Progress In Electromagnetics Research C, Vol. 7, 137–150, 2009

OPTIMIZATION OF CHANNEL CAPACITY FOR INDOOR MIMO SYSTEMS USING GENETIC ALGORITHM

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Abstract—The geometrical parameters of antenna arrays in a multiple-input multiple-output (MIMO) link that will maximize the ergodic channel capacity of the system are investigated. The problem of selecting the optimum number of the elements at the base station antenna array is also included. The genetic algorithm technique is applied for the optimisation process, using the ergodic capacity as a fitness function. A discrete model based on statistical distribution of the Angle of Arrival (AoA) of the incoming rays is considered. Searching for more compactness in the antenna size and higher system capacity, different array configurations with non-uniform inter-element spacing are considered, such as linear array, circular array and multi-circular array (Star) geometries. Numerical examples that illustrate new designs of non-uniform spaced arrays are presented to show the capability of the developed procedures to optimize the capacity of indoor MIMO systems.

1. INTRODUCTION

MIMO wireless communication systems offer a large increase in the system capacity compared to single antenna systems [1]. However, spatial fading correlation has its direct effect on the channel capacity of indoor MIMO systems. Therefore, multiple antenna arrays used in the transmitter and the receiver should be accurately designed to reduce the spatial correlation and achieve higher capacity. In the antenna design process, spatially correlated MIMO channels are typically modeled under certain assumptions about the scattering in the propagation environment and the geometry of the antenna arrays as in [2–11]. The proper modeling of channel is a challenging task and important part of the antenna design procedure as it can affect the performance of the wireless system under consideration. The channel model employed in this work is a discrete model based on statistical distribution of the AoAs of the incoming rays as in [11] for the receiver part. The spatial correlation matrix at the transmitter will be modeled as in [12–14]. Rician fading channel [15] is also included in the model to consider LOS and NLOS propagations. Previous MIMO antenna design optimization studies are reported in [16] and [17] where the issue of how to appropriately select the number of antennas at the asymmetric base station and mobile units is studied. In [18], for optimizing the MIMO system capacity unequal costs of implementing antennas at both channel ends are considered. However, in this work the cost function is expressed using approximated asymptotic expression for the ergodic capacity calculations. On the other hand, from the geometry selection point of view, Uniform Linear Array (ULA) is the most common geometry in cellular systems. However, Uniform Circular array (UCA) is as an alternative geometry with its enhanced properties. The analysis of fading correlation was carried out for UCA in [12–14]. The results show that the spatial correlation decreases for UCA compared to ULA on average for small and moderate angle spread (AS) for similar aperture sizes. On the other hand, ULA has less spatial correlation than UCA for near broadside angle-of-arrivals with moderate AS. Regarding non-uniform geometries designs, the free standing linear arrays (FSLA) optimized using particle swarm optimization (PSO) was introduced in [19]. Throughout this work, a MIMO channel model based on electric fields including mutual coupling effects is presented. Increased capacity was observed for different 3 dimensional (3D) uniform and non-uniform array configurations compared to conventional 0.5λ length ULA. Another study of outdoor capacity in urban city street grid was performed in [20]. Uniform rectangular array (URA) and uniform cubic array (UCuA) in addition to ULA and UCA with fixed inter-element spacing are considered. The investigations are carried out based on 3D spatial multi-ray realistic physical propagation channel model. It is shown that ULA shows superiority to the other geometries under certain LOS and NLOS propagation conditions and this superiority depends largely on the array azimuthal orientation.

Finding novel antenna array designs to optimize MIMO ergodic channel capacity remains under investigations. Therefore, this paper addresses the issue of investigating new MIMO antenna configurations and develops a design tool for this purpose. Three different geometries are considered, which are Non-uniformly spacing linear array (NULA), Non-uniformly spacing circular array (NUCA) and Non-uniformly spacing multi-circular array (N-Star). The three configurations are shown in Fig. 1, and these arrays will be optimized at the receive end for uplink indoor MIMO system while we assumed ULA at the transmitter as practical and convenient choice at the mobile unit. The genetic algorithm method (GA) was chosen as the most propitious approach. The optimization will include the selection of the minimum number of radiating elements for the considered arrays.

