

DIRECT RADIATING ARRAYS FOR SATELLITE COMMUNICATIONS VIA APERIODIC TILINGS

A. F. Morabito and T. Isernia

Dipartimento di Ingegneria Informatica, Matematica
Elettronica e Trasporti (DIMET)
Università Mediterranea di Reggio Calabria
Via Graziella, Reggio Calabria 89100, Italy

M. G. Labate and M. D’Urso

Giugliano Research Center, SELEX Sistemi Integrati
Via Circumvallazione Esterna, Giugliano, Napoli 80014, Italy

O. M. Bucci

Dipartimento di Ingegneria Biomedica, Elettronica e delle
Telecomunicazioni (DIBET)
Università di Napoli Federico II
Via Claudio 21, Napoli 80125, Italy

Abstract—This paper presents an innovative Direct Radiating Array (DRA) architecture exploiting aperiodic tilings of the plane. In particular, a *pinwheel tiling* has been selected in order to fix positions of the different radiating sources, which are constituted by properly shaped elements. Such a choice allows to achieve a good aperture efficiency and very low *pseudo-grating* lobes while using only two different kinds of radiating elements. Preliminary results are shown and discussed with reference to both cases wherein the single tiles are not fully populated and wherein *ad-hoc* sub-array radiators are used. The very encouraging results achieved leave open the way for further interesting possibilities.

Corresponding author: T. Isernia (tommaso.isernia@unirc.it).

1. MOTIVATIONS

Array antennas are very well established solutions in order to achieve directive (or even shaped) patterns while guaranteeing the capability to steer the pattern, or even to reconfigure it at all. As well known, usual architectural solutions are based on positioning the different elements on a regular (usually quadrantal, or triangular) grid of points on a plane. In order to avoid grating lobes entering into the visible region, the spacing amongst the different elements cannot exceed a given fraction of wavelength (depending on the constraints at hand) [1]. Such a circumstance, in turn, implies that a very large number of radiating elements is required, which is the price one pays for achieving simple scanning and/or reconfigurability.

In communications from geostationary satellites, wherein multi-beam antennas generating several pencil beams are of interest, the requirement on spacing can be somehow relaxed.

As a matter of fact, simple calculations show that a maximum spacing amongst the different elements as large as 3.5λ guarantees that no grating lobe will appear inside the earth cone [1, 2]. It is also worth to note that, by using such a large spacing, the single radiating elements are themselves directive, thus usefully contributing to the overall link budget. Notably, such a kind of solution dramatically reduces the number of elements and henceforth of control points (i.e., amplifiers and phase shifters) with respect to traditional half-wavelength spacing solutions. In fact, a reduction of a factor roughly equal to $(3.5/0.5)^2 \simeq 50$ is expected [1–3]. On the other side, notwithstanding the fact that relatively large (and hence directive) elementary bricks can be used, grating lobes of the array factor could significantly lower directivity (and henceforth the gain) of the overall array. Being power a very precious resource on satellites, this is a non-tolerable circumstance. Also, the number of antennas (and control points) keeps anyway quite large (something around 900 elements for a circular shape array with radius 60λ).

Therefore, an interest has recently raised in devising alternative architectural solutions for array antennas devoted to transmission (and reception) from satellites [2, 4–7]. In particular, the goal is to design multi-beam antennas such to fulfil all the given constraints in terms of directivity masks for the different beams while using a number of control points as small as possible. In order to have an efficient power exploitation on board, it is also required that all the amplifiers work at the same (optimal) level.

At present time, there seems to be in the open literature three different kinds of architectures in order to address such a problem,

i.e.: **sparse arrays** [4, 5, 8–12] (i.e., arrays whose uniformly excited elements are properly located onto a non regular grid), **thinned arrays** [13–17] (wherein, starting from an otherwise regular arrays, the required performances are achieved by properly withdrawing a certain number of elements), and **clustered arrays** [1, 6, 7, 18–20] (wherein the overall array is subdivided into a number of possibly different uniformly excited sub-arrays). By noting that a number of **hybrid solutions** [21–23] are also possible, we will focus our attention on the third chance. In this latter, in order to exploit all the available power, the final amplitude excitation of each single radiating element will be a *quantized* [1] one (so that the sum of the squared amplitudes of the excitations on each sub-array gives back a unitary power).

