RESONANT DIFFRACTION FROM A GRATING ON A PARAMAGNETIC LAYER WITH FREQUENCY DISPERSION

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Abstract—Theoretical results on the plane electromagnetic wave diffraction from a structure as a strip periodic grating on a paramagnetic layer, the permeability of which possesses negative real part in the microwave band, are obtained using analytical regularization based on the solution to the Riemann-Hilbert problem. The effect of the resonant transmission accompanied by extremely high absorption is thoroughly studied across the frequency band of the surface waves of the paramagnetic layer placed in the biasing magnetic field. This effect is caused by the surface waves of the layer excited resonantly by the plane incident wave with the diffraction grating present. The resonant frequency is electronically tuned by the biasing magnetic field.

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1. INTRODUCTION

Artificially structured materials have the exciting potential to revolutionise communication technology because they can be engineered for applications in microwave and optical devices. Among such composite materials that are very popular now are the so called metamaterials [1, 2], which at certain frequency are capable of exhibiting anomalous refraction with negative real-valued permittivity and permeability due to their frequency dispersion. Also materials with one negative constitutive parameter (permittivity or permeability) have attracted much interest [3, 4].

The ever growing demand for better operating characteristics, extended capabilities and new functions of today's microwave devices is the driving force behind the integration of composite materials in modern microwave technology. Although periodic structures such as diffraction gratings have a long history, their resonant behaviour in combination with composite media is rather novel and theoretically interesting as well as exciting for technological developments [5–7]. A rather novel study carried out in papers [8–10] has shown that the incident wave energy can be effectively absorbed in a resonant way by the structure consisting of a diffraction grating and absorbing medium with one of the constitutive parameters negative. For simplification the frequency dispersion of the constitutive parameters was not considered in those works.

In the present paper, we study diffraction of a plane wave by a strip grating lying on an absorptive layer whose permeability is frequency dependent and has a negative real part in the microwave band. This diffraction problem is mathematically solved using an *Analytical Regularization* [11, 12] based on the Riemann-Hilbert problem method [13], and the obtained solution thus admits a rigorous and provably accurate numerical treatment in the resonant domain.

We consider a crystal of ruby as a material of layer, which, being a paramagnetic material placed in the biasing magnetic field, demonstrates a frequency dispersion of its permeability. Note the frequency dispersion is effectively controlled by the biasing magnetic field. In spite of the rather simplified character of the dispersion form chosen, this dispersion wholly satisfactorily describes the real processes in the paramagnetic materials, and moreover the general character of this frequency dispersion is typical for plenty of new composite materials. A numerical modeling of the microwave diffraction in the grating resonance band (where the wave length is comparable to the grating period) reveals that the layer supports surface waves. The grating transforms the incident plane wave into the superposition of the

partial waves of spatial spectrum. Upon certain conditions the grating surface harmonics from this superposition are able to excite the surface waves of the layer. When this happens we observe the energetically pronounced resonant regimes with interesting applied properties; one of them, the regime of resonant transmission with extremely high absorption, is studied here. The property of this regime can be used for creating novel devices, non-reflecting coatings, and microwave antenna components, the characteristics of which are electronically tunable by the biasing magnetic field.

2. PROBLEM FORMULATION

The geometry of the structure is shown in Fig. 1(a). A periodic grating of infinitely thin and perfectly conducting strips parallel to the OX axis lies on the plane layer of paramagnetic material with frequency dispersive relative permeability $\mu = \mu(\omega)$ and fixed relative permittivity ε. The grating is placed in the plane $z = 0$, its period is l, and the slot width is d. The thickness of paramagnetic layer equals h.
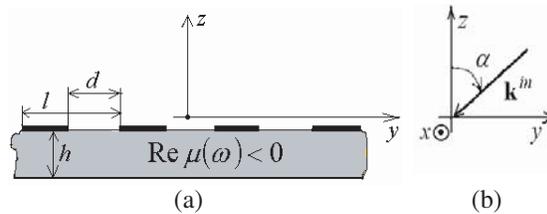

Figure 1. The structure geometry (a) and the wave incidence (b).

