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BROADBAND EXPERIMENTAL CHARACTERIZATION OF ARTIFICIAL MAGNETIC MATERIALS BASED ON A MICROSTRIP LINE METHOD

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Abstract—A broadband method is introduced to measure the effective constitutive parameters of artificial magnetic materials. The method is based on the microstrip line topology, thus making it easy to retrieve the constitutive parameters over a wide band of frequencies. To demonstrate the effectiveness of this method, artificial magnetic materials with Fractal Hilbert inclusions are fabricated and characterized. Good agreement between the experimental and numerical simulation results verifies the accuracy of the proposed method.

1. INTRODUCTION

Metamaterials have attracted enormous interest among electromagnetic research groups as they provide electromagnetic properties which do not exist in natural materials. In low-loss applications at microwave frequencies, natural materials are limited to nonmagnetic dielectrics. When relatively high permeability is required, the choices are limited to ferrite composites which provide high level of magnetic loss [1–3]. Therefore, artificial magnetic materials are designed to provide any desirable permeability and permittivity at microwave frequencies [4–10].

Accurate retrieval of the constitutive parameters of artificial media is typically considered a challenging task due to the inherently strong resonant behavior of such media. This is true in general, but more so in antenna applications. Since antennas are resonant devices as well, a slight error in the prediction of the frequency variation of constitutive parameters can render the antenna inoperable [10].

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So far several experimental methods have been reported for retrieval of parameters of artificial structures such as the resonator method [11, 12], the free space method [13–15], and the waveguide method [16, 17]. Each of these methods has its own advantages and disadvantages. The resonator method provides high accuracy but it is inherently narrowband, and individual measurement setups should be prepared for retrieval of the constitutive parameter at each single frequency; therefore, it is not a good candidate for measuring artificial magnetic materials which are dispersive in nature.

Acceptable accuracy was reported for the free space approach; however, an expensive setup of two horn antennas, combined with lens assemblies mounted on the horns, are required to generate plane waves [14]. Furthermore, Since standard horn antennas have limited operation bandwidth, to test artificial magnetic materials operating at different frequency bands, different setups are needed (for example, an antenna used for testing a structure which operates at 2 GHz cannot be used to test another structure which operates at 3 GHz).

In the waveguide method, the sample of artificial media is placed at the cross section of a waveguide and its constitutive parameters are calculated from reflected and transmitted waves [16, 17]. The setup needed for this method is more affordable when compared to free space method, but again to test artificial magnetic materials operating at different frequency bands, different setups are needed. In addition, a large size sample of artificial magnetic materials is needed to fill the entire cross section of the waveguide. This would be a severe requirement when testing artificial materials operating at the lower band of microwave region, say few hundred MHz (for an artificial material operating at 500 MHz, the sample size would be approximately $0.5\,\mathrm{m} \times 0.2\,\mathrm{m}$).

In this work, a broadband microstrip line-based method is proposed to measure the constitutive parameters of artificial magnetic materials. The advantage of this method over previously developed techniques is that the characterization can be performed using a simple inexpensive microstrip cell, and the same cell can be used for artificial magnetic materials operating at different frequency bands. In this method, there are fewer restrictions on the size of the sample under test.

Various microstrip line-based retrieval methods with different configurations were reported in the literature for characterization of natural materials [18–23], however, to the authors' knowledge, no microstrip line-based method is reported for characterization of artificial magnetic materials which are in general biansotropic and dispersive.

The organization of this paper is as follows: In Section 2, the retrieval method is introduced and explained. In Section 3, the proposed method is used to measure constitutive parameters of artificial magnetic materials with Fractal Hilbert inclusions. Measurement results are presented and comparison is made with the numerical simulation results.

2. CHARACTERIZATION METHOD

Artificial magnetic materials are in general anisotropic and dispersive; therefore a retrieval method should determine the permittivity and permeability tensors at different frequencies. The currently available artificial structures are realized by using planar inclusions with a specific topology in, say the x-y plane. These planar structures provide enhanced permeability only in the direction normal to the plane of the inclusion and enhanced permittivity in the directions tangent to the plane (for a discussion on the design aspects of artificial magnets, the reader is referred to [4–9] and references therein).