Also, the placement of the array elements will be optimized for the three array geometries under investigations. The GA search will be subject to propagation environment parameters such as AoAs and AS of the received signal and to antenna array size constraints. The rest of the paper is organized as follows, uplink MIMO channel model is descried in Section 2; Section 3 presents the GA procedures for MIMO antenna array design; Section 4 shows the numerical analysis and discussion; and the paper is concluded in Section 5.

2. SYSTEM MODEL AND SPATIAL CORRELATION CALCULATIONS

The uplink MIMO scenario considered implies that the transmitter is located at the Mobile Unit (MU) and the receiver at the Base Station (BS). The channel was modelled as a multi-clusters scattering environment which means that the signal will arrive at the BS from multiple angles of arrival (AoA), each with angle spread (AS) that is a measure of the angle displacement due to the non-line of sight (NLOS) propagation. Defining $R_{st}(p,q)$ and $R_{sr}(m,n)$ as the spatial correlation due to the transmitter and receiver antennas respectively, and assuming that the two links are statistically independent, then the link spatial correlation can be simplified and divided into the transmit and receive parts correlations as follows:

$$R_s(mp,nq) = R_{sr}(m,n) \times R_{st}(p,q) \tag{1}$$

For the ULA at the MU (transmitting side), it is assumed that the mean angle of departure θ_t is uniformly distributed over $[0, 2\pi]$, as given in [12] as follows:

$$R_{st} = J_o\left(2\pi\frac{(q-p)D_t}{\lambda}\right) \tag{2}$$

where D_t is the distance between elements p and q. However, at the BS (receiving) side the UCA and Star configurations were assumed to be adopted: these are technically better choices than the ULA, but the ergonomics of mobile devices dictates that a linear array must be adopted in that case, in order to fit into a convenient thin, flat device. The spatial covariance matrix ($\boldsymbol{R}_{sr}$) was computed for different antenna array configurations, since $\boldsymbol{R}_{sr}$, a discrete model, was used, based on statistical distribution of the AoAs of the incoming rays. A large number of rays was simulated in order to converge to the IEEE 802.11n channel model (which assumes continuous power angular spectrum (PAS), i.e. angle of arrival of rays). Moreover, antenna arrays consisting of dipole antenna elements were considered, with uniform amplitude excitation and no coupling between array elements. With these assumptions, we can express the spatial correlation matrix for each ray as a product of steering-vectors. Simulating P rays per cluster, the covariance matrix of the cluster is derived as:

$$R_{sr} = \sum_{k=0}^{n} a\left(\phi_o - \phi_p\right) a^H\left(\phi_o - \phi_p\right) \tag{3}$$

where $a(\phi)$ is the steering vector for a generic AoA (ϕ); ϕ_o is the mean AoA of the cluster and ϕ_p is the AoA offset with respect to ϕ_o for the pth ray. Note that ϕ_p is a random variable with Laplacian probability distribution function. This formula, when substituted in Equation (1), provides the MIMO channel matrix used for the simulations described in the next section. Hereafter the expression of the steering vectors is reported at the AoA ϕ for different array geometries, as illustrated in Figure 1, according to the following definitions:

- Uniform and Non-uniform spaced Linear Array (ULA & NULA)):

$$a(\phi) = \left[e^{jkd_1 \sin\phi}, \ldots, e^{jkd_M \sin\phi}\right]^T \tag{4}$$

where are the spacings for the M antennas with respect to the origin of the reference system. We choose the antenna spacings such that the

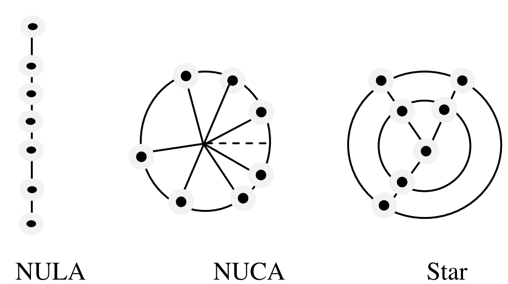

Figure 1. Non-Uniform spaced Linear Array, Non-Uniform spaced Circular Array and Star configurations.

total array aperture coincides with the length of the ULA for a fixed number of elements ($\sum_{i=1}^{M} d_i = Md$). If $d_1 = d_2 = \ldots = d_m = d$ then we get the ULA case.

- UCA and NUCA:

$$a(\phi) = \left[e^{jk\rho\cos(\phi-\phi_1)}, \ldots, e^{jk\rho\cos(\phi-\phi_M)}\right]^T \tag{5}$$

where ρ being the radius of the circular array and ϕ_m the angle of the mth array element with respect to the reference angle as shown in Figure 1.