The paper is organized as follows. In the next section, the main idea is presented and the concept of aperiodic tiling recalled. Moreover, the problem of choosing the most convenient tiling is discussed and proper selecting criteria are introduced and developed. Then, in Section 3 we test the proposed architecture in both cases wherein the single tiles are fully filled or not, and performances with scanning are also tested.

Conclusions, including a discussion on the possibility to achieve a Tx/Rx DRA, follow.

2. THE BASIC IDEA, AND THE PROPOSED SPECIFIC ARCHITECTURE

A very basic idea in order to reduce the number of control points in a large array amounts to use a reduced number of antennas [4, 5, 7], (which could be by themselves *brick-arrays* [20]). Then, gathering of the different brick antennas into larger sub-arrays (each fed by a different amplifier working in the same optimal conditions) can allow the desired aperture tapering (by means of quantized excitations on each radiator) and, in turn, far field shaping [7, 20].

In so doing, for industrial reasons, the number of different kinds of brick-elements (or brick-arrays) should be as low as possible. Also, in order to have directivity patterns as large as possible, a good aperture efficiency has to be realized (i.e., one should not have significant *holes* throughout the aperture). These two requirements are fulfilled by the solutions proposed in [6, 18], wherein the so called *domino* and *polyomino* array antennas have been introduced.

However, in both these cases the *brick* sub-arrays are positioned onto a regular grid with a large spacing, so that grating lobes (or *pseudo grating lobes*) of the underlying array factor still induce limitations on performances. Also, the performances of the array depend on the

different arrangements one can give to the different tiles, so that they cannot be easily foreseen.

In order to tackle these problems, we pursue herein the idea, first proposed in [20], of exploiting *aperiodic tilings* of the plane. These latter (see [24] for a thorough discussion) are different sets of tiles such to cover the plane without holes or overlapping, while being devoid (opposite to periodic grids) of any translational symmetry. This latter *lack of periodicity* (exhibiting anyway some kind of order) makes these structures particularly interesting insofar realization of arrays with a large average spacing is concerned. In fact, the lack of periodicity induces a much increased robustness against pseudo grating lobes. Then, the idea we want to exploit herein is to adopt elementary brick radiators having the shapes of one of the sets of tiles such to be able to partition the plane in an aperiodic fashion. In this way, we will be able to contemporarily get a very good aperture efficiency and pseudo grating lobes which are (possibly much) reduced with respect to that of regular periodic grids. As discussed in [20], a (tolerable) price will be paid, whatever the aperiodic tiling, in terms of polarization performances, as only circular polarizations can be easily dealt with. Also, some difficulties will arise because of the need of properly nesting the tiles.

Amongst the different aperiodic tilings, one has to choose the most convenient ones. In so doing, one should take into account different criteria such as, for instance, the number of different kind of tiles which are required, their relative dimensions and shapes, which could be easy or difficult to be realized and nested. Last, but not least, the spectral properties of the different tilings [24] also play a key role. In fact, while one cannot define for this kind of arrays an array factor (as no element pattern can be factored out), the spectral properties of the different grids play a role analogous to the array factor, thus allowing to foresee some properties of the final pattern.

While in [20] the discussion was limited to analysis of a given tiling in order to assess potentialities, a careful selection has been done herein in order to take into account the above criteria. In particular, we have selected the *pinwheel tiling* [24], whose elementary bricks and way to partition the plane in a non periodic fashion can be deduced from [24] and from Fig. 1.

The reasons for such a choice can be summarized as follows. As in the well-known Penrose *thick and thin* tiles [20], relatively simple shapes (this time, triangles) are dealt with, and only two different kind of tiles (which are each the reflected along one side of the other one) are considered. Then, two other circumstance come into play.