In this paper, without any loss of generality, it is assumed that the inclusions lie in the x-z plane. In this case the permeability of the artificial structure is described by the following tensor:

$$\mu(\omega) = \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu_y(\omega) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Figure 1 shows the proposed setup for permeability retrieval consisting of a shorted microstrip line with the substrate of the line being the artificial media under test. By measuring the input impedance of the shorted microstrip line over the frequency range of interest, the permeability of the artificial media can be calculated. For the quasi-TEM dominant mode in the microstrip line (see Fig. 1,) the dominant magnetic field in the substrate will be in the y direction. Therefore this configuration can be used for the retrieval of μ_y. For the quasi-TEM mode, the dominant mode in the microstrip line, the input impedance of the shorted line is given by

$$Z_{in} = Z_{0M} \sqrt{\frac{\mu_{eff,y}}{\epsilon_{eff,x}}} \tanh\left(jl\beta_0\sqrt{\mu_{eff,y}\epsilon_{eff,x}}\right), \tag{2}$$

where l and β_0 are the length of the shorted microstrip line and the propagation constant of the air, respectively. $\epsilon_{eff,x}$, and $\mu_{eff,y}$ are the x-directed effective permittivity, and y-directed effective permeability of the microstrip cell respectively. Z_{0M} is the characteristic impedance of the microstrip cell for $\mu_r = \epsilon_r = 1$, given by [24]

$$Z_{0M} = \frac{120\pi}{\dfrac{w}{h} + 1.393 + 0.677 \ln\left(1.444 + \dfrac{w}{h}\right)} \tag{3}$$

where w is the width of the microstrip line and h is the height of the substrate. The $\epsilon_{eff,x}$ in Eq. (2) can be calculated as [25]

$$\epsilon_{eff,x} = \begin{cases} \dfrac{1+\epsilon_x}{2}\left(\dfrac{A}{A-B}\right)^2 & \dfrac{w}{h} \leq 2 \\ \epsilon_x \left(\dfrac{C-D}{C}\right)^2 & \dfrac{w}{h} > 2 \end{cases} \tag{4}$$

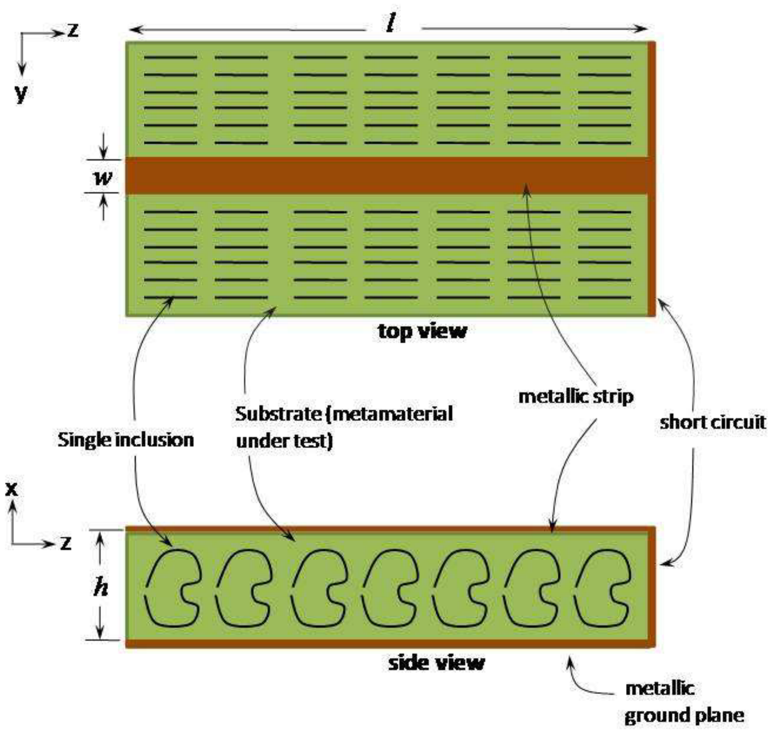

Figure 1. The setup configuration used for extraction of the permeability of the artificial media.

where

$$
\begin{aligned}
A &= \ln\left(\frac{8h}{w}\right) + \frac{1}{32}\left(\frac{w}{h}\right)^2 \\
B &= \frac{1}{2}\left(\frac{\epsilon_x - 1}{\epsilon_x + 1}\right)\left[\ln\left(\frac{\pi}{2}\right) + \frac{1}{\epsilon_x}\ln\left(\frac{4}{\pi}\right)\right] \\
C &= \frac{w}{2h} + \frac{1}{\pi}\ln\left(2\pi e\left[\frac{w}{2h} + 0.94\right]\right) \\
D &= \frac{\epsilon_x - 1}{2\pi\epsilon_x}\left[\ln\left(\frac{\pi e}{2}\left(\frac{w}{2h} + 0.94\right)\right) - \frac{1}{\epsilon_x}\ln\left(\frac{e\pi^2}{16}\right)\right],
\end{aligned}
$$

and ϵ_x is the x-directed permittivity of the artificial media. ϵ_x will be measured using a parallel plate metamaterial capacitor as proposed in [8]. By measuring the capacitance, the permittivity can directly be calculated.