- Star and N-star Array:

$$\begin{aligned} a(\phi) = & \Big[1, e^{jk\rho\cos(\phi-\phi_1)}, \ldots, e^{jk\rho\cos(\phi-\phi_3)}, \\ & e^{jk2\rho\cos(\phi-\phi_1)}, \ldots, e^{jk2\rho\cos(\phi-\phi_3)}\Big]^T \end{aligned} \tag{6}$$

where star array is designed with $(M_r - 1)/2$ branches and M_r array elements. The correlated Rician Fading MIMO channel Matrix (T) with dimensions ($M_t \times M_r$) at one instance of time can be modelled as a fixed (constant, LOS) matrix and a Rayleigh (variable, NLOS) matrix.

$$T = \sqrt{\frac{K}{1+K}}\overline{H_{ric}} + \sqrt{\frac{K}{1+K}} R_{sr}^{1/2} H_w R_{st}^{1/2} \tag{7}$$

where H_w are zero mean and unit variance complex Gaussian random variables that presents the coefficients of the variable NLOS matrix. K is the Rician K-factor, and R_r and R_t are the $M_r \times M_r$ and $M_t \times M_t$ correlation matrices that include all possible coefficients of spatial correlations between the channel links seen at transmit and receive elements respectively.

3. GA OPTIMIZATION OF ANTENNA CONFIGURATION

Genetic algorithms (GA) provide powerful solutions for antenna design problems. The gene is the basic building block in the GA optimization. Genes are generally the code representation of the optimization parameters. A string of these genes produces the *chromosome*: in this paper chromosomes are binary coded. A set of trial solutions in the form of chromosomes is assembled as a population, which is the unit that the GA optimizer searches in to reach an optimum solution.

The optimization iterations in GA are called *population*, which is the unit that the GA optimizer searches in to reach an optimum solution. The optimization iterations in GA are called *generations*. Pairs of individuals called *parents* are selected from the population in a probabilistic manner, depending on their fitness function. The objective function defining the optimization goal is called the *fitness function,* which is a means of assigning a value to each individual in the population. It is a link between the physical problem and the GA optimization process. Children are then generated from the selected pair of parents by applying crossover and mutation processes. Figure 2 shows the design procedure of a GA optimizing technique.

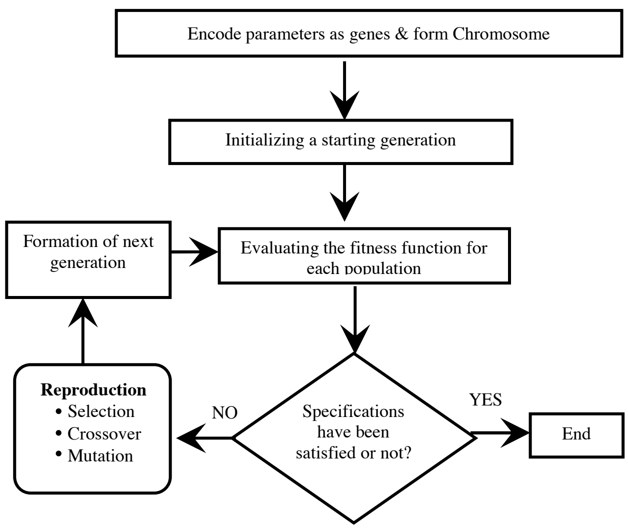

Figure 2. GA optimization procedures.

The optimizing parameters (genes) are the number of array elements at the base station (M_r) and the mobile unit (M_t) and the angles of the elements (ϕ) that is related to the spacings between them for NUCA/star configurations. Also the radii (ρ) of the receiver circular antenna are included in the numerical results that will be shown in the next section. These variable parameters of the array are placed in the chromosome vector. The fitness function of GA to evaluate the channel capacity in terms of the optimization parameters is:

$$\text{Fitness} = c = \log_2 \left[det \left(I_{Mr} + \frac{\text{SNR}}{Mt} T T^H \right) \right] \tag{8}$$

where T is the complex matrix obtained from the realization of (7);

SNR is the average signal to noise ratio; and I_{Mr} is the identity matrix with dimensions $M_r \times M_r$. Here we assume equal power transmission across the array elements at the transmitter. Using this procedure, an optimization tool is developed to design arrays with elements whose number, locations and inter-spacing are optimized for higher MIMO channel capacity. The design tool capability will be demonstrated in the following section.

