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UNIFORM SCATTERED FIELDS OF THE EXTENDED THEORY OF BOUNDARY DIFFRACTION WAVE FOR PEC SURFACES

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Abstract—In this paper, the uniform scattered fields from a perfectly conducting (PEC) half plane are studied with the *extended* theory of the boundary diffraction wave. A *new* vector potential of the boundary diffraction wave is found by considering the Fermat principle for the PEC surfaces. This vector potential is applied to the Helmholtz-Kirchhoff integral, and the theory of the boundary diffraction wave is *extended* to the PEC surfaces. The *extended* theory of the boundary diffraction wave is then applied to the scattering problem for the PEC half plane. The total scattered fields are compared numerically with the exact solution for the same problem. The numerical comparisons given in the paper show that the solution of the *extended* theory of the boundary diffraction wave is *very close* to the *exact* solution.

1. INTRODUCTION

The theory of boundary diffraction wave is taken into consideration to be the refinement of Young's ideas on the nature of diffraction [1]. The first formulation of the theory of boundary diffraction wave (TBDW) was introduced by Maggi-Rubinowicz by considering the Young's ideas [2, 3]. They independently showed that Helmholtz-Kirchhoff integral can be converted into a line integral representing the edge diffracted fields. The general case of the Maggi-Rubinowics formulation was expressed by Miyamoto and Wolf for various incident fields [4, 5].

The theory of boundary diffraction wave is a widely used approach for calculating the diffracted fields from aperture systems [6–9]. TBDW

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solutions are composed of the diffracted fields of the shadow region because, in the theory, the aperture surface is taken into consideration solely. Therefore, with this method, only transmitted fields can be calculated, whereas the reflected fields cannot.

TBDW can be easily applied to various diffraction problems, but it does not give satisfactory solutions for diffracted fields from PEC and impedance surfaces, since the theory is solely based on opaque screen. TBDW was studied by Otis and Lit [10] for the problem of diffracted Gaussian laser beams from the edge of the half plane. The problem of the diffraction from the opaque half plane was studied for normal and oblique incidence with TBDW approach in [11, 12]. The potential function of TBDW was obtained for the impedance surfaces by the asymptotic reduction considering the modified theory of physical optics (MTPO) in [13]. The uniform line integral representation for TBDW was found by using the MTPO in [14]. The following studies have been recently focused on the application of the TBDW [15, 16].

The advantage of the boundary diffraction wave theory is based on the line integral reduction in the surface integrals. For the problems with complex geometries, the evaluation of the surface integrals needs high computation times [17]. However, it is well known that the surface of the scatterer contributes to the geometrical optics waves, which can be easily evaluated by taking into account the classical method of geometrical optics. Thus, the line integral reduction of the surface integrals enables one to directly evaluate the edge diffracted waves, by integrating the reduced integrand along the edge contour, and as a result, the computation time decreases drastically.

In this paper, the modified vector potential of TBDW is defined by using a *different* approach other than methods available in the literature [10–14]. Here, this vector potential is applied to the Helmholtz-Kirchhoff integral, and TBDW is *extended* for PEC surfaces. Verification of the *extended* method is performed by applying it to the problem of diffraction from the PEC half plane. The total uniform diffracted fields are obtained by this approach, and the total scattered fields are compared numerically with the exact solution of the same problem.

A time factor $e^{j\omega t}$ is assumed and suppressed throughout the paper.

2. THE EXTENDED THEORY OF THE BOUNDARY DIFFRACTION WAVE FOR PEC SURFACES

The field disturbance at any point P within a volume ν bounded by any closed surface S according to Helmholtz-Kirchhoff integral is expressed as

$$U(P) = \oiint\limits_S \vec{V}(Q,P) \cdot \vec{n} dS \tag{1}$$

$U(P)$ is the magnetic or electric field, and it is the solution of homogeneous Helmholtz equation. Here, $\vec{n}$ is the unit inward normal vector and Q is the variable point on the surface S. Using the Stokes theorem, Helmholtz-Kirchhoff integral can be split into two terms:

$$U(P) = U_B(P) + U_{GO}(P). \tag{2}$$

The first term represents the boundary diffraction wave from the boundary Γ of the aperture surface (see Figure 1).