The interaction of electromagnetic field with paramagnetic material under the condition of electron-spin resonance results in the frequency dispersion of its permeability. The frequency behaviour of the complex valued permeability follows from the macroscopic model of the medium based on the well-known Bloch motion equation, see for example [14, 15]. In the so-called "slowly passing" resonance line approximation (i.e., for the stationary case) and for small amplitude of alternating field, the permeability of the paramagnetic material $\mu = \mu' + i\mu''$ can be presented in the following form [14, 16]:

$$\mu'(\omega) = 1 + \frac{2\pi\chi_0 \left(\frac{\omega}{\omega_0} - 1\right)}{\frac{1}{(\omega_0\tau)^2} + \left(\frac{\omega}{\omega_0} - 1\right)^2}, \quad \mu''(\omega) = \frac{2\pi\chi_0}{\frac{1}{\omega_0\tau} + \omega_0\tau\left(\frac{\omega}{\omega_0} - 1\right)^2}, \quad (1)$$

where ω is the frequency of the incident wave, $\omega_0 = \gamma B_0$ is the *Larmor* frequency of the electron-spin precession in the biasing magnetic field B_0, γ is the gyromagnetic ratio, χ_0 is the static magnetic susceptibility, and τ is a cross relaxation time.

Throughout the paper we suppose, that the paramagnetic layer has the following parameters: $\tau = 10^{-8}$ s, $\chi_0 = 1$, and $\varepsilon = 10.0$. An example of such a medium is ruby, where the electron-spin resonance phenomenon occurs for one of allowed levels. Suppose that an external magnetic field $B_0 = 2.697$ T causing precession with the frequency $\omega_0 = 4.71 \times 10^{11}$ rad s^{-1} is applied to the paramagnetic layer, and also suppose that the grating period is $l = 5 \times 10^{-3}$ m, and $d/l = 0.5$. Under these assumptions the frequency dispersion of permeability is shown in the Fig. 2 as a function of non-dimensional frequency $\Omega = \frac{l}{\lambda} = \frac{l}{2\pi}\sqrt{\varepsilon_0\mu_0}\,\omega$, where λ is the wave length in vacuum and the period l is given above. Thus, due to the magnetic interaction, in the long-wave ($l < \lambda$) and in the resonant ($l \sim \lambda$) ranges of the grating, the real part of layer permeability has negative value, while the positive imaginary part describes the losses in the medium.
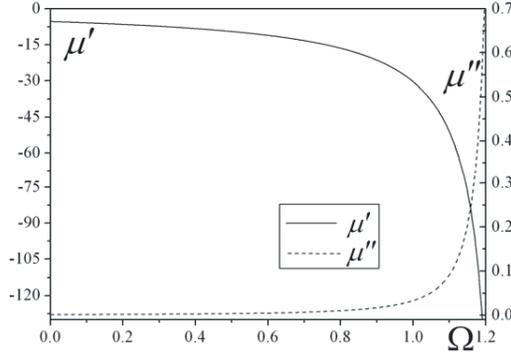
Figure 2. The frequency dispersion of the permeability $\mu = \mu' + i\mu''$ as a function of the non-dimensional frequency parameter $\Omega = \frac{1}{2\pi}\sqrt{\varepsilon_0\mu_0}\,\omega$ ($l = 5 \times 10^{-3}$ m).

The monochromatic plane wave of E-polarization ($\mathbf{E}||OX$): $\mathbf{E}^{in} = \mathbf{E}_0 \exp\left[i\left(\mathbf{k}^{in}\mathbf{r} - \omega t\right)\right]$, $\mathbf{H}^{in} = \mathbf{H}_0 \exp\left[i\left(\mathbf{k}^{in}\mathbf{r} - \omega t\right)\right]$ is obliquely incident on the grating such that

$$\mathbf{E}_0 = (\tilde{e},\ 0,\ 0)\,, \qquad \mathbf{H}_0 = \left(0,\ -\frac{\tilde{e}}{\rho}\cos\alpha,\ \frac{\tilde{e}}{\rho}\sin\alpha\right), \qquad (2)$$

where $\tilde{e}$ is a complex amplitude of the electric field, $\alpha \in [-\pi/2,\ \pi/2]$ is the angle between the incident wave vector $\mathbf{k}^{in} =$

$-\omega\sqrt{\varepsilon_0\mu_0}\,(0,\,\sin\alpha,\,\cos\alpha)$ and the OZ axis (see Fig. 1(b)), and $\rho = \sqrt{\mu_0/\varepsilon_0}$ is the impedance of free space. We seek the diffracted field.