4. NUMERICAL RESULTS

The system performance was analyzed in different propagation conditions: the capacity gain and space diversity advantage attainable with the antenna configurations are shown below. UCA and star configurations and optimization are considered at the BS whereas a ULA is applied at the MU, as shown in Figure 3. Figure 4 shows the ergodic capacity of the UCA and Star and the ULA as a function of Θ (AoA). This example is performed for 7×7 MIMO system with the following assumptions, elements spacing $D = 0.5\lambda$ for ULA and array radius $(\rho) = 0.75\lambda$ for UCA and outer Star radius (ρ_2) while the inner Star radius (ρ_1) is half of ρ_2 with elements distributed evenly. AS $= 20°$ and SNR $= 15\,\text{dB}$ is considered. As shown in Figure 4, the UCA and Star outperforms the ULA in particular at endfire $(\Theta = 90°)$ when

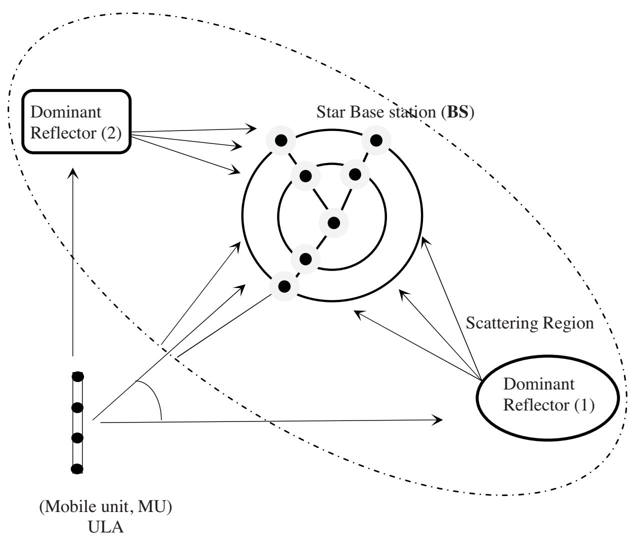

Figure 3. Geometry of the MIMO system model of $Mt \times Mr$ elements ULA/Star with signal arriving from multi-clusters in scattering channel.

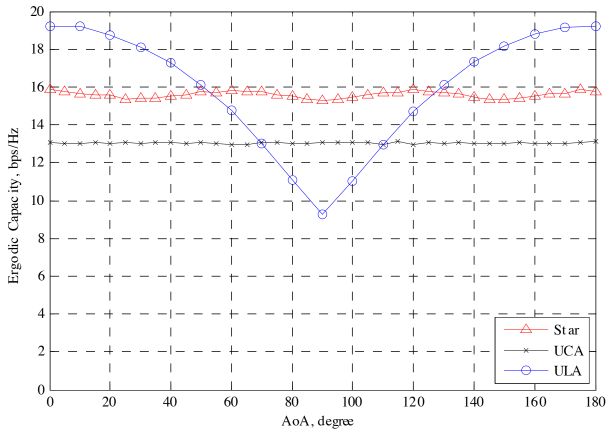

Figure 4. Ergodic capacity versus AoA, one cluster case, 7×7 MIMO ULA, UCA and star system, SNR $= 15$ dB.

both have the same aperture sizes. However, the ULA performance is better for certain angles-of-arrival near broadside of the ULA $\Theta = 0°$ and $180°$. Three different examples are presented as follows to show the optimization process results for maximizing MIMO system capacity at different propagation scenarios.

Example 1: The GA was first used to optimize the NUCA geometry and then find the location of elements at nonequal angles. The MIMO system is assumed to be a (7×7) ULA/NUCA with the following assumption: Rician fading channel with $K = 10$, AS $= 20$, SNR $= 15$ and $\rho = 0.5\lambda$. Population size is 200 for 20 generations, total ULA length is assumed to be 0.5λ. Here, six angle parameters are used as genes, and each is constrained by $\pm 2\pi/M_r$. Figure 5 shows the optimum NUCA geometry. Element locations are at [0, 66, 103, 154, 235, 257, 309] degrees, with system capacity of 11.4 bps/Hz. This shows a little improvement compared to the uniform circular array (UCA) with elements located at equally spaced angles that gives a capacity of 10.01 bps/Hz. However, this run indicates that the GA could be used to design the NUCA geometry effectively, and further investigations are needed for this geometry under different realistic scenarios, especially in cases where more compact arrays are required. Increasing the number of transmit or receive antennas will always increase system capacity. Also, large separations give reduced spatial correlations and higher capacity. Thus, GA optimizing tradeoffs among M_r, M_t and ρ are needed and will be discussed in the next two examples for both UCA and Star configurations.