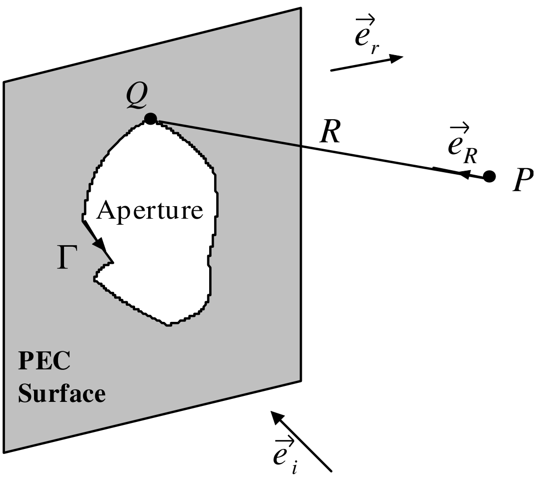


Figure 1. Geometry of the boundary diffraction wave.

Then the expression can be given as

$$U_B(P) = \int\limits_\Gamma \vec{W}(Q,P) \cdot \vec{l} dl. \tag{3}$$

$\vec{V}(Q,P)$ can be expressed as the curl of a vector potential $\vec{W}(Q,P)$. Therefore, the vector potential is symbolically given as

$$\vec{W}(Q,P) = \frac{1}{4\pi} \frac{e^{-jkR}}{R} \left[\vec{e}_R \times \frac{\nabla_Q}{(-jk + \vec{e}_R \cdot \nabla_Q)} U(Q) \right] \tag{4}$$

by considering [4, 5]. Here, $\vec{e}_R$ is the unit vector of the vector $\vec{R}$, and R denotes the distance between the observation point P and Q (see Figure 1).

The second term in Eq. (2) represents the contributions of geometrical-optics (GO) fields from the special Q_i points on the PEC surface or aperture. These points are singularities relating to the vector potential $\vec{W}(Q,P)$. These discrete singular points, $Q_1, Q_2, \dots Q_n$, are surrounded by small circles with radii σ_i, and the boundaries of these circles are called Γ_i $(i = 1, 2, \dots n)$. Hence, $U_{GO}(P)$ is symbolically described as [4,5]

$$U_{GO}(P) = \sum_i \lim_{\sigma_i \to 0} \int_{\Gamma_i} \vec{W}(Q_i, P) \cdot \vec{l} dl \tag{5}$$

where, $\vec{l}$ is the unit vector along the tangent of Γ_i, and dl is an element of Γ_i.

The vector potential $\vec{W}(Q,P)$ is not unique since *rot grad* $\equiv 0$, but the vector potential's arbitrariness does not affect the TBDW integral [18]. According to the Fermat principle, diffraction is a local phenomenon [19]. If the screen is not an opaque surface, $U(Q)$ given in Eq. (4) will be equal to the sum of the incident $U_i(Q)$ and the reflected $U_r(Q)$ fields at the secondary source point Q. The second term in Eq. (2) represents Geometrical Optics (GO) fields' contributions. These fields are related to singularities of the vector potential.

