LARGE PHASED ARRAYS DIAGNOSTIC VIA DISTRI-BUTIONAL APPROACH

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Abstract—A deterministic method for detecting faulty elements in phased arrays is proposed and tested against experimental and numerical data. The solution approach assumes as input the amplitude and phase of the near-field distributions and allows to determine both positions and currents of radiating elements. The corresponding non linear inverse problem is properly solved by exploiting the distributional approach, which allows to cast the initial problem to the solution of a linear one, whose solution is made stable by adopting a proper regularization scheme based on the Truncated Singular Value Decomposition tool. The results fully confirm accuracy of the proposed technique.

1. MOTIVATIONS

Many applications, ranging from sonar, radar and space communications, need for fully active phased arrays [1–3]. These antennas have several hundreds of radiating elements or proper sub-arrays [2, 3], and the possibility of their failure strongly increases. These element failures can cause sharp variations in the aperture field across the array aperture, thus increasing both the sidelobes and the ripple level of the far-field radiation pattern. In order to know which element or elements are damaged, active antennas can include calibration systems. These

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systems make an easy control of the system components, but it can fail if the calibration system is damaged too. Moreover, calibration systems can be also rejected because its inclusion means a critical increase in array volume, weight and costs.

An alternative solution consists in the location of the faulty elements starting from the measures of the far or near-field data. With these approaches one can determine the excitation coefficients of each radiating element and detects the failures by comparing the reconstructed ones with the nominal currents. Several deterministic and stochastic techniques have been developed [4–9] in the last years. Among the stochastic approaches, we point out the learning algorithms based on examples, such as neural networks [4, 5], and the genetic algorithms based approaches [6, 7]. These methods have the advantage to require small amount of samples of the radiated field and, in many cases, only amplitude data [7], but, due to the high size of search space, they can have poor performances.

Note the diffused enthusiasm for *physically inspired* optimization techniques has induced to neglect the fact that all global optimization algorithms are limited in their performances by the computational cost required to get, within a given precision, the actual solution. This cost grows very rapidly with the number of unknowns [12], i.e., with the phased array antenna size. As a consequence, in large scale problems, due to the necessity of stopping the search after a given amount of flops, it is likely that only sub-optimal solutions will be generally achieved, which can be significantly worse than the actual optimal ones.

Moreover, not only general global algorithms are computationally heavy: they are all essentially equivalent, as implied by the so called No Free Lunch Theorems [13]. These theorems state that a truly general-purpose universal optimization strategy does not exist [13]: on average the performances of any two optimization algorithms are the same across all possible optimization problems. Hence, for any algorithm, an elevated performance over one class of problems is exactly paid for in performance over another class. Now, for a given sufficiently general algorithm, neither it is practically possible to characterize the class of problems to which it is fitted, nor we can blindly refer to results obtained in a other area [13]. And so, the only way to devise an effective algorithm is to exploit the properties of the specific class of problems under consideration, thus possibly avoiding the use of global optimization schemes.

Among the deterministic methods, a simple and fast approach to estimate the array excitations from near-field measures is the Backward Transformation Method (BTM) [10], based on a proper exploitation of the Fast Fourier Transform (FFT) algorithm. A different method has

been proposed in [11] and has been compared to the BTM [10]. It has been shown that the method in [11] has better performance than BTM when the noise and the truncation error in the data can not be neglected (as happens in actual applications). Finally, in [15], an interesting comparison among several approaches for finding defective elements in large antenna arrays has been also performed.

Anyway, large experimental evidence of the good performance of the BTM in case of planar array diagnosis is present in literature, so that no doubt exists about the practical convenience of the BTM with respect to the proposed approach in case of planar array diagnosis. In this contribution, we consider a novel and effective determinist approach that properly overcomes the limitation of planar arrays and that has good performances in case of unknown measurement position error. In particular, in order to account of this last problem, we assume that as well as the excitation also the position of the radiating elements are unknown of the diagnostic tool. Different from traditional approaches discussed above, the problem at hand belongs to the class of nonlinear inverse problems [14]. Note the near-field data depend in a linear way on the excitation coefficients and in a non-linear way from the elements locations. Therefore, in principle, global optimization scheme should be exploited. To overcome the need to use global optimization schemes, we herein propose to linearize the inverse problem by properly exploiting the distributional approach adopted in [14–16].

