NEW CONCEPTS IN ELECTROMAGNETIC JERKY DYNAMICS AND THEIR APPLICATIONS IN TRANSIENT PROCESSES OF ELECTRIC CIRCUIT

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Abstract—In this paper, jerk function for a transient process of RL circuit is investigated. Some new concepts such as time rate of change of induced emf, time rate of change of displacement current, and Appell function have been introduced for the first time in electromagnetic jerky dynamics. The problems on Appell function of several simple models in electromagnetic jerky dynamics are discussed. In the last conclusions and remarks are also presented.

1. INTRODUCTION

Jerk is a familiar term in ordinary language, though its exact origin as a physical concept is obscure. In 1928, engineer Melchior defined the concept of jerk for the first time. Schot [1] and Sandin [2] also discuss the jerk. Jerk is a mechanical term for a specific aspect of motion: the rate of change of acceleration with time. The concept is a natural extension of a line of thinking that originated with Galileo. In 1997, Linz [3] and Sprott [4] generalize the conception of jerk into the general jerk function (the third-order time derivative of any independent variable). The jerk functions that reflect jerky motion are determined by third-order differential equations. The new systems of jerk function are called jerky dynamicical systems. As vonBaeyer [5] said, the articles with these funny titles of jerk function and jerky motion illustrate in a particularly vivid way the revolution that is transforming the ancient study of mechanics into a new science — one that is not just narrowly concerned with the motion of physical

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bodies, but that deals with changes of all kinds. As we all know, the instability often associated with a change of a change of a change. Even the important quantity of jerk for describing the financial well-being of a worker can faithfully portray a change of a change of a change of a person's financial history.

Following Linz, Sprott and vonBaeyer, in this paper, some new concepts in electromagnetic jerky dynamics will firstly be introduced. The problems of electromagnetic jerky dynamics will be discussed. That is to say, the concepts of jerky dynamics will be extended and discussed in the transient process of some circuit.

2. SOME NEW CONCEPTS IN ELECTROMAGNETIC JERKY DYNAMICS

2.1. The Jerk Function of a Transient Process

We consider a simple circuit with a resistor (R), an inductor (L), a battery (ε) and a switch (K) connected in a loop. When the switch is on, the current change from zero to the maximum value $I_m = \varepsilon/R$ for the electromagnetic inertia of the inductor. This is a kind of transient process of RL circuit. In the process being discussed, according to the second Kirchhoff law, the sum of the voltage drops around any loop is zero. Then

$$\varepsilon - L\dot{I} = IR \tag{1}$$

with the dot denoting the time derivative. Integrating the Equation, the formula of increasing current is as follow.

$$I = I_m \left(1 - e^{-\frac{R}{L}t}\right) \tag{2}$$

Contrasting the formula of increasing velocity in the model of terminal velocity

$$v = v_m \left(1 - e^{-\frac{\gamma}{m}t}\right) \tag{3}$$

Eq. (2) and Eq. (3) have the identical form. There is an analogy between increasing velocity and increasing current. Making twice time derivative to Eq. (3), the jerk function of the process of increasing velocity can be written as

$$j = \ddot{v} = -\frac{v_m \gamma^2}{m^2} e^{-\frac{\gamma}{m}t} \tag{4}$$

This shows that the process of increasing velocity is a kind of motion with variable acceleration. Making twice time derivative to Eq. (2),

the jerk function of the process of increasing current can be written as

$$j = \dddot{q} = \ddot{I} = -\frac{I_m R^2}{L^2} e^{-\frac{R}{L}t} \tag{5}$$

The third order time differential of charge is nonzero. This shows that the process of increasing current is also a kind of jerky motion.

2.2. Time Rate of Change of Induced Emf and Magnetic Appell Function

For further interpreting the physical meaning of jerk function, we make the time derivative of electromagnetic induction law $\varepsilon_i = -L\dot{I}$ and have the expression

$$\dot{\varepsilon}_i = -L\ddot{I} = -Lj \tag{6}$$

with $\dot{\varepsilon}_i$ denoting the time rate of change of emf. Eq. (6) is called the formula of time rate of change of induced emf. This is exactly the physical meaning of jerk function $j = \ddot{I}$ that is the electromagnetic induction changing with time. By the way, even having no RL circuit, there are the problems of electromagnetic induction changing with time. It embodies the time rate of change of eddy electric field and has special significance in the betatron.

