11, 413–424, 1970.
28. Chakrabarti, A. and S. Dowerah, "Traveling waves in a parallel plate waveguide with periodic wall perturbations," *Can. J. Phys.*, Vol. 62, No. 3, 271–284, 1984.
29. Kobayashi, K., "Combined perturbation and Wiener-Hopf analysis of the diffraction by a finite sinusoidal grating," *Microwave Opt. Technol. Lett.*, Vol. 4, No. 11, 495–501, 1991.
30. Kobayashi, K. and T. Eizawa, "Plane wave diffraction by a finite sinusoidal grating," *IEICE Trans.*, Vol. E74, No. 9, 2815–2826, 1991.
31. Kobayashi, K., "Some diffraction problems involving modified Wiener-Hopf geometries," *Analytical and Numerical Methods in Electromagnetic Wave Theory*, M. Hashimoto, M. Idemen, and O. A. Tretyakov (eds.), Chap. 4, Science House, Tokyo, 1993.