The paper is organized as follows. In Section II, the distributional approach is briefly recalled and applied to the problem at hand. In Section III the proposed method is experimentally compared with the classical method in [10] for the case of a fully active phased array and then numerically tested for the case of a conformal array and spherical measurement system. Conclusions follow.

2. THE PROPOSED SOLUTION SCHEME

Let us consider an array of N elements, located in unknown positions $\mathbf{r}_n(x_n, y_n, z_n)$. Let w_n and $\mathbf{f}_n(\theta, \phi)$ be the excitation coefficient and the electric-field radiation pattern of the n — the radiating element, respectively (Fig. 1). A probe having effective height $\mathbf{h}(\theta, \phi)$ is placed in a known spatial point $\mathbf{r}_m(x_m, y_m, z_m)$. The voltage at the probe
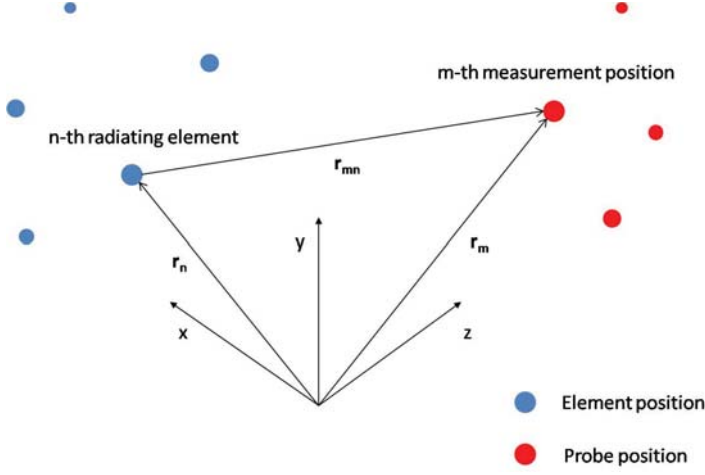
Figure 1. Geometry of the problem showing the $n - th$ radiating element position and the $m - th$ measurement position of the probe.

output can be expressed as

$$V(x_m, y_m, z_m) = \sum_{n=1}^{N} w_n \mathbf{f}_n(\theta_{m,n}, \phi_{m,n}) \cdot$$

$$\mathbf{h}(\theta_{n,m}, \phi_{n,m}) \frac{\exp(-\mathrm{j}\, k_0 R_{mn})}{4\pi R_{mn}} \tag{1}$$

where $\otimes$ denotes the usual dot product, $k_0 = \frac{2\pi}{\lambda}$ is the free-space wavenumber, and λ is the working wavelength. Moreover, $\theta_{m,n}$ and $\phi_{m,n}$ are the relative angles between the m-th measurement point and n-th element position defined as

$$\theta_{m,n} = \arccos\left[\frac{z_m - z_n}{R_{mn}}\right]$$

$$\phi_{m,n} = \arctan\left[\frac{y_m - y_n}{x_m - x_n}\right] \tag{2}$$

with $R_{mn} = |\mathbf{r}_m - \mathbf{r}_n|$. Furthermore, we assume the system of targets is composed by identical radiating elements, so that $\mathbf{f}_n(\theta_{m,n}, \phi_{m,n}) = \mathbf{f}(\theta_{m,n}, \phi_{m,n}) \forall n$.

Aim of the problem at hand is to determine both locations $\mathbf{r}_n$ and excitations w_n of the N radiating elements from a set of M samples of amplitude and phase near-field radiation pattern (or M values of

voltages given by Eq. (1). From Eq. (1), it can seen that the measured voltages (the data of the inverse problem) and the actual unknowns $(w_n, \mathbf{r}_n)$ are differently related each other. In particular, while the data of the problem are related to the excitation coefficients w_n with a linear relation, the unknown locations appear in Eq. (1) as the arguments of the exponential terms, thus making the overall diagnostic problem a non-linear inverse problem. To overcome such a problem and properly taking advantage from the linearity of the problem with respect to a part of the unknowns, inspired by the hybrid array synthesis method proposed in [19], a possible idea is to cast the global diagnostic problem to the solution of a more simple linear problem after representing the unknown elements positions by means of δ functions [14–16].