3. DIFFRACTION FROM THE PEC HALF PLANE

The diffraction geometry to be used in this paper is depicted in Figure 2. In this figure, a homogeneous plane wave is illuminating the PEC half plane. The homogeneous plane wave and the pseudo-reflected wave can be given as

$$U_i(P) = u_i e^{-j\vec{k}_i \cdot \vec{r}} \tag{6}$$

and

$$U_r(P) = u_r e^{-j\vec{k}_r \cdot \vec{r}} \tag{7}$$

for observation point P. Here $\vec{k}_i$ and $\vec{k}_r$ are equal to $k\vec{e}_i$ and $k\vec{e}_r$, respectively. $\vec{r}$ is the position vector of point P and can be given as

$$\vec{r} = (x\vec{e}_x + y\vec{e}_y + z\vec{e}_z). \tag{8}$$

The related unit vectors can be written as

$$\begin{aligned} \vec{e}_i &= -\cos\phi_0 \vec{e}_x - \sin\phi_0 \vec{e}_y \\ \vec{e}_r &= -\cos\phi_0 \vec{e}_x + \sin\phi_0 \vec{e}_y \end{aligned} \tag{9}$$

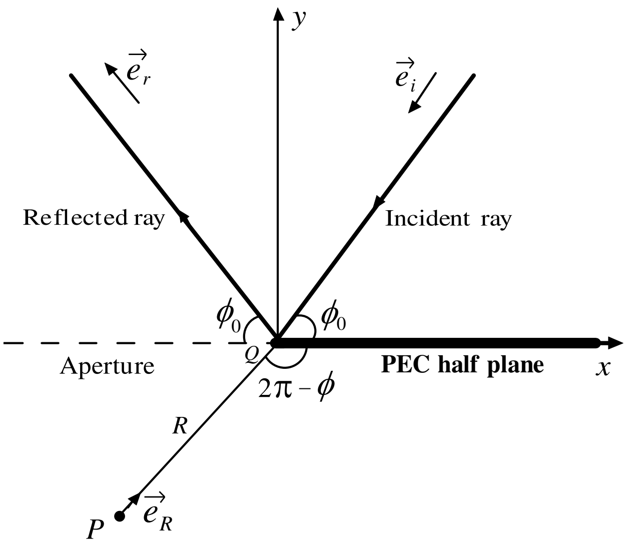


Figure 2. Geometry of the diffraction from PEC half plane.

by considering the geometry in Figure 2. The incident (or transmitted) field can be evaluated by using Eq. (6). By means of the variable change of $x = \rho \cos\phi$, $y = \rho \sin\phi$ in Eq. (8), one can obtain

$$U_i(P) = u_i e^{jk\rho\cos(\phi-\phi_0)} \tag{10}$$

by utilizing Eqs. (8) and (9) in Eq. (6). Similarly, the reflected field can be derived as

$$U_r(P) = -u_i e^{jk\rho\cos(\phi+\phi_0)} \tag{11}$$

by utilizing Eqs. (8) and (9) in Eq. (7) for the PEC half plane problem.

The diffracted field from the PEC half plane can be evaluated by the contribution of the first term in Eq. (2). In this study, firstly, the vector potential concerning to the PEC half plane will be found. Since the secondary source point Q is located at the origin, x' and y' is equal to zero for this case. Gradient of $U(Q)$ can be given as

$$\nabla_Q U(Q) = -jku_i(\vec{e}_i - \vec{e}_r) \tag{12}$$

at this point. So, the vector potential of the PEC half plane problem can be found as

$$\vec{W}(Q,P) = u_i \frac{1}{4\pi} \frac{e^{-jkR}}{R} \left(\frac{\vec{e}_R \times \vec{e}_i}{1 + \vec{e}_R \cdot \vec{e}_i} - \frac{\vec{e}_R \times \vec{e}_r}{1 + \vec{e}_R \cdot \vec{e}_r} \right) \tag{13}$$

by using Eqs. (6), (7), (9) and (12) in Eq. (4). The related unit vectors can be written as

$$\begin{aligned}\vec{e}_R &= -\cos\phi\vec{e}_x - \sin\phi\vec{e}_y\\ \vec{l} &= -\vec{e}_z\end{aligned}\tag{14}$$