First, the fact that (opposite to the Penrose tiles [20]) the pinwheel

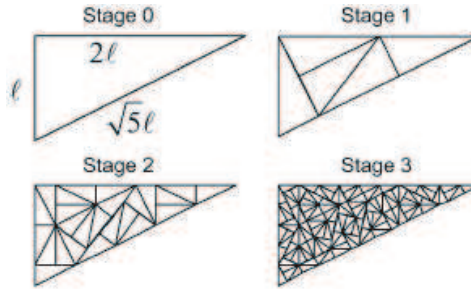


Figure 1. First three iterations generating the pinwheel tiling.

tiles have exactly the same area allows a better control of the tapering of the aperture field under the given (amplifier induced) quantization constraints on each tile or cluster of tiles.

Second, the spectral properties of the pinwheel tiling outperform those relative to other tilings exhibiting only two shapes (such as the Penrose couples) [24]. This latter circumstance is easily explained by recalling that while the Penrose tiling exhibits some kind of decagonal symmetry (and hence pseudo-grating lobes along ten different directions), no similar symmetry is found with the pinwheel tiling, with a consequent uniform spreading of energy along circles of constant radius in the spectral domain [24].

3. THE PROPOSED ARCHITECTURE AT WORK

In order to check the validity of the proposed architecture, we thought it worthwhile to try to synthesize a radiation pattern fulfilling the specifications of the ESA tender [25]. Roughly speaking, at a frequency of 20 GHz, these specifications require to design an array (with a maximum radius of 60λ , being λ the wavelength in free space) radiating at least 19 different spots located 0.56° each from the other. Each beam should have a gain of at least 43.8 dBi at the End Of Coverage, (EOC), positioned 0.325° from the maximum, and sidelobes 20 dB lower than the EOC gain in all directions out of the 0.795° cone from maximum [25] and the separation amongst neighbouring beams is ensured by either a sub-band or polarization diversity. Finally, requirements on the level of grating lobes outside the coverage zone and outside the earth are also present.

Following the approach suggested in [26], as a first step of the overall synthesis procedure, an optimal continuous aperture distribution acting as a reference was determined, by properly stating

the synthesis problem as a convex optimization one. By deferring the interested reader to [26, 27] for more details, we just note that a suitable non trivial extension of [28] to the case of directivity constraints and continuous sources is herein adopted. Fig. 2 and Fig. 3 show the optimal continuous circularly symmetric distribution and the corresponding directivity pattern.

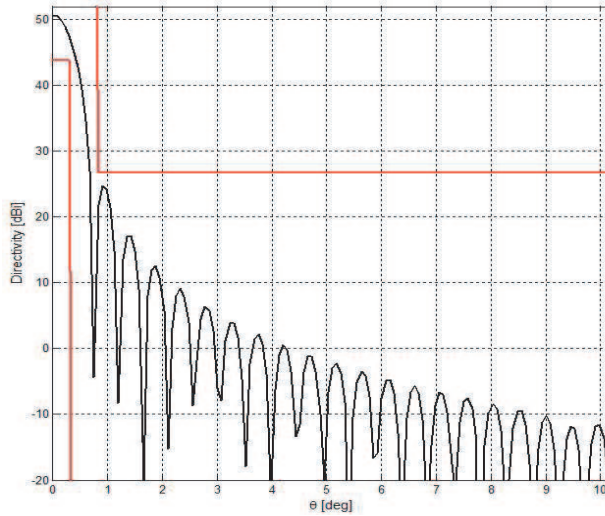


Figure 2. Optimal directivity pattern fulfilling the design constraints of [25].

It is worth to note that the optimal distribution, while being anyway easy to realize, is indeed different from the usually exploited Taylor distributions [1]. Then, this *optimal* continuous distribution was approximated by means of a staircase radial function with levels proportional to $1/\sqrt{2^n}$, and each brick antenna belonging to the i -th ring was assumed to be excited with the corresponding level of the staircase. Note that in this way only 3 dB dividers are required in each subarray. Fig. 4 shows the considered stepped aperture distribution, obtained by minimizing the quadratic norm of the difference between the reference distribution and the stepped one.

Adopting a pinwheel with a minor side $l = 4\lambda$, in a first instance, in order to keep things simple, we put a uniformly excited circular aperture (of the maximum possible diameter, equal to 3.1λ) in the in-centre of each tile, as shown in Fig. 5.