There is still an analogy between $j = \ddot{I}$ and $j = \dddot{v}$. $\dot{F} = m\ddot{v} = mj$ is called the formula of time rate of change of force whose physical meaning is the force changing with time. Integrating $\dot{F} = m\ddot{v} = mj$ with respective to the velocity, we have

$$\int_{v_0}^{v} \dot{F} dv = \int_{v_0}^{v} m\ddot{v} dv = \int_{\dot{v}_0}^{\dot{v}} m\dot{v} d\dot{v} = \frac{1}{2}m\dot{v}^2 - \frac{1}{2}m\dot{v}_0^2 = A_k - A_{k0} \tag{7}$$

Eq. (7) is the theorem of Appell function in mechanical jerky process of motion with variable acceleration. Where $A_{k0} = \frac{1}{2}m\dot{v}_0^2$ and $A_k = \frac{1}{2}m\dot{v}^2$ are respectively the initial and the final kinetic Appell function [10]. Following Eq. (7) and integrating Eq. (6) with respective to the current, we have

$$\int_{I_0}^{I} \dot{\epsilon}_i dI = \int_{I_0}^{I} -L\ddot{I} dI = \int_{\dot{I}_0}^{\dot{I}} -L\dot{I} d\dot{I} = -\left(\frac{1}{2}L\dot{I}^2 - \frac{1}{2}L\dot{I}_0^2\right) = -(A_m - A_{m0}) \tag{8}$$

Eq. (8) is the theorem of Appell function on magnetic jerky process in inductor L. Where $A_{m0} = \frac{1}{2}L\dot{I}_0^2$ and $A_m = \frac{1}{2}L\dot{I}^2$ are the initial and the final magnetic Appell function respectively. We know that the dimension of self inductance is $L^2MT^{-2}I^{-2}$ with L, M, T and I

denoting the length, the mass, the time and the current. Obviously there is an analogy between the magnetic Appell function and the kinetic Appell function $A_k = \frac{1}{2}m\dot{v}^2$. And they have the same dimension of L^2MT^{-4}. Because the magnetic and kinetic Appell function have the same dimension, they can be classified into the state function of jerky dynamics.

2.3. Time Rate of Change of Displacement Current and Electric Appell Function

In electromagnetism the capacitance (C) with voltage (U) has electric energy $W_e = \frac{1}{2}CU^2$. And the inductance (L) with current (I) has magnetic energy $W_m = \frac{1}{2}LI^2$. Following definition of magnetic Appell function $A_m = \frac{1}{2}L\dot{I}^2$ from Eq. (8), can we define electric Appell function? The answer is affirmative.

Now we consider the charging and discharging process of capacitor. Let the electric Appell function be $A_e = \frac{1}{2}C\dot{U}^2$, we find that the dimension of A_e is L^2MT^{-4}. Where the dimensions of the capacitance (C) and the time rate of change of voltage $(\dot{U})$ are $L^{-2}M^{-1}T^4I^2$ and $L^2MT^{-4}I^{-1}$ respectively. The dimension of A_e is exactly the dimension of A_k and A_m. Following Eqs. (7) and (8) and integrating time rate of change of displacement current $(\dot{I}_d)$ with respective to voltage (U), we have

$$
\int_{U_0}^{U} \dot{I}_d dU = \int_{U_0}^{U} C\frac{d\dot{U}}{dt} dU
$$

$$
= \int_{\dot{U}_0}^{\dot{U}} C\dot{U}d\dot{U} = \frac{1}{2}C\dot{U}^2 - \frac{1}{2}C\dot{U}_0^2 = A_e - A_{e0} \qquad (9)
$$

Eq. (9) is the theorem on Appell function in electric jerky process in capacitor C. Where $A_{e0} = \frac{1}{2}C\dot{U}_0^2$ and $A_e = \frac{1}{2}C\dot{U}^2$ are respectively the initial and the final electric Appell function. That is to say, the process of charging and discharging of capacitor is also classified into jerky motion. So Appell function can be defined as common measurement of jerky motion.