by considering the geometry in Figure 2. Then, one can obtain

$$\frac{(\vec{e}_R\times\vec{e}_i)\cdot\vec{l}}{1+\vec{e}_R\cdot\vec{e}_i} = \frac{-\sin(\phi-\phi_0)}{1+\cos(\phi-\phi_0)} = -\tan\left(\frac{\phi-\phi_0}{2}\right)\tag{15}$$

and

$$\frac{(\vec{e}_R\times\vec{e}_r)\cdot\vec{l}}{1+\vec{e}_R\cdot\vec{e}_r} = \tan\left(\frac{\phi+\phi_0}{2}\right)\tag{16}$$

by considering Eqs. (10) and (15). Hence, the diffracted field integral can be found by utilizing Eqs. (13), (15) and (16) in Eq. (3). One obtains

$$U_B(P) = -u_i\frac{1}{4\pi}\left[\tan\left(\frac{\phi-\phi_0}{2}\right) - \tan\left(\frac{\phi+\phi_0}{2}\right)\right]\int_\Gamma \frac{e^{-jkR}}{R}dl.\tag{17}$$

where, R is equal to $[x^2+y^2+(z-z')^2]^{1/2}$ and $dl = dz'$ for this problem. Therefore, the diffracted field's integral can be rewritten as

$$U_B(P) = -u_i\frac{1}{4\pi}\left[\tan\left(\frac{\phi-\phi_0}{2}\right) - \tan\left(\frac{\phi+\phi_0}{2}\right)\right]\int\limits_{z'=-\infty}^{\infty} \frac{e^{-jkR}}{R}dz'.\tag{18}$$

The integral expression in Eq. (18) defines a Hankel function [20]

$$\int_c e^{-jkch\gamma}d\gamma = \frac{\pi}{j}H_0^{(2)}(k\rho)\tag{19}$$

by using the variable change of $(z-z') = \rho sh\gamma$, where ρ is equal to $[x^2+y^2]^{1/2}$. As a result, Eq. (18) gives

$$U_B(P) = -u_i\frac{1}{4j}\left[\tan\left(\frac{\phi-\phi_0}{2}\right) - \tan\left(\frac{\phi+\phi_0}{2}\right)\right]H_0^{(2)}(k\rho).\tag{20}$$

Debye's asymptotic expansion of the second kind Hankel function can be given as

$$H_0^{(2)}(kv) \approx \sqrt{\frac{2}{\pi}}\frac{e^{-j[kv-(\pi/4)]}}{\sqrt{kv}}\tag{21}$$

for $k\rho \to \infty$. Hence, the diffracted field can be found as

$$U_B(P) \approx -\frac{u_i}{2\sqrt{2\pi}} \left[\tan\left(\frac{\phi-\phi_0}{2}\right) - \tan\left(\frac{\phi+\phi_0}{2}\right) \right] \frac{e^{-jk\rho - j\pi/4}}{\sqrt{k\rho}} \tag{22}$$

for the PEC half plane. The diffracted field in Eq. (22) approaches to infinity at the transition region. Otherwise, the uniform diffracted fields are finite in regions where the non-uniform solution goes to infinity. Hence, the first part of the diffracted field can be derived as

$$U_{Bi}(P) \approx -\frac{u_i e^{-j\pi/4}}{2\sqrt{\pi}} \sin\left(\frac{\phi-\phi_0}{2}\right) \frac{e^{-j2k\rho\cos^2\left(\frac{\phi-\phi_0}{2}\right)}}{\sqrt{2k\rho}\cos\left(\frac{\phi-\phi_0}{2}\right)} e^{jk\rho\cos(\phi-\phi_0)} \tag{23}$$

by utilizing the trigonometric identity of $1 = 2\cos^2(A) - \cos(2A)$. Hence, Eq. (23) can be written as

$$U_{Bi}(P) \approx u_i \hat{F}(\xi_i) \sin\left(\frac{\phi-\phi_0}{2}\right) e^{jk\rho\cos(\phi-\phi_0)}. \tag{24}$$