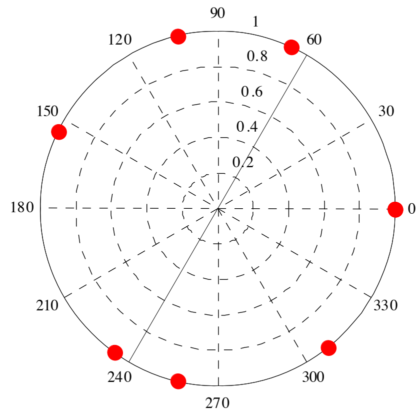

Figure 5. Optimized non uniformly spaced circular array for 7×7 ULA/NUCA MIMO system configuration for AS = 20 and SNR = 15 dB. Elements are at [0 66 103 154 235 257 309], Capacity = 11.4 bps/Hz.

Example 2: we study the optimization of a ULA/UCA system by GA numerical calculation and select the best M_r and M_t numbers for ULA at MU and UCA at BS, also to find the minimum array radius for the UCA to have compact size and space array. We assume SNR = 15 dB and 10 dB, two cases are considered AoA are 0° and 90°. Three constraints are considered here, ρ (UCA radius) $< 0.5\lambda$, $M_t \leq 9$ and $M_r \leq 9$. The Genes which construct the chromosomes are M_b, M_r and ρ. The algorithm has initial number of generations equals 20 and a population size of 200 chromosomes. Table 1 shows the results of the optimum (ULA/UCA) numbers that are (4×9) for all cases and the optimum UCA radius is 0.5λ. It is noted that this case gives higher capacity than the case of (9×9) which is 11.02 bps/Hz and 8.21 for the SNR = 10 and 15 dB respectively.

Table 1. UCA configuration optimization table ($\rho_{\max}$).

SNR	AoA	As	Nt (ULA)	Nr (UCA)	ρ	Channel capacity
15	0	20	9	4	0.491	12.01
15	90	20	9	4	0.5	12.05
10	0	20	9	4	0.5	8.78
10	90	20	9	4	0.5	8.76

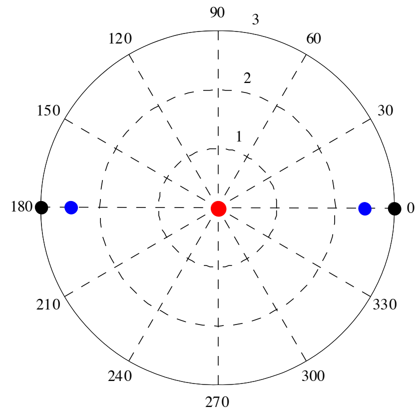

Figure 6. The optimum Star array configuration when As $=$ 5, AOA $=$ 90 case.

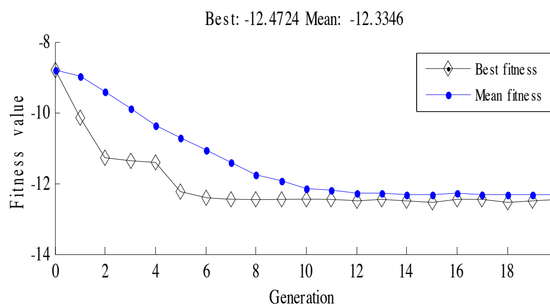

Figure 7. The convergence results for the GA run with 20 generations and population size $=$ 200.

The Genes which construct the chromosomes are (Mr, Mt, r1,r2).

The algorithm has initial number of generations equals 20 and a population size of 200 chromosomes.