The incident field is x-independent, and the grating extends infinitely in the x-direction, thus the problem is solvable in two-dimensional terms ($\partial/\partial x \equiv 0$). The constitutive parameters of media are scalar permittivity and permeability. Therefore, the considered problem can be separately solved for E- or H-polarization ($\mathbf{H}\|OX$) cases. One can prove that the diffracted field has the same polarization as the incident wave.

For solution existence and uniqueness, the conditions to satisfy are: Maxwell's equations, radiation condition, boundary conditions, quasiperiodicity condition, and condition of the field energy finiteness within any confined volume of space.

3. SOLUTION

The grating periodicity along the OY axis allows us to expand the solution in a Fourier series. The series substitution in the Helmholtz equation gives the field representation which coincides with the Rayleigh expansion of the diffracted field in an infinite series of partial waves of the spatial spectrum. Let the index $j = 1$ correspond to the upper and lower half-spaces, and $j = 2$ to the paramagnetic layer, then the propagation constant of the n-th harmonic is $\xi_n = 2\pi n/l - k_1\sin\alpha$ in the y-direction and $\zeta_n^{(j)} = \sqrt{(k_j)^2 - (\xi_n)^2}\,\left(\mathrm{Im}\left(\zeta_n^{(j)}\right) \geq 0\right)$ in the z-direction, where $k_1 = \omega\sqrt{\varepsilon_0\mu_0}$ and $k_2 = k_1\sqrt{\varepsilon\mu(\omega)}$. For example, the field electrical component (which is parallel to the OX-axis) is

$$E^x = \tilde{e}\exp(-ik_1(y\sin\alpha + z\cos\alpha)) + \sum_{n=-\infty}^{+\infty} a_n\exp\left(i\zeta_n^{(1)}z\right)\exp(i\xi_n y)$$
$$0 < z;$$

$$E^x = \sum_{n=-\infty}^{+\infty}\left[x_n^+\exp\left(i\zeta_n^{(2)}z\right) + x_n^-\exp\left(-i\zeta_n^{(2)}z\right)\right]\exp(i\xi_n y),\quad -h < z < 0;$$

$$E^x = \sum_{n=-\infty}^{+\infty} g_n\exp\left(-i\zeta_n^{(1)}z\right)\exp(i\xi_n y),\qquad\qquad z < -h, (3)$$

where a_n, $x_n^\pm$, and g_n are the unknown complex amplitudes of the n-th spatial harmonics in the corresponding partial domains.

Applying the boundary conditions to each surface ($z = 0$ and $z = -h$) relates the sought Fourier coefficients in the all partial

domains. Also using the boundary conditions provides the system of
functional equations to determine the unknown coefficients, which is a
system of dual series equations involving trigonometric functions. This
system is a particular case of the systems obtained and solved in work
[7]. It is equivalent to the operator equation of the first kind [17] in the
Hilbert space given by the Meixner condition [18]. Thus, the system is
ill-conditioned [19] and, generally speaking, the truncation technique
cannot be employed [20]. A direct usage of the methodological results
from [7] converts the system to the following form

$$
\begin{cases}
\displaystyle\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} X_n \exp\left(in\varphi\right) + \theta X_0 = 0, & \delta < |\varphi| < \pi \\[2em]
\displaystyle\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{|n|}{n} X_n \exp(in\varphi) + \theta X_0 = \sum_{n=-\infty}^{\infty} p_n X_n \exp(in\varphi), & |\varphi| < \delta \\[2em]
\displaystyle\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(-1)^n}{n+\theta} X_n + X_0 = 0, & \varphi = \pi
\end{cases}
\tag{4}
$$