As a matter of fact, in each radial coordinate one determines the value (amongst the admissible ones) which is closest to the ‘ideal’ one.

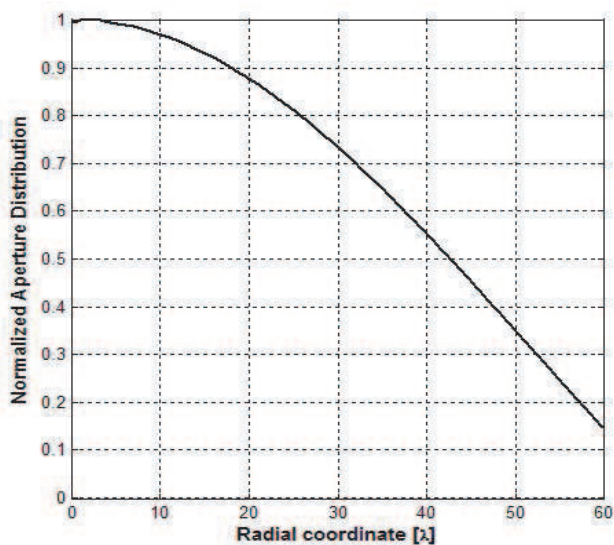


Figure 3. Aperture distribution corresponding to the directivity pattern of Fig. 2.

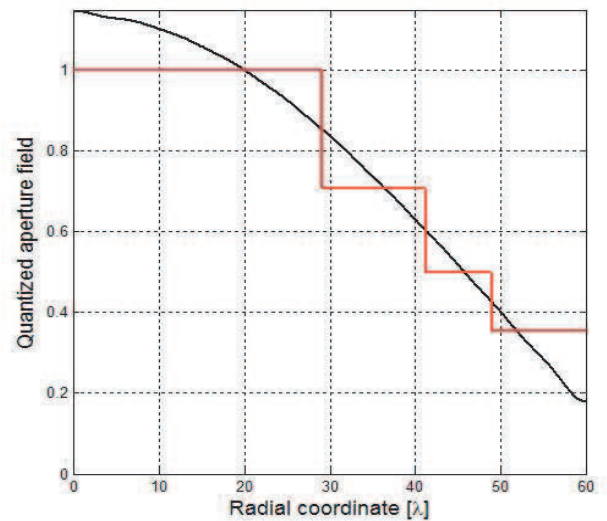


Figure 4. Reference solution quantized into the levels $1/\sqrt{2^0}$, $1/\sqrt{2^1}$, $1/\sqrt{2^2}$, $1/\sqrt{2^3}$, according to a *simple best-fitting* scheme.

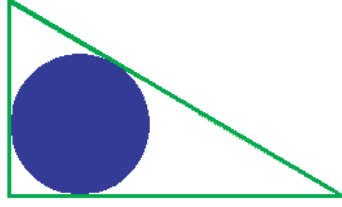


Figure 5. Single circular aperture of diameter 3.1λ in tile in-centre.

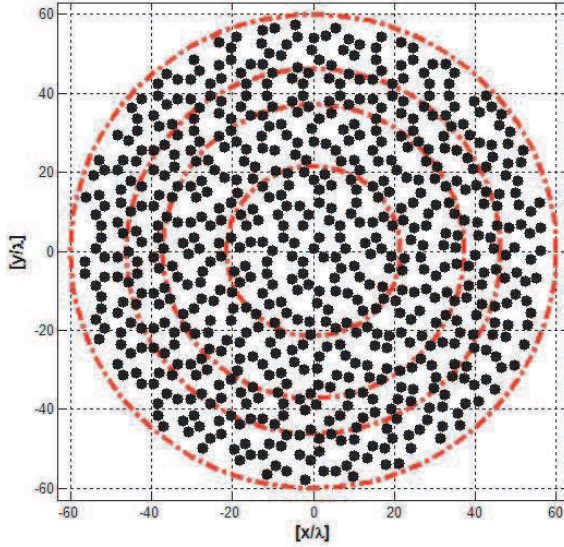


Figure 6. Locations of the radiating sources and rings radii. The single circular source has a diameter of 3.1λ .