2.4. The Appell Function of Electric Field and Magnetic Field

According to the electric Appell function $A_e = \frac{1}{2}C\dot{U}^2$ and using a parallel plate capacitor, we further have the Appell function of electric

field. That is as follow

$$A_e = \frac{1}{2}C\dot{U}^2 = \frac{1}{2}\frac{\epsilon s}{d}\dot{E}^2 d^2 = \frac{1}{2}\dot{D}\dot{E}V \tag{10}$$

where $V = sd$ is the volume between two plates, and ϵ denoting permittivity. From Eq. (10), the density of Appell function of electric field can be defined as

$$a_e = \frac{A_e}{V} = \frac{1}{2}\dot{D}\dot{E} \tag{11}$$

and from Eq. (11), we can compute Appell function of electric field for any area

$$A_e = \int_V a_e dV \tag{12}$$

By the same token, according to the magnetic Appell function $A_m = \frac{1}{2}L\dot{I}^2$ and applying to a solenoid, we further have the Appell function of magnetic field. That is as follow

$$A_m = \frac{1}{2}L\dot{I}^2 = \frac{1}{2}\mu n^2 V \dot{I}^2 = \frac{1}{2}\dot{B}\dot{H}V \tag{13}$$

with V denoting the volume of the solenoid, μ denoting permeability and n denoting the number of turns per unit length. From Eq. (13), the density of Appell function of magnetic field can be defined as

$$a_m = \frac{A_m}{V} = \frac{1}{2}\dot{B}\dot{H} \tag{14}$$

and from Eq. (14), we can compute Appell function of magnetic field for any area
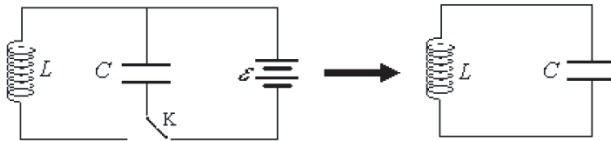
$$A_m = \int_V a_m dV \tag{15}$$

Figure 1. Scheme for LC electromagnetic oscillation.

3. SIMPLE MODELS IN ELECTROMAGNETIC JERKY DYNAMICS

3.1. LC Electromagnetism Oscillation

We consider a simple circuit with inductor (L), capacitors (C), a battery (ε) and a switch (K) connected in a loop. See Figure 1. We take the three variables, $I = i_L = i_C$, $U = v_C$ and $q = q_C$. The differential equations of LC electromagnetic oscillation is

$$\begin{cases} L\dfrac{dI}{dt} = -U \\ C\dfrac{dU}{dt} = I \end{cases} \tag{16}$$

and $CU = q$, where the constants L and C denote the inductance and the capacitance respectively.

The first-order differential equations (16) can be cast into the second-order differential equation as follow

$$\frac{d^2I}{dt^2} + \omega^2 I = 0 \tag{17}$$

with $\omega^2 = \frac{1}{LC}$. Eq. (17) is the jerk function of LC electromagnetic oscillation. The general solution of the system of Eq. (17) is

$$I(t) = a\cos(\omega t + \phi) \tag{18}$$

and

$$U(t) = (a/C\omega)\sin(\omega t + \phi) \tag{19}$$

where a and ϕ is constants controlled by initial conditions. We can give the electric energy

$$W_e = \frac{1}{2}CU^2 = \frac{a^2}{2C\omega^2}\sin^2(\omega t + \phi) \tag{20}$$

and the magnetic energy

$$W_m = \frac{1}{2}LI^2 = \frac{a^2}{2C\omega^2}\cos^2(\omega t + \phi) \tag{21}$$

and the total electromagnetic energy is conserved

$$W = W_e + W_m = \frac{a^2}{2C\omega^2} \tag{22}$$

Now we consider the problems in jerky dynamics, the jerk is

$$j = \dddot{q} = \ddot{I} = -a\omega^2 \cos(\omega t + \phi) \tag{23}$$

correspondingly the electric Appell function will be

$$A_e = \frac{1}{2}C\dot{U}^2 = \frac{a^2}{2C}\cos^2(\omega t + \phi) \tag{24}$$

and magnetic Appell function will be

$$A_m = \frac{1}{2}L\dot{I}^2 = \frac{a^2}{2C}\sin^2(\omega t + \phi) \tag{25}$$

and the total electromagnetic Appell function is also conserved

$$A = A_e + A_m = \frac{a^2}{2C} \tag{26}$$

In Figure 2, jerk, energy and Appell function changing with the time have been plotted for certain parameters.