where $X_n = s_n x_n^+$; the coefficients s_n and p_n are expressible in terms
of structure parameters and frequency. The value $\theta \in [-0.5, 0.5)$ is
chosen so that $-\theta = m_0 + \Omega \sin\alpha$, where m_0 is the nearest integer
to $-\Omega \sin\alpha$. In (4), we introduced $\varphi = 2\pi y/l$, $\delta = \pi d/l$; thus the
inequality $|\varphi| < \delta$ indicates the grating slot, and $\delta < |\varphi| < \pi$ refers to
the strip. Upon s_n and p_n presentations which can be obtained from
[7], it can be shown that

$$
p_n X_n \underset{|n|\to\infty}{=} \sigma_n n^{-2},
\tag{5}
$$

where $\sum |\sigma_n|^2 < \infty$. The rest of the unknown complex amplitudes in
all domains can be expressed in terms of x_n^+.

The representation (5) indicates that the series in the right-hand
side of system (4) is uniformly and rapidly convergent. Thus for the
given problem the singularity (i.e., the slowly convergent series) of the
problem operator is separated to the left-hand side of the equations,
and the rapidly convergent series is on the right-hand side. This system
is equivalent to the Riemann-Hilbert problem [13]. Applying the
well-known analytical regularization method [11, 13] yields the infinite

system of linear algebraic equations

$$\theta x_0^+ = \sum_{p=-\infty}^{+\infty} \gamma_p^0 V_{0p} x_p^+ + b_0, \quad x_n^+ = \sum_{p=-\infty}^{+\infty} \gamma_p^n V_{np} x_p^+ + b_n, \quad (6)$$

where the values γ_p^n, V_{np}, and b_n are given in [7, 13]. From the asymptotic estimates of the coefficients $\gamma_p^n \sim 1/p^2$, and from the behaviour of V_{np} (e.g., for $n \neq p$ we roughly have $V_{np} < \frac{\text{const}\sqrt{|p|}}{\sqrt{|n||n-p|}}$ as $|n|, |p| \to \infty$), it follows that (6) is equivalent to a Fredholm system of the second kind. As known, such a system can be solved by a truncation procedure with any preassigned accuracy. Thus the analytical regularization allows us to get rid of the ill-conditioning and arrive at the form admitting effective numerical treatment.

4. NUMERICAL RESULTS

Let us introduce the reflection (R_0) and transmission (T_0) coefficients in the zero order of the spectrum; these determine the relative fractions of scattered energy density spread from the structure to the upper and lower half-spaces by the propagating zero harmonics with the corresponding wave vectors $\mathbf{k}_0^{R,T} = \left(0, \xi_0, \pm\zeta_0^{(1)}\right)$.

In the frequency band $\Delta\Omega \in [0, 1/(1+\sin\alpha)]$, only the zero order harmonics spread energy in the half-spaces, all others ($n \neq 0$) have imaginary part of $\zeta_n^{(1)}$, and thus exponentially decay from the structure. These decaying harmonics travel along the grating (axis OY) with the phase velocities $v_n^y = c\Omega/(n - \Omega\sin\alpha)$, where c is the velocity of light in vacuum; the totality of these surface harmonics does not transfer the field energy even along the surface. The paramagnetic layer with negative permeability does not support traveling volumetric waves; all the harmonics in (3) for the cdomain $-h < z < 0$ exponentially decay away from the layer boundaries. The frequency band $\Delta\Omega$ determines the single-mode regime of diffraction which we actually consider here. For this regime in the far field region, the diffracted field is represented by only one propagation plane wave with energy fraction R_0 (for the half-space $z > 0$) and T_0 (for $z < -h$). We suppose below that the paramagnetic layer is thin (i.e., $h/\lambda \ll 1$), and let $\tilde{e} = 1$; thus the energy dissipation in the paramagnetic layer is defined by the loss coefficient $L_0 = 1 - (R_0 + T_0)$ for the single-mode regime.

