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MODE DEGENERACY IN CIRCULAR CYLINDRICAL RIDGE WAVEGUIDES

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Abstract—Studies of dual channel, polarisation agile, quad-ridge and octo-ridge feeds suggest that mode degeneracy in multi-ridge structures severely constricts operational bandwidths, for a large range of ridge dimensions. Mode characteristics in dual-ridge, quad-ridge and octo-ridge waveguide are examined in this paper, with a view to identifying both the nature of the degeneracy, and its implications for bandwidth. The results presented have been generated using a full-wave finite element electromagnetic field simulator.

1. INTRODUCTION

Ridge waveguides, independent of the form of the enclosure, which can be rectangular, square, or circular in cross-section, have evolved because of their wide bandwidth potential. Here we define bandwidth as the ratio of the difference frequency (Δf) to the centre frequency (f_0). Single ridge and double ridge waveguides in rectangular enclosures were first described by Cohn [1] some sixty years ago, while comprehensive design equations and graphs were later provided by Hopfer [2]. Since then, a range of irregular and asymmetrical ridge waveguides has been described from time to time in the literature, and much of this work is well summarised by Rong [3].

The need for dual-polarised radiating elements for planar phased arrays, and compact dual-polarised feeds for reflector antennas, has been the main impetus for the development of quadruple ridge square waveguide [4], while quad-ridge circular waveguides designed for similar applications have been described by Chen [5,6] and Rong [3]. Recent developments in wide band multi-channel single antenna systems, have led to the investigation of an octo-ridge circular waveguide, as a possible feed system with dual-channel, polarisation agile, capability. In essence octo-ridge waveguide can be viewed as two quad-ridge waveguides within the same envelope. Attempts to elucidate the modal characteristics of this structure, using a full-wave finite element solver, has resulted in an improved understanding of the issue of ridge mode degeneracy, which is presented in detail here. This phenomenon does not seem to have been considered elsewhere.

2. RIDGE WAVEGUIDE OPTIONS

The dimensional constraints associated with ridge waveguide as we move from the dual-ridge geometry (upper semi-circle when mirrored in the horizontal diameter), to the quad-ridge geometry (lower left quadrant when mirrored in the horizontal and vertical diameters), and then to the octo-ridge geometry (lower right quadrant suitably mirrored), is indicated in Fig. 1. While dual-ridge waveguide is devoid of a possible adjacent ridge propagation mechanism influencing mode behaviour, as we will see this is not true of quad-ridge and octo-ridge geometries. However, for octo-ridge waveguide some additional

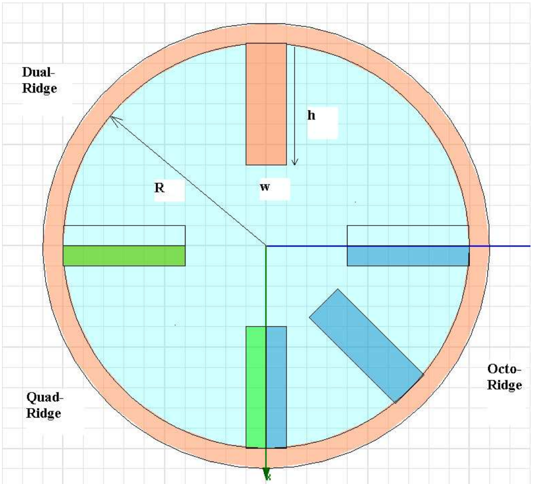


Figure 1. Quad-ridge waveguide — dimensional options.

geometrical constraints apply which obviously do not occur with dual-ridge arrangements. The lower left and lower right quadrants of Fig. 1 give an indication of the significant dimensions and the size restraints. Just looking at the diagram, and given that wide bandwidth requires small gaps ($= 2g$ where $g = R - h$), the concern is the corner-to-corner separation between adjacent ridges for the quad-ridge and octo-ridge geometries, since electromagnetic fields tend to concentrate at metallic edges ('edge effect'). This effect can obviously be limited in the octo-ridge structure, by employing equal ridge width (w) to tip spacing (corner-to-corner). This means that each ridge tip must subtend an angle of no more than 22.5° at the guide centre. Unfortunately, the ridge height is also limited by this criterion, which in turn prevents the adoption of structures with small gaps.