Analytical models and simulation results have shown that the permittivity of artificial magnetic materials is independent of frequency [4–9]. Therefore a low-frequency measurement of a parallel-plate metamaterial capacitor would give a good estimate of the high-frequency permittivity if the host dielectric itself is non-dispersive. Additionally, according to the classical image theory,using only one period of the artificial unit cells in the area between the two metallic parallel plates can mimic the behavior of an infinite array of unit cells, which is the default assumption in the analysis and design of artificial structures. These aspects, therefore, makes the method presented in [8] highly robust and well-suited for metamaterial characterization. It is interesting to note, however, that the authors of [8] observed a rather large difference between the analytically estimated and measured permittivity values (a difference of more than 30%). The large discrepancy is in fact due to the approximations used in the derivation of the analytical formula. For the same artificial structure measured in [9], we have performed a full wave numerical simulation using the simulation setup discussed in [9] and obtained a difference between simulation and measurements of less than 7%.

Next, the input impedance of the shorted microstrip line is measured, and by solving Eq. (2), the unknown parameter, $\mu_{eff,y}$ is determined. The effective permeability, in general, is a complex number, and Eq. (2) should be solved for both the real part, $\mu'_{eff,y}$, and the imaginary part, $\mu''_{eff,y}$. Then, after finding the effective permeability, the relative permeability of the substrate, μ_y, is calculated by solving the following equations [25]:

$$\mu'_{eff,y} = \begin{cases} \dfrac{2\mu'_y}{1+\mu'_y}\left(\dfrac{A-B'}{A}\right)^2 & w/h \leq 2 \\ \mu'_y\left(\dfrac{C}{C-D'}\right)^2 & w/h > 2 \end{cases} \tag{5}$$

$$\mu''_{eff,y} = \mu''_y \frac{\mu'_{eff,y} - \left(\mu'_{eff,y}\right)^2}{\mu'_y - \left(\mu'_y\right)^2} \tag{6}$$

where A and C were introduced before and B' and D' are given by:

$$B' = \frac{1}{2}\left(\frac{1-\mu'_y}{1+\mu'_y}\right)\left[\ln\left(\frac{\pi}{2}\right) + \mu'_y \ln\left(\frac{4}{\pi}\right)\right]$$

$$D' = \frac{1-\mu'_y}{2\pi}\left[\ln\left(\frac{e\pi}{2}\left(\frac{w}{2h}+0.94\right)\right) - \mu'_y \ln\left(\frac{e\pi^2}{16}\right)\right],$$

where μ'_y and μ''_y are the real and imaginary parts of the permeability of the artificial media respectively.

Eq. (5), and Eq. (6) are derived in [24, 25] using conformal mapping technique where the permeability, μ'_y, was assumed positive. Therefore, these equations cannot be used at over the frequency range where the permeability is negative. However, since artificial magnetic materials are designed to operate at frequencies over which the permeability is positive, characterization of the permeability behavior at those frequencies would be sufficient.

Eq. (5), and Eq. (6) are nonlinear equations and can be solved by several numerical methods such as the Gauss-Newton method, the Levenberg-Marquardt method, the Trust-Region Dogleg method, ... etc. [26]. In this work, the Trust-Region Dogleg method is used. The process of characterization is summarized in the chart shown in Fig. 2.

3. MEASUREMENT RESULTS

To test the accuracy of the proposed method for extracting the permeability of artificial magnetic materials, we consider the structure with Fractal Hilbert2 inclusions reported in [9]. One unit cell of this structure is shown in Fig. 3. The inclusion consists of a conducting trace having a width of $w = 0.180$ mm and separation between traces of $s = 0.180$ mm. This artificial media was fabricated and measured, and