In the case of normal incidence of the plane wave, the frequency dependence of the specular reflection and transmission of the zero order harmonics is presented in Fig. 3 for the single layer, the single

grating, and the structure. In what follows, the truncation number of system (6) is 25. We observe the sharp minimum of the modulus of reflection coefficient at $\Omega_0 \approx 0.154$ (see Fig. 3(a)) for diffraction from the structure. At the "long wave" range $l/\lambda \ll 1$, the single grating nearly completely reflects the E-polarized incident wave, while the single layer is almost transparent there.

The reflection minimum for the structure corresponds to the resonance wave transmission with high absorption ($\approx 37\%$) in the rather thin paramagnetic layer ($h/\lambda \approx 3 \times 10^{-3}$), the losses of which are characterized by the dissipation factor $\tan\delta = |\mu''/\mu'| \approx 2.82 \times 10^{-4}$ (see Fig. 2, and Fig. 3(b)). The wave losses in the layer without grating are only about $2 \times 10^{-3}\%$ for this frequency Ω_0 (see Fig. 3(a)).

The paramagnetic layer can be considered as an open resonant structure possessing eigen-frequencies and relevant eigen-oscillations. The study of the eigen-value problem for the paramagnetic layer shows that it has a surface eigen-oscillation in the form of surface waves [10, 21]. These surface waves cannot be excited by an incident plane wave. Being a periodical inhomogeneity, the grating converts the incident plane wave into an infinite superposition of spatial harmonics, and thus makes possible the excitation of oscillations in the layer [22]. On the other hand, the grating makes different oscillations interact with each other and so establishes an electrodynamical coupling between different Floquet harmonics both inside and outside the layer. In a condition of synchronism, when, for a certain frequency, the phase velocities of one of the grating surface harmonics and the surface wave of the layer are close enough, effective excitation of the layer surface oscillations takes place. On the whole, the realization of this
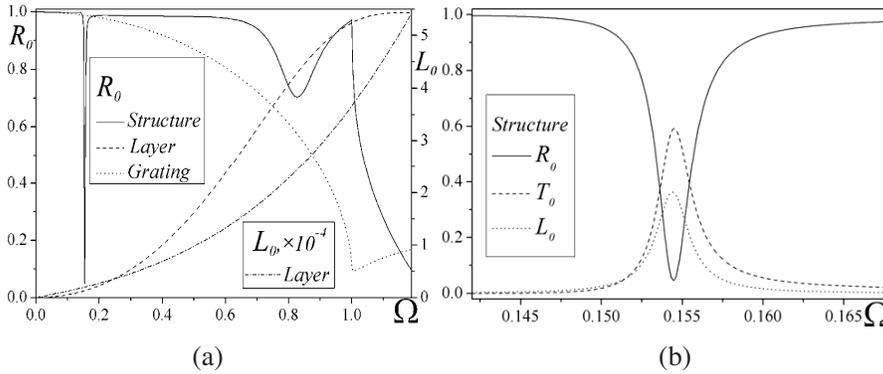
Figure 3. Reflection (R_0), transmission (T_0) and loss (L_0) coefficients versus frequency Ω ($d/l = 0.5$, $h/l = 0.02$, $\alpha = 0°$).
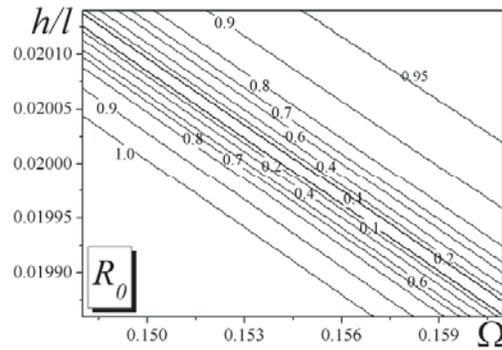
Figure 4. Reflection coefficient R_0 (contours of equal magnitude) versus frequency Ω and relative thickness h/l of the paramagnetic layer $(d/l = 0.5, \alpha = 0°)$.
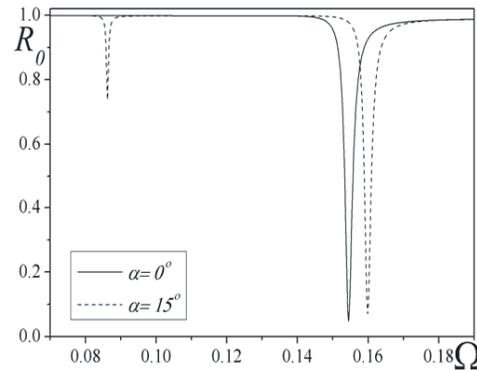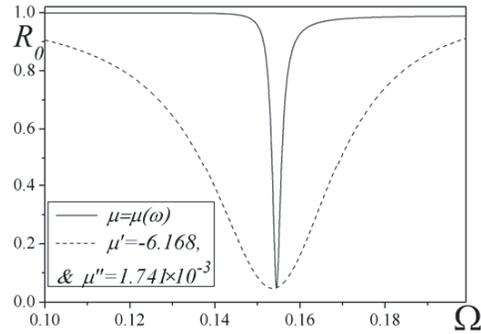