3.2. RLC Electromagnetism Oscillation

We consider a circuit with resistors (R), inductor (L), capacitors (C), a battery (ε) and a switch (K). The simplest circuit has one element of each connected in a loop. See Figure 3.
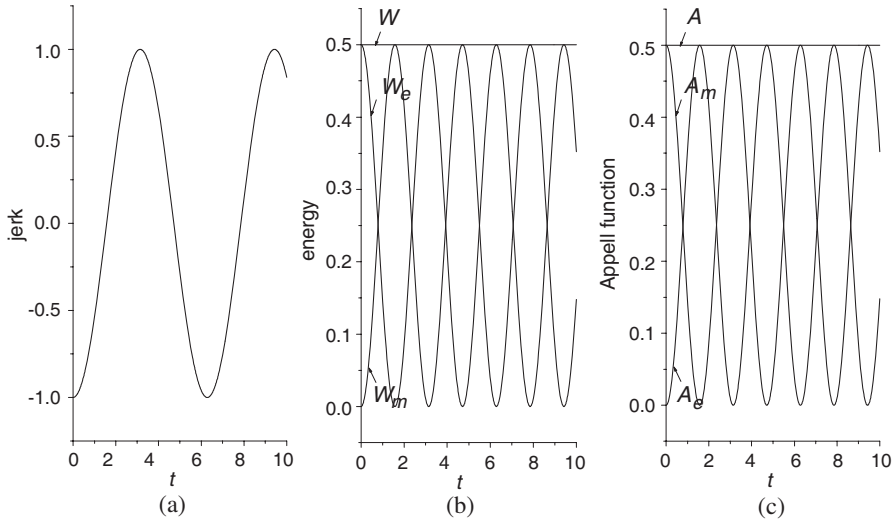

Figure 2. (a) Jerk, (b) energy and (c) Appell function when $L = 1\,\mathrm{H}$, $C = 1\,\mathrm{F}$, and $\varepsilon = 1\,\mathrm{V}$.
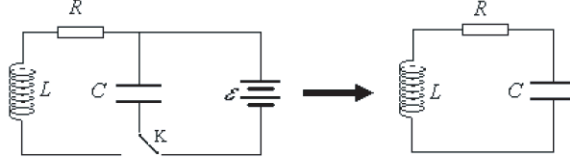
Figure 3. Scheme for RLC electromagnetic oscillation.

We take the three variables, $I = i_R = i_L = i_C$, $U = v_C$ and $q = q_C$. The differential equations of RLC electromagnetic oscillation is

$$\begin{cases} L\dfrac{dI}{dt} = -RI - U \\ C\dfrac{dU}{dt} = I \end{cases} \tag{27}$$

and $CU = q$, where the constants R, L and C denote the resistance, the inductance and the capacitance respectively. The first-order differential equations (27) can be cast into the second-order differential equation as follow

$$\frac{d^2I}{dt^2} + \gamma\frac{dI}{dt} + \omega^2 I = 0 \tag{28}$$

with $\gamma = \frac{R}{L}$, $\omega^2 = \frac{1}{LC}$. Eq. (28) is the jerk function of RLC electromagnetic oscillation.

The general solutions of the system of Eq. (28) are as follows: Case I: if $\gamma^2 = 4\omega^2$, then

$$I(t) = a_0 e^{-\gamma t/2} + b_0 t e^{-\gamma t/2} \tag{29}$$

and

$$U(t) = \frac{(2a_0\gamma + 4b_0 + 2b_0\gamma t)}{C\gamma^2} e^{-\gamma t/2} \tag{30}$$