In particular, after introducing the function

$$\gamma(\mathbf{\acute{r}}) = \sum_{n=1}^{N} w_n \delta(\mathbf{\acute{r}} - \mathbf{r}_n) \tag{3}$$

which takes accounts for the position and the excitation coefficients of the radiating elements, it is possible to rewrite Eq. (1) as follows

$$V(x_m, y_m, z_m) = \int_D \mathbf{f}(\theta_{m,\mathbf{\acute{r}}}, \phi_{m,\mathbf{\acute{r}}})$$
$$\cdot \mathbf{h}(\theta_{\mathbf{\acute{r}},m}, \phi_{\mathbf{\acute{r}},m}) \frac{\exp(-\mathrm{j}\, k_0 R_m)}{4\pi R_m} \gamma(\mathbf{\acute{r}}) \mathrm{dV'} \tag{4}$$

where $\theta_{m,\mathbf{\acute{r}}}$ and $\phi_{m,\mathbf{\acute{r}}}$ are defined as

$$\theta_{m,\mathbf{\acute{r}}} = \arccos\left[\frac{z_m - z'}{R_m}\right]$$
$$\phi_{m,\mathbf{\acute{r}}} = \arctan\left[\frac{y_m - y'}{x_m - x'}\right] \tag{5}$$

with $R_m = |\mathbf{r}_m - \mathbf{\acute{r}}|$, $\mathbf{\acute{r}}(x', y', z') \in D$, and D the spatial domain containing all the radiating elements. As it can be seen from Eq. (4), by using Eq. (3), the determination of locations and excitation coefficients of the radiating elements is cast to the inversion of the linear operator defined by the Eq. (4), with values in the Hilbert space $L^2(O)$ of square integrable functions defined over the observation domain O, i.e., the spatial domain where we measure the voltage V.

Note that, by using Eq. (4), the function $\gamma(\mathbf{\acute{r}})$ becomes the actual unknown of the linear inverse problem in Eq. (5).

As well known [14], due to the presence of noise in the measured data, the solution of the above problem generally does not exist

and a generalized solution has to be searched [14]. Moreover, the lack of independent data due to the truncation of the measurement domain and the adoption of sub-optimal sampling points, (see [20] for a detailed discussion on this issue) makes the inverse problem at hand an ill-posed problem [14]. In order to stabilize the solution, a regularization procedure has to be adopted. In the following, we adopt the Truncated Singular Value Decomposition method described in [14], which effectively allows to obtain a trade-off between accuracy and stability of the solution [14].

In detail, if $\{u_n, \sigma_n, v_n\}_{n=0}^{\infty}$ denotes the singular system [14] of the radiation operator in Eq. (4), the estimated version of the unknown $\gamma(\acute{\mathbf{r}})$ function, achieved by using a standard TSVD [14] based procedure can be expressed as [14]

$$\widehat{\gamma(\mathbf{r})} = \sum_{n=0}^{N_T} \frac{\langle V(\mathbf{r}), v_n \rangle}{\sigma_n} u_n \tag{6}$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product in the space $L^2(O)$ while N_T is the TSVD truncation index [14]. Notably, the proposed formulation can be applied for arbitrary geometries (e.g., conformal arrays) and/or measurement domain (e.g., spherical measurement systems). In the following, both traditional planar array with usual planar measurement system and conformal array with spherical measurement system are considered.
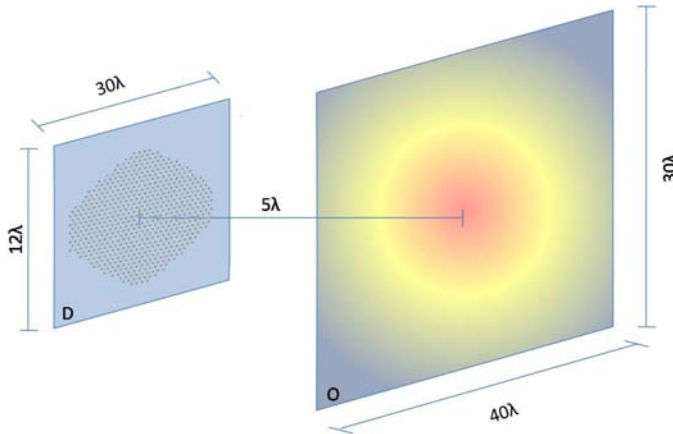
Figure 2. The adopted experimental set-up and the considered phased array layout.