Figure 5. Reflection coefficient R_0 versus frequency Ω in the oblique wave incidence case $(d/l = 0.5, h/l = 0.02)$.

resonant condition corresponds to the eigen-regime of the grating-layer structure.

The value of reflection coefficient R_0 as a function of the non-dimensional frequency and the relative thickness of the paramagnetic layer h/l is represented in Fig. 4. In this picture we observe a lengthy domain of the resonant transmission with high absorption. The transmission frequency increases nearly linearly as the layer thickness decreases.

In Fig. 5, we observe the splitting of the absorption resonance into two ones, in the oblique incidence case. In this situation, the $+n$ and
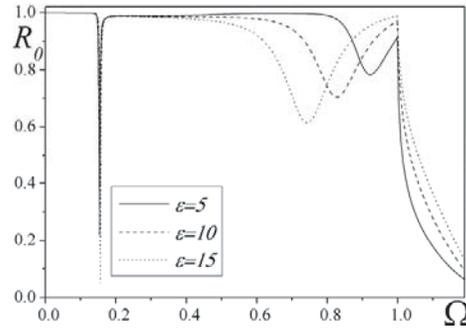
Figure 6. Reflection coefficient R_0 versus frequency Ω in the dispersive and non dispersive cases ($d/l = 0.5$, $h/l = 0.02$, $\alpha = 0°$).

$-n$ harmonics have different phase velocities v_n^y (since $\alpha \neq 0°$), and both are able to excite layer surface waves at certain frequencies, so that two absorption resonances appear instead of one.

The influence of the frequency dispersion of the layer permeability, given in the form (1), on the development of the absorption resonance is compared to the reflection obtained with constant constitutive parameters, which values give the same resonance, in Fig. 6. Varying the dispersion character of the permeability can make the reflection minimum considerably sharper, because of the essentially resonant nature of the phenomenon. Ignoring the dispersion of the permeability results in omitting important spectral features of the paramagnetic layer; in this case we mainly observe the contribution of resonant properties of the grating only. The combination of the two strongly resonant systems, the grating and the layer, into one structure, significantly complicates the spectral characteristics and changes the reflection minimum width.

The frequency dependence of the reflection coefficient for the layer permeability, given by (1), is shown in Fig. 7 for different values of layer permittivity. The frequency of the absorption resonance depends slightly on the layer permittivity; the spectrum of the surface waves in the layer is principally determined by the permeability. As follows from (1), the layer permeability, and thus the frequency of absorption resonance, can be effectively controlled by electronically adjusting the biasing magnetic field B (see Fig. 8).