Example 3: a third example is presented to show the GA applications of finding the optimum ULA/Star array configuration (M_r, M_t, $\rho 1$, $\rho 2$) for which $\rho 1$, $\rho 2$ are the inner and outer radii for the star array. We assume SNR $= 15\,\mathrm{dB}$; two cases are considered AoA $= 0°$ and $90°$; As $=$ 5, 10, 20 cases and the constraint $0.1 < \rho 1 < 2.9$ $\rho 2 = 3$ (Star radii) $< 0.5\lambda$ and M_t and $M_r \leq 9$. The Genes which construct the chromosomes are (M_r, M_t, $\rho 1$, $\rho 2$). The algorithm has initial number of generations equal to 20 and a population size of 200 chromosomes. The results are shown in Figures 6 to 9 and Table 2. Figures 6 and 8 illustrate the element locations for the optimum solutions obtained from the GA runs. Figures 7 and 9 show

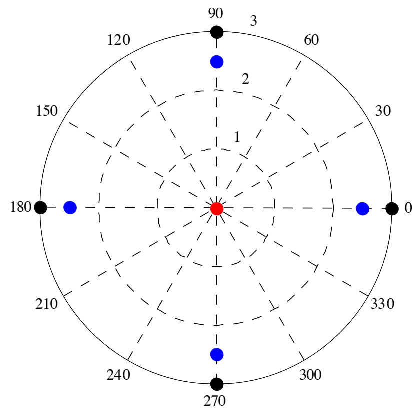

Figure 8. The optimum Star array configuration when As $= 5$, AOA $= 0$ case.

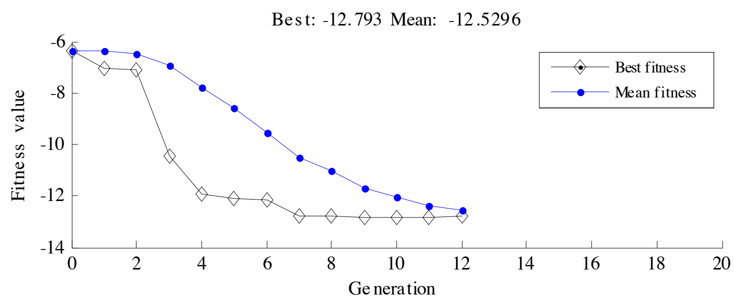

Figure 9. The convergence results for the GA run with 20 generations and population size $= 200$.

the convergence results for the GA run. The figures illustrate the best and the mean fitness functions for each generation. While initial set up is 20 generations, converges conditions are included to stop the GA when the mean fitness value becomes equal to the best fitness value over the entire population. For (AoA=0) case study, GA is executed and the output converges in 12 generations as shown in Figure 9.

Table 2 present the results for all runs at different AS and AOA. As shown, for reasonable capacity 8×5 MIMO gives reasonable economy in the array cost M_t and (9×9) which saves in the array cost and size. It should be noted that when the AoA is $90°$, the M_r of the star array is 5 which leads to a NULA configuration where the incoming rays will be normal to the array line (broadside case): this case has low spatial correlation.

Table 2. Star configuration optimization table ($\rho_{\max}$).

SNR	AoA	As	Mt (ULA)	Mr (star)	$\rho1$	$\rho2$	Channel capacity
15	0	20	8	7	1.521	3	15.5836
15	0	10	8	7	2.565	3	14.557
15	0	5	8	9	2.523	3	12.868
15	90	20	9	5	1.500	3	15.586
15	90	10	8	5	1.571	3	14.8618
15	90	5	7	5	2.500	3	12.4724

5. CONCLUSION

This paper brings forward the issue of maximising the system capacity of uplink indoor MIMO systems when different antenna configurations are used at both transmission link ends. An abstract channel model is employed, which is appropriate for modeling narrow-band Rician fading in fixed wireless systems. In this model, spatial correlation matrix is derived with discrete function for different geometries at the receiver, whereas mathematical function for spatial fading correlation of the transmitting antenna is utilized. The search for optimum geometries, under constraints of practicality, and selection of the number of elements have been shown to be performed successfully using genetic algorithm methods for different propagation scenarios, by taking the channel capacity as a fitness function. The main constraint of practicality is the requirement that the mobile unit must have a linear array in order to be convenient and acceptable to mobile users that is why we assumed ULA for transmitting antenna. The number of the array elements in the base station and mobile unit may change with SNR, angle of arrival and angle spread of received rays, but compromise fixed values obviously have to be chosen. At the base stations, the Star and uniform circular array topologies are found to be preferable to other configurations for systems that suffer from high spatial correlations when compact arrays are necessarily employed. The GA method has demonstrated its ability to find realistic and economical MIMO array designs requiring only modest numbers of elements at each end of the link.

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