3. MODAL CHARACTERISTICS AND BANDWIDTH

3.1. Dual-ridge Waveguide

In the ensuing discussion the following mode nomenclature is introduced to identify ridge waveguide modes as they evolve from the modes in empty circular waveguide. The nomenclature TE_{nmW} or TM_{nmR}, with additional subscript (W), implies a mode in the ridge waveguide, which remains essentially a mode of circular cylindrical waveguide with the ridges having little influence on the mode, or where the increasing ridge depth merely results in a reduction in the effective diameter of this 'waveguide'. On the other hand, TE_{nmW} or TM_{nmR}, with additional subscript R, implies a mode in the ridge waveguide, which is strongly influenced by the ridge height and ridge separation and becomes a quasi-TEM mode at large h/R. In Figs. 2, 3 and 4 the normalised cut-off frequencies of the first few significant modes of dual-ridge, quad-ridge, and octo-ridge waveguide are presented. The modal cut-offs are all normalised to the cut-off frequency of the TE_{11} mode in a guide with $h/R = 0$. Results are present as a function of h/R and for clarity we have held w/R constant at a typical value of $w/R = 0.1$.

In a dual-ridge circular cylindrical waveguide, at $h/R = 0$, the first three modes in cut-off frequency terms are the TE_{11} mode at unity (on the normalised frequency scale), the TM_{01} mode at 1.306 and the TE_{21} mode at 1.655. In fact, because of the circular symmetry of the guide there are five modes, a degenerate pair of TE_{11} modes with spatially orthogonal fields, the TM_{01} mode, and a degenerate pair of TE_{21} modes, again with spatially orthogonal fields. In dual-ridge waveguide the ridges influence the degenerate modes differently largely depending on whether or not the ridges significantly disturb

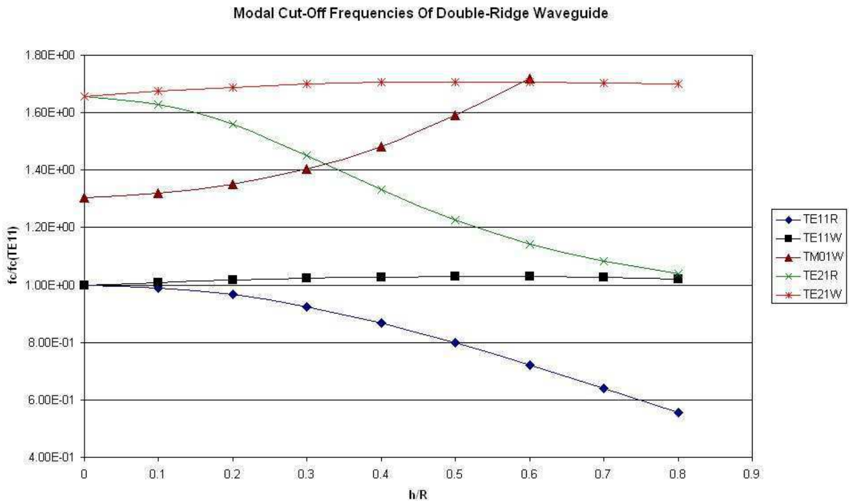


Figure 2. Normalised cut-off frequency (f_{cnm}/f_{c11}) as a function of h/R for dual-ridge cylindrical waveguide ($w/R = 0.1$).