The realization of the regime of the resonance transmission with high absorption may be readily illustrated by Fig. 9 which shows the modulus of the first five harmonics in the upper half-space (see field presentation (3) for the domain $0 < z$). As we can observe

Figure 7. Reflection coefficient R_0 versus frequency Ω for different values of layer permittivity ε ($d/l = 0.5$, $h/l = 0.02$, $\alpha = 0°$).
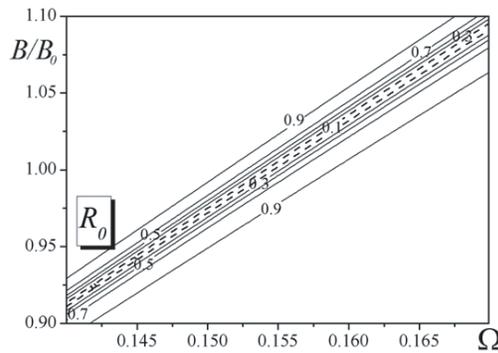
Figure 8. Reflection coefficient R_0 (contours of equal magnitude) versus frequency Ω and relative biasing magnetic field B/B_0 ($B_0 = 2.697$ T, $d/l = 0.5$, $h/l = 0.02$, $\alpha = 0°$).

from the picture, at the resonant frequency Ω_0, the amplitudes of the surface harmonics ($n \neq 0$) have maxima, some of which significantly exceed the unit amplitude of the incident wave; the zero volumetric harmonic corresponding to the reflected plane wave has a minimum amplitude. Under synchronism condition described above, one of the surface grating harmonics excites the layer surface wave, and as a result we observe the resonant maximum of the harmonic amplitude. Due to the electrodynamical coupling established by the grating, all harmonics demonstrate the amplitude extremum at this frequency. Thus a resulting field of high magnitude can be formed near the grating-layer interface. This high-power field is effectively absorbed on the layer surface and partly penetrates into the lower half-space;
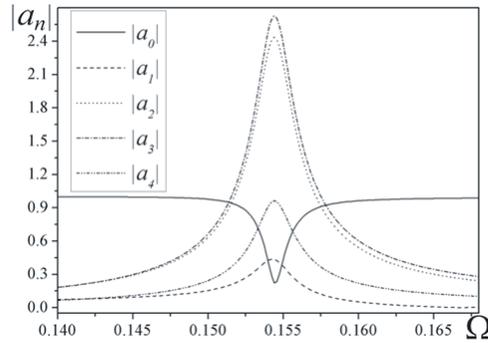
Figure 9. Modulus of the first complex amplitudes $|a_n|$ of the reflected field versus frequency Ω ($d/l = 0.5$, $h/l = 0.02$).

as a result we observe the regime of the resonance transmission with high absorption. This is a qualitative model of the regime; a detailed quantitative analysis of this complicated resonant regime may be based on the rigorous solution to the spectral problem for the structure, and this is the subject of our next paper.

The diffraction process for the structure with conventional magnetodielectric with positive real parts of the constitutive parameters is quite different to that of paramagnetic materials with negative real permeability. In the conventional magnetodielectric, at least one (the zero) volumetric harmonic always propagates, and the main contribution to the diffraction process (both spectral and absorption features) is made just by the propagating volumetric harmonics.

5. CONCLUSION

Using the analytical regularization based on the Riemann-Hilbert problem method, we have solved the problem of plane electromagnetic wave diffraction from the structure consisting of a strip grating and paramagnetic layer with frequency dispersive material, which has permeability of negative real part in the microwave band. The layer with such dispersion possesses surface waves, which can be excited by surface harmonics produced by the grating.

The diffraction regime of the resonance transmission with extremely high absorption was observed and thoroughly investigated. The regime is realized by a synchronism condition between the phase velocities of one of the surface grating harmonics and the layer

surface wave. This phenomenon is of resonance character, and it is a response to the oscillatory excitations which are close to the structure eigenmodes. The powerful surface field localized at the grating-layer interface in the resonance is responsible for transmission and extremely high absorption, which occurs just at the surface but not in the layer volume.

The necessity of considering the dispersion character of the constitutive parameters for adequate description of electromagnetic process was demonstrated. In the oblique incidence case, the phenomenon of splitting of the absorption resonance was shown.

The quality factor of the resonant regime considered is quite high. Such structures, with a narrow and tunable pass-band, are promising for application, for example, as filters or absorbing devices with electronically tunable frequency.

ACKNOWLEDGMENT

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