Case II: if $\gamma^2 > 4\omega^2$, then

$$I(t) = a_1 e^{\left(-\gamma - \sqrt{\gamma^2 - 4\omega^2}\right)t/2} + b_1 e^{\left(-\gamma + \sqrt{\gamma^2 - 4\omega^2}\right)t/2} \tag{31}$$

and

$$\begin{aligned} U(t) = \ & \frac{a_1}{2C\omega^2}\left(-\gamma + \sqrt{\gamma^2 - 4\omega^2}\right) e^{\left(-\gamma - \sqrt{\gamma^2 - 4\omega^2}\right)t/2} \\ & + \frac{b_1}{2C\omega^2}\left(-\gamma - \sqrt{\gamma^2 - 4\omega^2}\right) e^{\left(-\gamma + \sqrt{\gamma^2 - 4\omega^2}\right)t/2} \end{aligned} \tag{32}$$

Case III: if $\gamma^2 < 4\omega^2$, then

$$I(t) = e^{-\gamma t/2} \left[a_2 \cos\left(\sqrt{\omega^2 - \gamma^2/4}\, t\right) + b_2 \sin\left(\sqrt{\omega^2 - \gamma^2/4}\, t\right) \right] \quad (33)$$

and

$$U(t) = e^{-\gamma t/2} \left[\left(a_2\gamma + 2b_2\sqrt{\omega^2 - \gamma^2/4}\right) \sin\left(\sqrt{\omega^2 - \gamma^2/4}\, t\right) \right.$$
$$\left. + \left(b_2\gamma - 2a_2\sqrt{\omega^2 - \gamma^2/4}\right) \cos\left(\sqrt{\omega^2 - \gamma^2/4}\, t\right) \right] / \left(2C\omega^2\right) \quad (34)$$

If the solutions of I and U are obtained, we can give the electric energy

$$W_e = \frac{1}{2}CU^2 \quad (35)$$

and the magnetic energy

$$W_m = \frac{1}{2}LI^2 \quad (36)$$

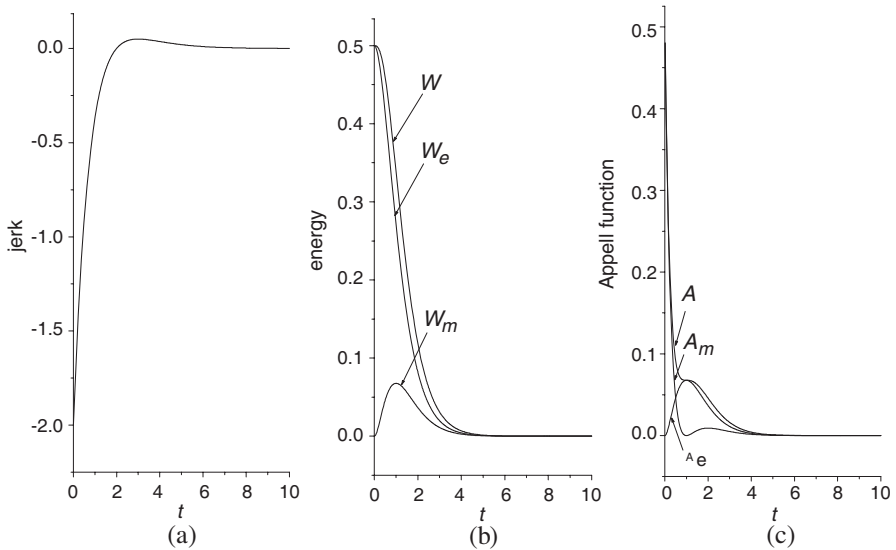but the total electromagnetic energy is not conserved because the resistor is a kind of dissipative element.