(a) Azimuth cut   (b) Elevation cut
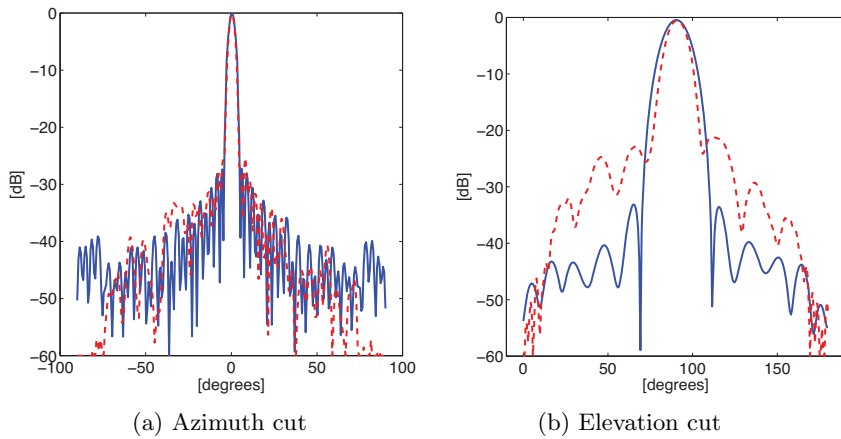
Figure 3. A comparison between the theoretical radiation pattern (blue solid line) and the measured one (red dot-line) of the phased array layout in Fig. 2.

3. TESTING AGAINST EXPERIMENTAL AND NUMERICAL DATA

As first example, the proposed diagnostic tool has been tested for the case of $N = 604$ horn elements located as shown in Fig. 2. The data for the inversion have been collected using a traditional planar scanning placed at distance 5λ from the antenna under test aperture, and a fixed number of radiating elements have been forced off. The dimensions of the spatial domain $D = [-7\lambda, 5\lambda] \times [-15\lambda, 15\lambda]$ and of the measurement plane $O = [-15\lambda, 15\lambda] \times [-20\lambda, 20\lambda]$ are input data for the problem, see Fig. 2. The voltages (amplitude and phase) have been collected in $M = 61 \times 81$ points. The probe is a small horn antenna. All the measurement set-up is available at the Research and Development Department of SELEX Sistemi Integrati.