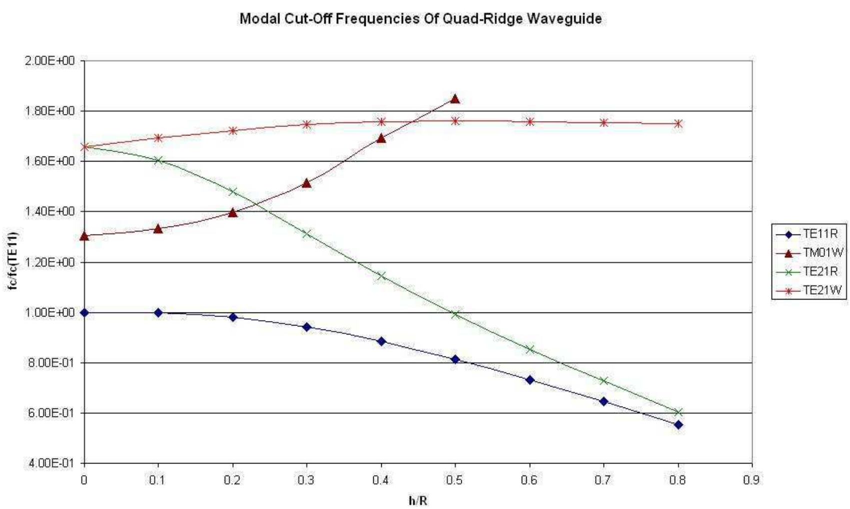


Figure 3. Normalised cut-off frequency (f_{cnm}/f_{c11}) as a function of h/R for quad-ridge cylindrical waveguide ($w/R = 0.1$).

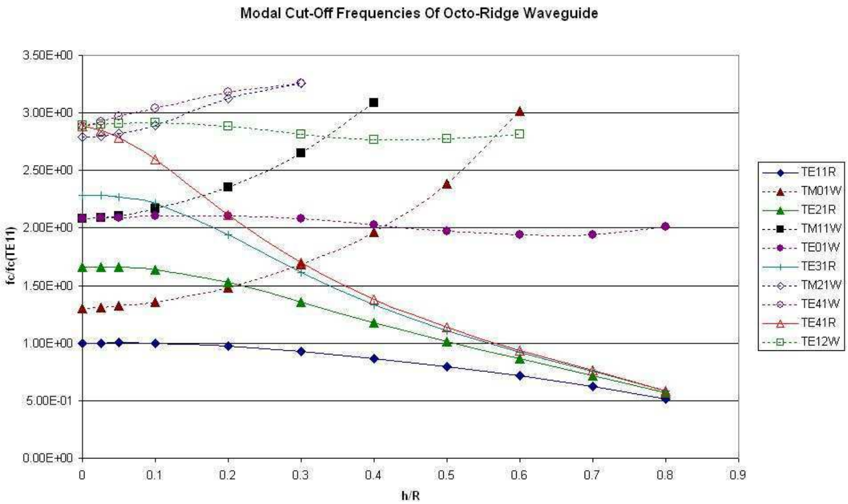
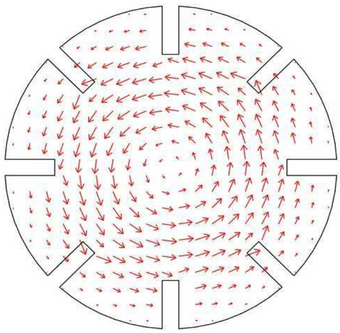
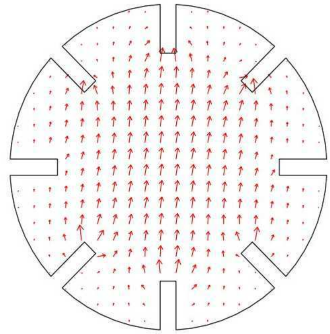


Figure 4. Normalised cut-off frequency (f_{cnm}/f_{c11}) as a function of h/R for octo-ridge loaded cylindrical waveguide ($w/R = 0.1$).

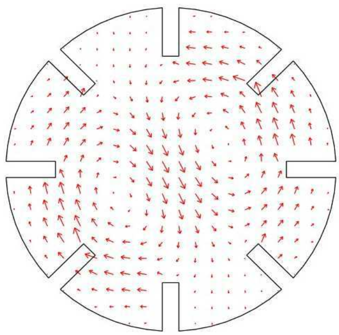
the radially directed electric field components of the mode of interest. In the case of the TE_{11} mode, the degenerate component with the electric field aligned with the ridges, is 'shorted' by the ridges, as they are increased in length, with the E-field becoming trapped between the ridge tips (c.f. 6 o'clock/12 o'clock ridges in Fig. 5(b)). The mode becomes a quasi-TEM mode. The cut-off frequency of this mode drops quickly as h/R is increased (Fig. 2 — diamond annotated curve). On the other hand the TE_{11} mode degeneracy with its E-field normal to the ridges (Fig. 5(b) 3 o'clock/9 o'clock ridges), is hardly influenced by the ridges at all and becomes the TE_{11W} mode for the dual-ridge waveguide (Fig. 2 — curve annotated with squares). The TM_{01W} mode is little different from the empty waveguide mode even for quite deep ridges, particularly if the ridges are narrow. This is simply because the E-fields for this mode (Fig. 5(f)) are weak at large radii, and the ridges tend to have minimal influence on the fields and hence the modal cut-off frequency, at least for moderate values of h/R. The cut-off frequency increases with h/R quite simply because the ridges reduce the effective diameter of the cylindrical waveguide. The TE_{21} mode behaves in a manner similar to the TE_{11} mode with one degenerate element, which has its E-fields predominantly normal to the ridge faces, showing a cut-off frequency which is relatively independent of ridge depth (h/R). The other spatially orthogonal component, whose E-fields are increasingly