Figure 4. case $\gamma^2 = 4\omega^2$: (a) jerk, (b) energy and (c) Appell function when R=2 Ω, $L = 1$ H, $C = 1$ F, and $\varepsilon = 1$ V.
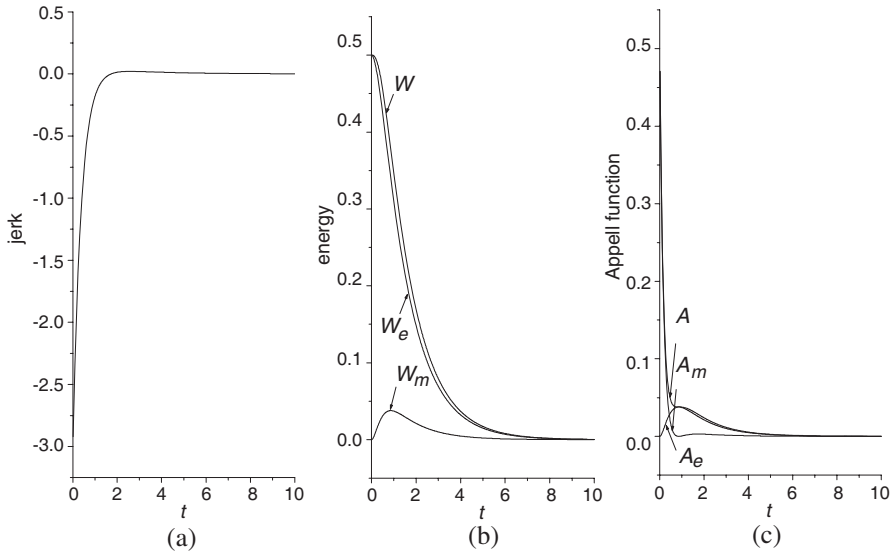
Figure 5. case $\gamma^2 > 4\omega^2$: (a) jerk, (b) energy and (c) Appell function when $R = 3\,\Omega$, $L = 1\,\text{H}$, $C = 1\,\text{F}$, and $\varepsilon = 1\,\text{V}$.
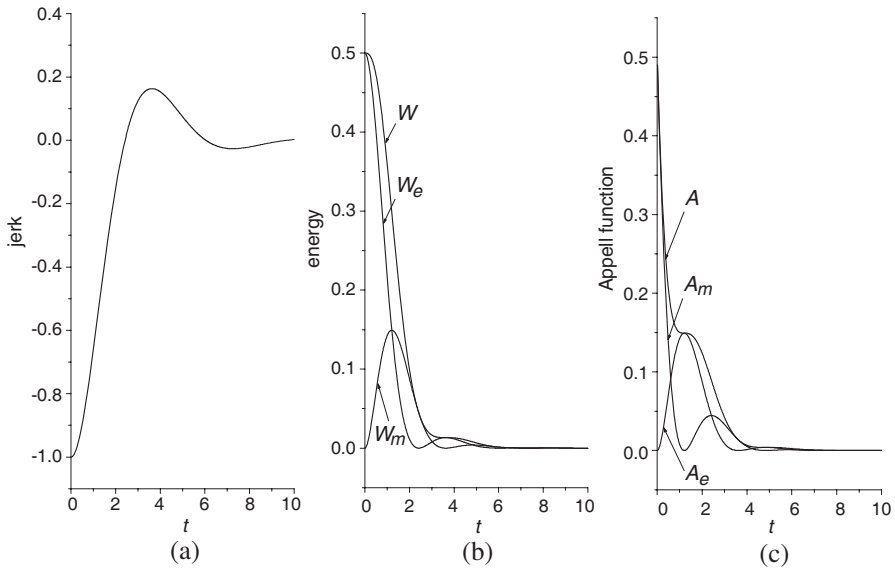

Figure 6. case $\gamma^2 < 4\omega^2$: (a) jerk, (b) energy and (c) Appell function when $R = 1\,\Omega$, $L = 1\,\text{H}$, $C = 1\,\text{F}$, and $\varepsilon = 1\,\text{V}$.

When we consider the problems in jerky dynamics, the jerk is

$$j = \dddot{q} = \ddot{I} \tag{37}$$

Similarly, the electric Appell function will be

$$A_e = \frac{1}{2}C\dot{U}^2 \tag{38}$$

and magnetic Appell function will be

$$A_m = \frac{1}{2}L\dot{I}^2 \tag{39}$$

but the total electromagnetic Appell function is also not conserved.

In Figures 4–6, jerk, energy and Appell function changing with the time have been plotted for certain parameters in different situations.