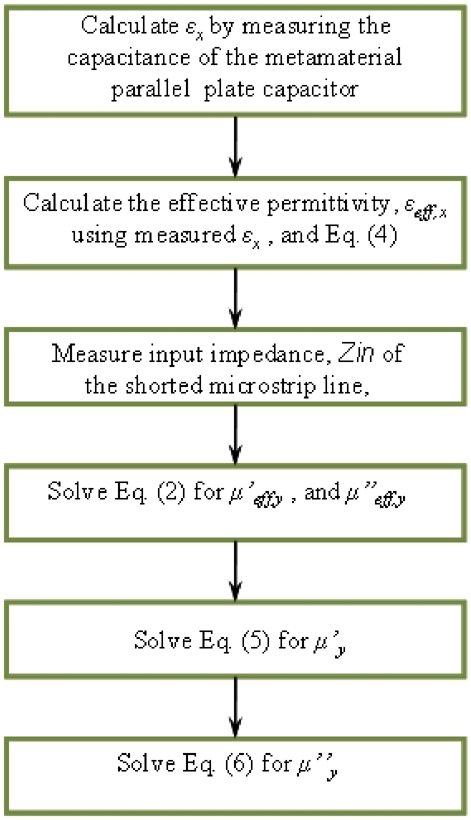

Figure 2. Characterization process.

in this section, we compare the measurement results with numerical simulation results.

The numerical results are obtained using Ansoft HFSS10, a full wave numerical simulation tool based on the finite element method, and the numerical setup reported earlier in [9]. In the numerical setup, a unit cell of the artificial material combined with periodic boundary conditions are used to mimic an infinite slab of artificial materials. For numerically extraction of constitutive parameters, plane wave analysis is used, and parameters are extracted from the reflected and transmitted waves from the unit cell [9].

Using printed circuit technology, a strip of 6 unit cells of Fractal Hilbert2 inclusions were fabricated on an FR4 substrate with $\epsilon_r = 4.4$, and $\tan\delta = 0.02$ (See Fig. 4). Forty of these strips were then stacked in the y direction to provide a three-dimensional substrate. Due to

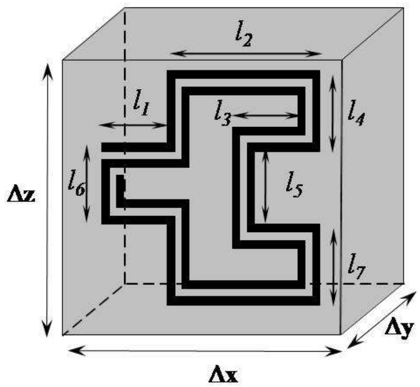

Figure 3. Fractal Hilbert2 inclusion used for constructing artificial magnetic material. $l_1 = l_3 = l_4 = l_5 = l_6 = l_7 = 3.03\,\text{mm}$, $l_2 = 6.06\,\text{mm}$, $\Delta y = 3,028\,\text{mm}$, $\Delta x = \Delta z = 11\,\text{mm}$.

Figure 4. A single strip containing 6 unit cells of inclusions fabricated using printed circuit board technology.

the thickness of the metal inclusions, an average air gap of 50 μm develops between the strips in the stacking process. The air space while unavoidable in the fabrication process is nevertheless measurable, so it can be easily included in the design. Using this substrate and conducting plates, a parallel-plate metamaterial capacitor was fabricated, and by measuring its capacitance, the permittivity was extracted. According to this measurement, the permittivity for this artificial magnetic substrate is calculated as $\epsilon_x = 9.6$. The numerical full-wave simulation results in $\epsilon_x = 9.58$, demonstrating strong agreement with measurements. The 50 μm air gap is included in the simulation.

The same substrate that is used for permittivity measurement is used as a substrate of the shorted microstrip line to extract the permeability. The fabricated fixture used for permeability measurement is shown in Fig. 5. The fabricated substrate has dimensions of 12, 8.2, and 1.1 cm in the y, z, and x directions, respectively. For the quasi-TEM dominant mode, the **H** and **E** fields in

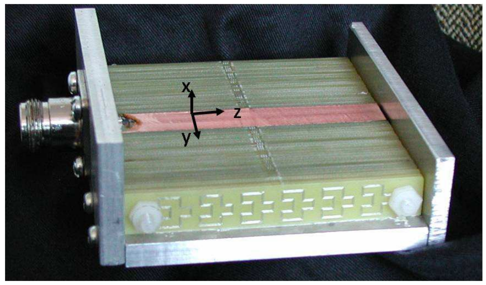

Figure 5. The fabricated fixture used for permeability measurement.