As first step, let us observe the desired and measured radiation patterns reported in Fig. 3. The first one has been achieved by using a Taylor current distribution such to fulfill the design constraints. The measured one is obtained by standard near-field far-field transformation [21] starting from the measured near-field data. As it can be seen, according to the above hypothesis, the measured radiation pattern and the theoretical one are sensibly different. The overall pattern shapes and the sidelobes level are different, and a proper investigation aimed to detect possibly faulty elements becomes mandatory. To this aim, the measured near-field data have been first
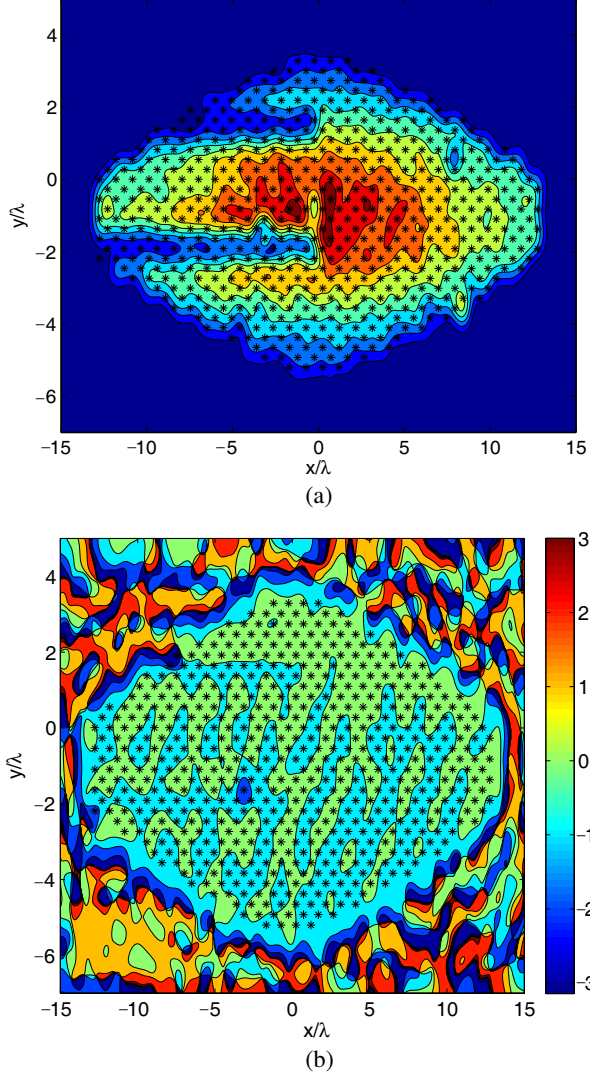
Figure 4. Reconstructed amplitude (a) and phase (b) distributions by using the method in [10].

processed by using the BTM method of [10]. Fig. 4(a) and Fig. 4(b) show the amplitude and phase distributions of the excitation coefficient respectively. Then, the available data have been processed by adopting the diagnostic tool herein proposed (see Fig. 5). As it can be seen and as we expected for this case (planar geometry and planar measurement
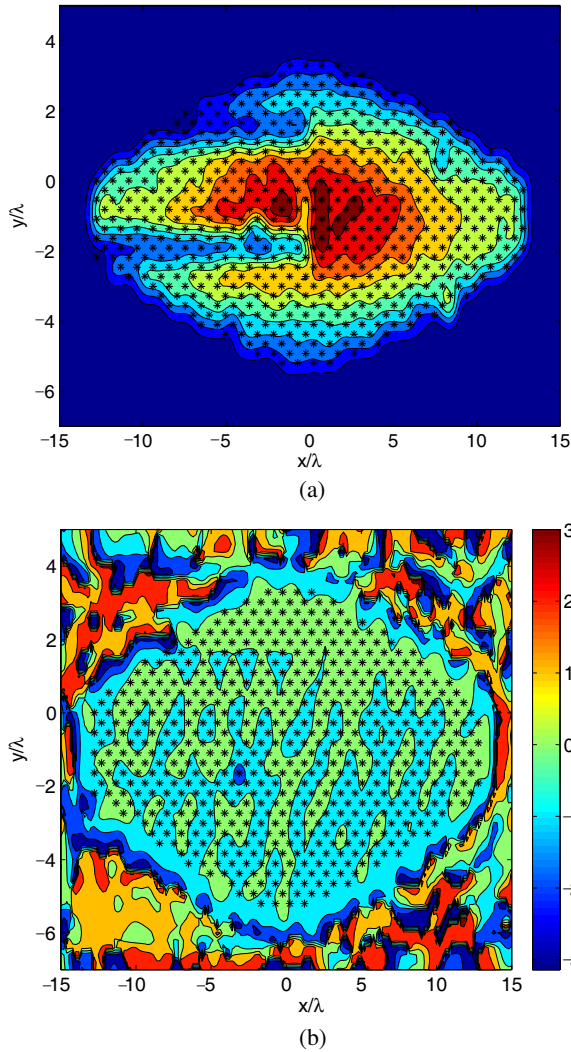
Figure 5. Reconstructed amplitude (a) and phase (b) distributions by using the proposed method.

system), the two methods obtain very similar results. Indeed, both BTM [10] and method herein proposed allow to identify that the main problem is an error of the elements of two semi-row of the phases array antenna under test. Moreover, Figs. 4 and 5 show a mismatch along the y-axis between the system reference centered on the AUT and the system reference centered on the measured domain O.