4. THREE-ORDER LAGRANGE EQUATION RELATING TO MAGNETIC APPELL FUNCTION

Since three-order Lagrange equation has been introduced by Mei F. X., people make certain relational research on the equation [6–9]. When the jerk function on electric current has been applied in a transient process of above-mentioned RL circuit, we can obtain a three-order Lagrange equation relating to magnetic Appell function whose form is as follows.

$$\frac{d}{dt}\frac{\partial A_m}{\partial \dot{I}} - \frac{1}{2}\frac{\partial A_m}{\partial I} = \dot{\varepsilon}_i \tag{40}$$

On the one hand, from the left side of Eq. (40), we see

$$\frac{d}{dt}\frac{\partial}{\partial \dot{I}}\left(\frac{1}{2}L\dot{I}^2\right) - \frac{1}{2}\frac{\partial}{\partial I}\left(\frac{1}{2}L\dot{I}^2\right) = \frac{d}{dt}\left(L\dot{I}\right) = L\ddot{I}. \tag{41}$$

On the other hand, only making time derivative to the Kirchhoff expression of this transient process, i.e., $\varepsilon - \varepsilon_i = IR$, the right side of Eq. (40) may have the following form

$$\dot{\varepsilon}_i = -R\dot{I} = -\frac{I_m R^2}{L}e^{-\frac{R}{L}t} \tag{42}$$

Substituting Eqs. (41) and (42) into Eq. (40), we obtain the same result of Eq. (5).

5. CONCLUSIONS AND REMARKS

What have we learned from the above discussion? First, the concept of jerk has been generalized into jerky dynamics. The mode of thinking enlightens us to extend some new concepts in electromagnetic jerky dynamics. The motion aroused by the time rate of change of force, induced emf, and displacement current can be summed up as the jerky motion. Second, Appell function is the important state function and general measurement of jerky motion. The birth of jerky dynamics symbolizes that Appell function is the important physics quantity. Just like energy is the important physics quantity in physics. Any physics quantity with the dimension of $L^2MT^{(-4)}$ can be called Appell function here. Third, we give some simple models in electromagnetic jerky dynamics and discuss the features of Appell function.

Appell function is a special quantity. We can find the discussion about Appell function in some Ref. [10, 14–16]. Besides Linz [11] gave a conserved quantity $K = \frac{1}{2}(\ddot{x} + A\dot{x})^2 + \frac{1}{2}B(\dot{x} + Ax)^2$ whose dimension is that of Appell function in special Newtonian jerky dynamics. In this paper, we only discuss the transformation and conservation of electromagnetic Appell function. We also find that there are some other Appell functions. For example, simple harmonic vibration of spring has potential Appell function $A_p = \frac{1}{2}k\dot{x}^2$ (where k is the stiffness constant, and $\dot{x}$ is the velocity) whose dimension is exactly that of kinetic Appell function $A_k = \frac{1}{2}m\dot{v}^2$. Obviously the mechanical Appell function $A = A_k + A_p$ is conserved. Then are there the transformation and conservation of extensive Appell function? Even after the hyperjerk system [12, 13] is introduced. Can the concept of hyperAppell function $\frac{1}{2}m(\frac{d^n x}{dt^n})^2$ be introduced? These perhaps are some important problems to be studied.

In the vocabulary of physics, the term of Appell function is not well known. But Appell function is neither moribund nor speculative. The word was used in science for a long time. Now the concept has been linked to some of the topics in jerky dynamics. It would be a pity if Appell function is ignored since its name is not well known.

Comparing the electromagnetic jerky dynamics in this paper with S (Sprott) L (Linz) G (Gottlieb) jerky dynamics, we can find the difference. SLG jerky dynamics has transcend traditional physics. It generalized the classical mechanical concept (jerk) into the mathematical jerk function or jerk equation and discussed the regular [17, 18], chaotic [3, 4] solutions and the relative rules [11, 19]. Some summarizations and remarks have been given in Ref. [20, 21]. However, in this paper, the jerk function is the third order time differential of charge ($\dddot{q}(t)$). This is the analogy and extension of

the jerk in traditional electromagnetism. This paper is devoted to generalizing and extending the concepts and rules of physics, such as from time rate of change of force $(\dot{F})$ to time rate of change of induced emf $(\dot{\varepsilon}_i)$ and time rate of change of displacement current $(\dot{I}_d)$, from mechanical Appell function to electric and magnetic Appell function. In addition, the paper did not involve the nonlinear character.

ACKNOWLEDGMENT

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