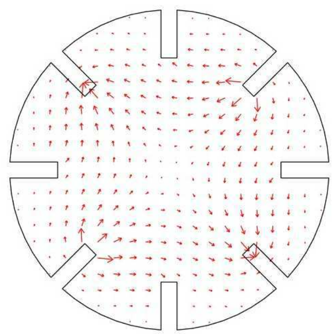
(a) TE_{01} mode



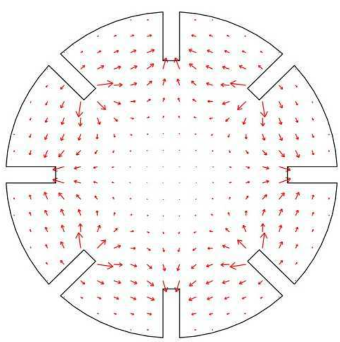
(b) TE_{11} mode



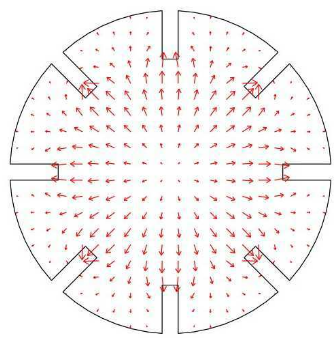
(c) TE_{12} mode



(d) TE_{21} mode



(e) TE_{41} mode



(f) TM_{01} mode

Figure 5. Modal electric field patterns for octo-ridge loaded cylindrical waveguide.

shorted by the deepening ridges, displays a decreasing cut-off frequency with growing h/R, until at $h/R > 0.8$ it becomes degenerate with the TE_{11W} mode. This degeneracy occurs because as h/R tends to unity the dual ridge waveguide becomes two semi-circular cross-section waveguides. At the h/R limit the TE_{21R} mode and the TE_{11W} mode fields are identical within each semi-circular region, but while the TE_{21R} mode fields are in anti-phase across the closed ridge, the TE_{11W} mode fields are in-phase. Hence they are degenerate.

The close proximity of the cut-off frequencies at low values of h/R for the TE_{11W} mode and the TE_{11R} mode, does not mean the guide bandwidth is negligible here. At low h/R the behaviour of the two components of the TE_{11} mode is little different to the situation in empty waveguide, where the orthogonal component of the mode does not limit bandwidth, but may cause modal precession if it is excited — by a flaw in the waveguide, for example. At $h/R < 0.3$, the bandwidth of the dual-ridge guide is dictated by the difference in cut-off frequency between the TE_{11R} mode and the TM_{01W} mode, while for $h/R > 0.3$ the TE_{21R} mode is the significant higher order mode. For $h/R > 0.8$ the bandwidth can be very large indeed as f_{c11R} tends to zero.

3.2. Quad-ridge Waveguide

For quad-ridge cylindrical waveguide (Fig. 3), the modal characteristics of the critical low order modes, are not too dissimilar to the dual-ridge modal pattern, except for two important instances. Firstly, the four-fold symmetry of the quad-ridge structure means that the degenerate pair of TE_{11} modes in the empty circular waveguide does not produce different modes in the ridged structure. The TE_{11R} mode also comprises a degenerate pair, since it can exist in two forms, with fields which are spatially orthogonal. However, the more crucial change, from a bandwidth perspective, is the quite different behaviour of the TE_{21} modes. The TE_{21W} mode, can again set-up with a strong radially directed E-field in the inter-ridge spaces, and thus it continues to display a cut-off frequency which is independent of ridge depth. However, the TE_{21R} mode with its E-field concentrated in the ridge gaps, as with dual-mode ridge waveguide, no longer becomes degenerate with the TE_{11W} mode as $h/R \Rightarrow 1$, since this mode no longer exists. It becomes degenerate with the TE_{11R} mode. This difference has a crucial impact on bandwidth. As $h/R \Rightarrow 1$, instead of the bandwidth expanding in the manner of the fundamental mode in dual-ridge guide, the ridge mode in quad-ridge waveguide displays negligible bandwidth. In fact for this structure optimum bandwidth ($\approx 40\%$) occurs at $h/R \approx 0.23$.