Here, $\hat{F}(\xi_i)$ is equal to $e^{-j(\xi_i^2+\pi/4)}/(2\sqrt{\pi}\xi_i)$. ξ_i, the argument of the Fresnel function, represents the detour parameter [21,22]. The detour parameter gives the phase difference between the incident (or reflected) and diffracted fields. This can be given symbolically as $\xi = -\sqrt{\psi_{i,r} - \psi_d}$, where $\psi_{i,r}$ is the phase function of the incident (or reflected) field and ψ_d is the phase function of the diffracted field. Thus, the detour parameter associated with the incident field can be easily obtained as

$$\xi_i = -\sqrt{2k\rho}\cos[(\phi-\phi_0)/2] \tag{25}$$

by considering Eqs. (10) and (22). So, the first uniform part of the diffracted field can be written as

$$U_{Bi}(P) \approx u_i F(|\xi_i|)\mathrm{sgn}(\xi_i) \sin\left(\frac{\phi-\phi_0}{2}\right) e^{jk\rho\cos(\phi-\phi_0)} \tag{26}$$

by using the asymptotic relation of the Fresnel function for large arguments, i.e., $\hat{F}(\xi_i) \approx F(|\xi_i|)\mathrm{sgn}(\xi_i)$. Here, $\mathrm{sgn}(\xi_i)$ shows the signum function. Fresnel integral $F(\xi_i)$ can be given as

$$F(\xi_i) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int_{\xi_i}^{\infty} e^{-jt^2} dt. \tag{27}$$

Similarly, the second uniform part of the diffracted field can be found as

$$U_{Br}(P) \approx -u_i F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right)e^{jk\rho\cos(\phi+\phi_0)} \tag{28}$$

Here, ξ_r is the detour parameter associated with the reflected field and can be obtained as

$$\xi_r = -\sqrt{2k\rho}\cos[(\phi+\phi_0)/2] \tag{29}$$

by considering Eqs. (11) and (22). As a result, the uniform total diffracted field can be found as

$$\begin{aligned} U_B(P) &= U_{Bi}(P) + U_{Br}(P) \\ &= u_i\left[F(|\xi_i|)\mathrm{sgn}(\xi_i)\sin\left(\frac{\phi-\phi_0}{2}\right)e^{jk\rho\cos(\phi-\phi_0)}\right. \\ &\quad \left.-F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right)e^{jk\rho\cos(\phi+\phi_0)}\right] \end{aligned} \tag{30}$$

for the PEC half plane.

4. DISCUSSION AND NUMERICAL RESULTS

In this section, the *extended* TBDW total scattered fields are compared with the exact solution of the perfectly electric conducting (PEC) half plane problem. The exact solution can be given as [23]

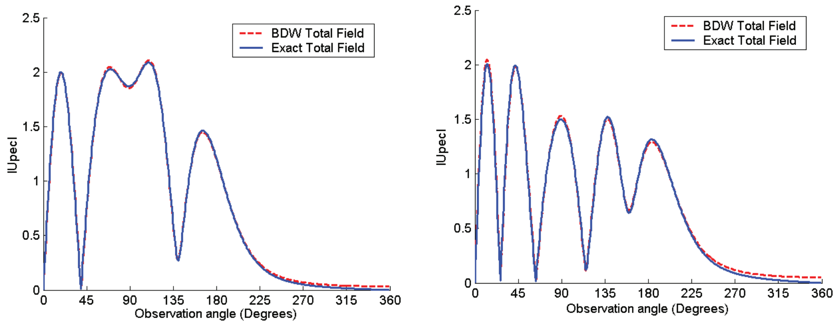
$$U_t^{(pec)}(P) = 2u_i\sum_{n=1}^{\infty} e^{jn\pi/4}J_{n/2}(k\rho)\sin\left(\frac{n\phi}{2}\right)\sin\left(\frac{n\phi_0}{2}\right). \tag{31}$$