(a) Azimuth cut   (b) Elevation cut
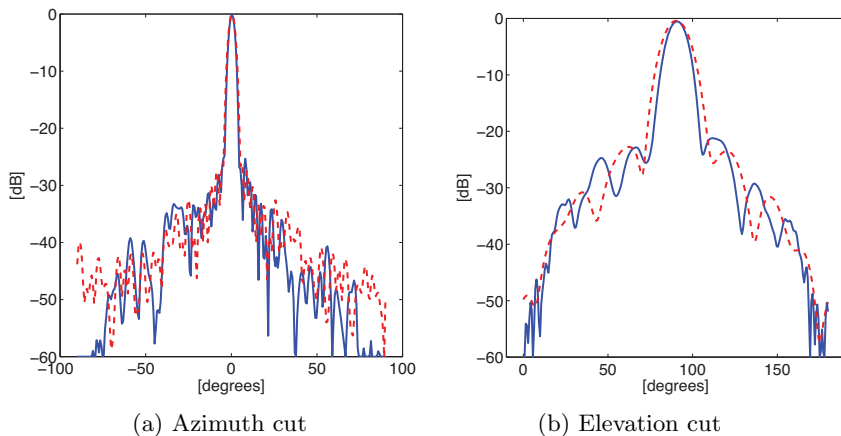
Figure 6. A comparison between the measured (blue solid line) and reconstructed (red dot-line) radiation pattern of the antenna under test. The reconstructed one has been achieved by using the retrieved currents in Fig. 5.

In order to emphasize the capability of the proposed method of detecting array failure, as well as the possibility to evaluate the actual currents, the far-field of the considered phased array has been evaluated starting from the reconstructed currents. The achieved result has been compared with the measured one, see Fig. 6. The good agreement between the two curves fully confirms the accuracy and usefulness of the developed tools.

As further comment, note the proposed method is characterized by an higher computational cost and memory requirements than BTM one [10], being based on the evaluation of a Singular Value Decomposition of the radiation operator relating the unknowns of the problem to the (measured) near-field. On the other hand, the BTM method [10] does not allow to detect faulty elements in non planar arrays. These kind of geometries can be instead considered by using the proposed diagnostic tool. Moreover, due to the adopted regularization scheme, the proposed method exhibits an increased robustness against noise with respect to [10].

In order to point out the effectiveness of the proposed approach for generic geometry arrays and measurement systems, we consider the case of a conformal array composed by $N = 33$ Hertzian dipole located as shown in Fig. 7(b) (the large black dot). The data for the inversion have been simulated by using Computer Simulation Technology Microwave Studio (CST MWS) [22], a commercial full wave

(a)