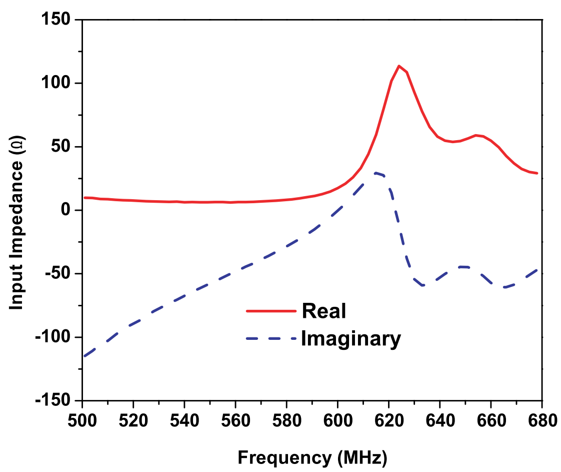

Figure 6. Measured input impedance.

the substrate will be in the y and x directions, respectively. Therefore this configuration can be used for retrieval of μ_y.

Using a vector network analyzer, the complex input impedance of the shorted microstrip line shown in Fig. 5 is measured over the frequency range of 500–680 MHz (See Fig. 6). Then using the measured value of $\epsilon_x = 9.6$ and measured impedance, μ_y is extracted by the method explained in Section 2. The real and imaginary parts of the measured y-directed permeability are shown in Fig. 7, and Fig. 8. In these figures, the measurement results are compared with the numerical simulation results. The 50 µm air gap was also included in the simulation.

The shaded area in Fig. 7 and Fig. 8 determines the frequencies over which the real part of the permeability is negative. Over this frequency range corresponding to frequencies higher than 638 MHz,

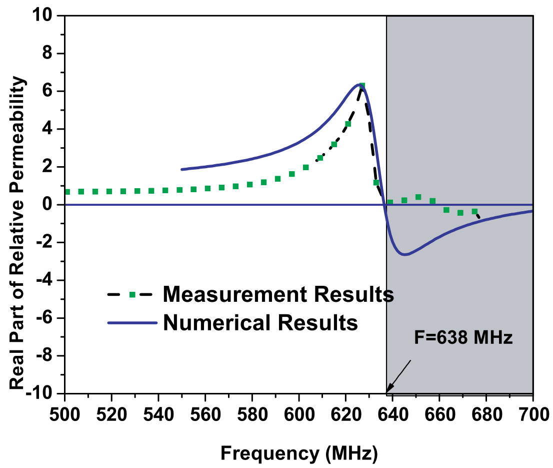

Figure 7. The measured and numerically simulated real part of the permeability for the artificial magnetic material shown in Fig. 5.

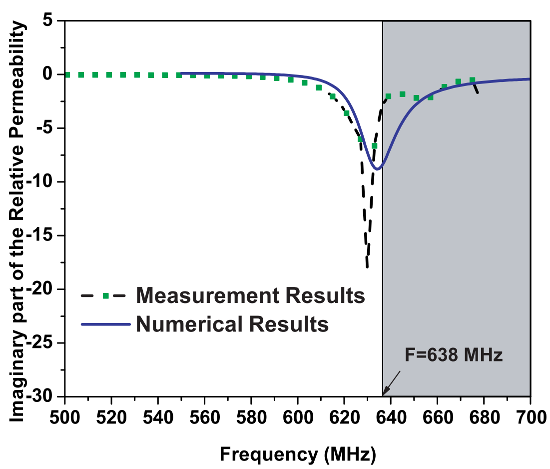

Figure 8. The measured and numerically simulated imaginary part of the permeability for the artificial magnetic material shown in Fig. 5.

as explained before, the measurement results are not valid. Over the frequency range where the real part of the permeability is positive, good agreement is observed between the simulation and measurement results.

As briefly discussed above and more extensively in [9], in the numerical analysis used in [9], periodic boundary conditions are used to mimic an infinite number of unit cells. However, in practice we can realize only finite number of unit cells. For example in the setup used in this work (see Fig. 5), the fabricated substrate contains 6 unit cells of inclusions in the z direction, and only one unit cell in the x direction. It can be predicted that increasing the number of unit cells provide higher homogeneity in the fabricated substrate which will result in a better agreement between numerical and measurement results. On the other hand, in a wide class of applications such as antenna miniaturization, only one unit cell is used in the x direction to avoid high profile substrates [8, 10].

4. CONCLUSION

A method based on the shorted microstrip line was proposed for characterization of artificial magnetic materials. The retrieval method for permeability extraction was introduced and explained. An artificial substrate based on Fractal Hilbert inclusions was fabricated and characterized using the proposed method. Measurement results were presented and compared with numerical results of a full wave simulation. Numerical results were obtained from a plane wave analysis using a finite element simulation tool. A strong agreement was observed between measurement and numerical results.

The primary advantage of the method introduced in this paper, when compared to previous methods, is its relatively low cost (in comparison to the free space or the waveguide methods) and its capability of extracting the permeability over a wide band of frequencies (in comparison to the resonator method).

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