The *extended* TBDW total scattered fields is derived as

$$\begin{aligned} U_{Bt}^{(pec)} &= u_i\left\{e^{jk\rho\cos(\phi-\phi_0)}u(-\xi_i) - e^{jk\rho\cos(\phi+\phi_0)}u(-\xi_r)\right. \\ &\quad +\left[F(|\xi_i|)\mathrm{sgn}(\xi_i)\sin\left(\frac{\phi-\phi_0}{2}\right)e^{jk\rho\cos(\phi-\phi_0)}\right. \\ &\quad \left.\left.-F(|\xi_r|)\mathrm{sgn}(\xi_r)\sin\left(\frac{\phi+\phi_0}{2}\right)e^{jk\rho\cos(\phi+\phi_0)}\right]\right\} \end{aligned} \tag{32}$$

by using Eqs. (10), (11), and (30) for PEC half plane problem.

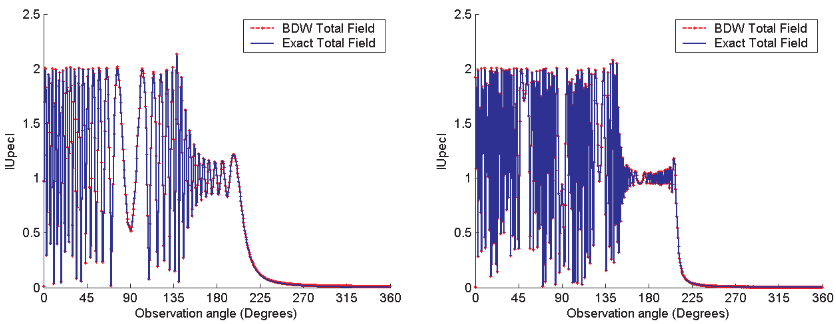
The variations of the extended TBDW total scattered fields, Eq. (32), and the exact solution of the Helmholtz equation, Eq. (31), with observation angles are given in Figure 3.



(a) ($\phi_0 = \pi/6$) (b) ($\phi_0 = \pi/4$)

Figure 3. Comparison of total scattered fields for the angles of edge incidence.

Here, u_i is the selected as unit amplitude, and $k\rho$ is taken as 10. The angles of the edge incidence (ϕ_0) are selected as $\pi/6$ and $\pi/4$ in Figure 3. It is seen from Figure 3 that the *extended* TBDW total scattered fields are very close to the exact solution. It should be noted that the same observation is valid for all the angles of the edge incidence (ϕ_0).



(a) (k = 100) (b) (k = 1000)

Figure 4. Comparison of total scattered fields for the values of $k\rho$.

The variations of the extended TBDW total scattered fields and the exact solution of the Helmholtz equation for different $k\rho$ values are also given in Figure 4. Here, u_i is selected as unit amplitude, and the angle of edge incidence ϕ_0 is taken as $\pi/6$. $k\rho$ values are selected as in Figure 4. It is observed that the *extended* TBDW total scattered fields approximate to the exact solution *successfully* for all observation angles. It should be noted that the same observation holds true also for all values of $k\rho$.

5. CONCLUSION

The contribution of this study relies mainly on the extension of the theory of boundary diffraction wave (TBDW). In the paper, a new vector potential is proposed for TBDW. By using this new vector potential in Helmholtz-Kirchhoff integral, TBDW is extended for the perfectly electric conducting (PEC) surfaces. The *extended* TBDW is then applied to the problem of scattering from a PEC half plane. The diffraction integral is obtained and evaluated asymptotically by using Debye's asymptotic expansion of the Hankel function. The *extended* TBDW solutions of diffracted fields are made uniform by using the detour parameter. The total uniform scattered fields are compared numerically with the exact solution of the same problem. The numerical comparisons given in this study prove that the *extended* TBDW and the exact solutions are in *good agreement*.

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