3.3. Octo-ridge Waveguide

For octo-ridge waveguide the modal relationships become more complex, largely because empty waveguide modes of a higher order than TE_{21} become significant (Fig. 4). Now, TE_{31} and TE_{41} modes have an impact on bandwidth. It is clear from Fig. 4 (where subscript W modes are denotes by dotted curves) that TE_{11R}, TM_{01W} and TE_{21R} modes are substantially unchanged by the additional ridges in the octo-ridge structure. On the other hand, the TE_{21W} mode has disappeared since, no matter how it is orientated, it is suppressed by the extra ridges. The E-fields of the TE_{01} and TM_{11} modes in the empty cylindrical waveguide are relatively weakly disrupted by the ridges, and hence they form the waveguide type modes TE_{01W} and TM_{11W} in the octo-ridge structure. The really significant new modes are the TE_{31R} and TE_{41R} modes. That the TE_{31} mode, which is the next higher mode in empty cylindrical waveguide, should be formed in the octo-ridge geometry is a surprise because the modal symmetry and the structure symmetry are seemingly incompatible. Nevertheless, it acquires a field pattern in the octo-ridge structure which becomes quasi-degenerate with the TE_{11R} mode at large values of h/R (Fig. 4 — curve with + markers). For $h/R \Rightarrow 0$, the TE_{12} and TE_{41} modes in the empty cylindrical waveguide are degenerate. However, as the ridge height (h) is increased it is evident that the TE_{12} mode fields are much less distorted by the ridges than those of the TE_{41} mode (see Figs. 5(c) and (e)). Consequently the TE_{12W} cut-off curve (curve with box markers) initially drops very slowly with increasing h/R, then levels off. On the other hand the TE_{41} mode, with every radial E-field branch (Fig. 5(c)) 'shorted' by the ridges, is heavily 'damped' by the ridge structure and its cut-off frequency drops rapidly as h/R is enlarged (curve with unfilled triangle annotation). At $h/R > 0.8$ it also becomes quasi-degenerate with the TE_{21R} and TE_{11R} modes. These degenerate modes at $h/R \Rightarrow 1$ all have essentially quasi-TEM mode field patterns in the ridge gaps. They differ only in E-field direction between adjacent ridges. For example, while TE_{41R} has the field reversing from one ridge to the next (8 times), the TE_{11R} mode has only two reversals. Hence the observed modal degeneracy in octo-ridge waveguide as $h/R \Rightarrow 1$. Apart from the noted field reversals, the similarities between the E-field distributions, for the TE_{11R} , TE_{21R} and TE_{41R} modes, in the vicinity of the ridges (see Fig. 5(b), (d), (e)) are quite evident, (it becomes more apparent as the ridges are enlarged), and these figures provide pictorial confirmation of the nature of the degeneracy, highlighted in this paper.

4. CONCLUSIONS

Full-wave electromagnetic modelling of ridged cylindrical waveguide has shown that degeneracy of quasi-TEM ridge modes severely impairs bandwidth. Whereas dual-ridge cylindrical waveguide can provide very wide bandwidth performance when the fundamental mode is the quasi-TEM mode, this mode requires deep ridges. However, electromagnetic simulations demonstrate that in quad-ridge and octo-ridge structures it is no longer possible to operate with deep ridges because of previously unrecorded modal degeneracies as $h/R \Rightarrow 1$, which severely limit bandwidth. In fact the optimum single mode operating bandwidth for the quad-ridge structure occurs at $h/R \approx 0.23$ while for the octo-ridge geometry is occurs at $h/R \approx 0.21$. These values are approximate since they are weakly dependent on ridge width (w).

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