(b)
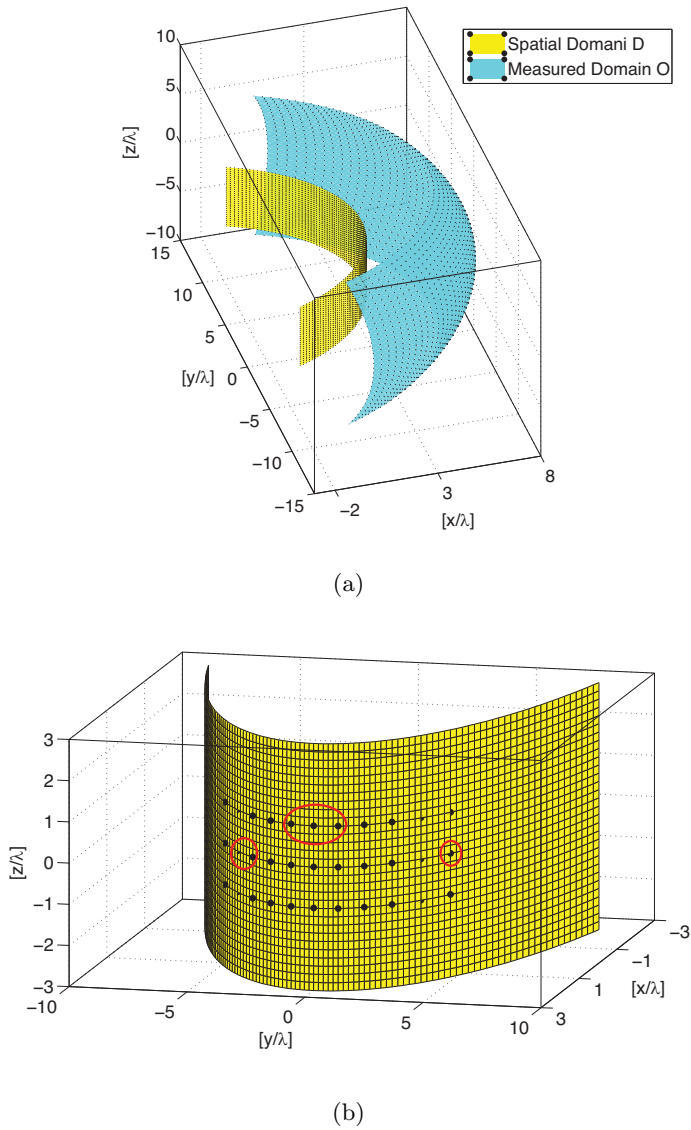
Figure 7. The adopted measurement configuration (a) and the considered conformal phased array layout (b) for the numerical simulation.
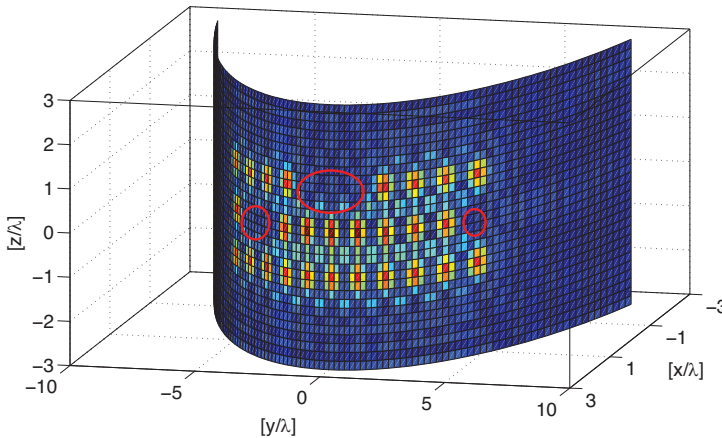
Figure 8. Reconstructed amplitude distributions by using the proposed method.

3D electromagnetic (EM) simulator based on the Finite-Integration Technique. In particular, the spatial domain D is a bounded surface domain of a cylinder described by equations $\rho = 10\lambda$, $\phi \in [-30, 30]$ deg, $z \in [-3\lambda, 3\lambda]$ and the measurement domain O is a bounded surface domain of a sphere described by equations $\rho = 15\lambda$, $\theta \in [60, 120]$ deg, $\phi \in [-60, 60]$ deg (see Fig. 7). The voltages (amplitude and phase) have been collected in $M = 31 \times 62$ points of the measurement domain O. The red circled radiating elements in Fig. 7 (b) have been forced off and the remaining ones are excited by a unitary current amplitude. The calculated data have been processed by adopting the proposed diagnostic tool (see Fig. 8). As it can be seen, all the turned off elements are well localized and the amplitude of the excitation coefficients is very well reconstructed for all the radiating elements. So, this confirms the relevance of the proposed method that extend the use to more complex geometry arrays and measurement systems.

4. CONCLUSIONS

A novel and effective deterministic method for detecting faulty elements in large phased arrays of generic geometry has been proposed and tested against experimental data measured at SELEX Sistemi Integrati and numerical data obtained using CST MSW simulation tool. The proposed strategy assumes as input the amplitude and phase of the near-field distribution and allows to determine both positions and currents of all radiating elements, thus detecting location and

number of faulty antennas. By properly adopting a distributional approach, the overall problem has been cast to the solution of a simple linear problem, whose solution is made stable by adopting a proper regularization scheme based on the Truncated Singular Value Decomposition tool.

The achieved results fully confirm accuracy, usefulness and robustness against noise of the proposed technique.

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