This results in a best L_2 approximation (under the given quantization constraints) of the source and hence, because of the Parseval theorem, of the corresponding spectrum. In order to have an accurate estimation of the power pattern and of the total radiated power (and hence of the directivity pattern), field samples have been computed on a very dense grid, corresponding to more than 16 samples per wavelength in the spectral plane U-V. The achieved results, corresponding to an actual radius of 58λ and 657 ‘elementary’ antennas, are shown under Fig. 6 and Fig. 7, and only exploit 251 control points. In fact, a control point corresponds herein to an entry point feeding (with an overall unity power, and a common phase) one or more antennas (tiles)

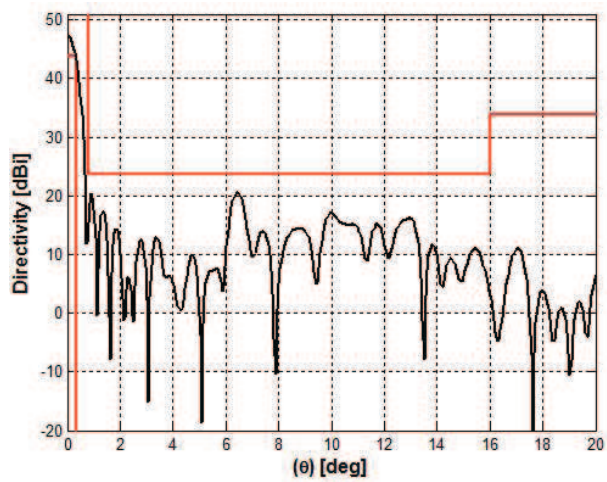


Figure 7. Directivity pattern corresponding to the DRA array layout reported in Fig. 6. The DRA working frequency in *Tx* mode is 20 GHz.

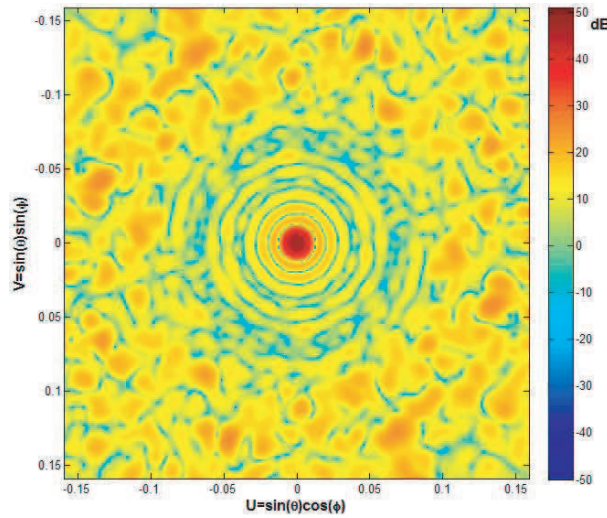


Figure 8. U-V representation of the directivity pattern reported in Fig. 7.

contemporarily. Then, as one has a control point for each tile in the central part, one control point every two tiles in the first annular ring, and so on and so forth (according to the staircase behavior of Fig. 4), a

total of 251 control points is finally achieved. In Fig. 6 the locations of the different antennas and the borders of the different rings are given, while Fig. 7 shows a cut of the corresponding directivity pattern, which, by virtue of the properties of the tiling, is circularly symmetric but for unessential far away asymmetries (see Fig. 8).

As can be seen, the proposed architecture, while fulfilling the constraints on the side lobes (22.6 dB lower than the EOC gain on the worst cut), gives back an EOC directivity of about 43.4 dBi, which is slightly below the requirement. However, the radiating source only covers roughly 46% of each tile (corresponding to an aperture efficiency of -3.4 dBi on each tile), so that the EOC directivity decrease with respect to the maximal achievable one (46.8 dBi, see Fig. 2) exactly checks with the above decrease of the tile aperture efficiency.

Such a circumstance implies that the performances achieve the theoretical limitations (see Fig. 2) established by the continuous reference source of Fig. 3 but for the aperture efficiency of the single tile.

Accordingly, (see Fig. 11 for a cut of the essentially circularly symmetric pattern) we can foresee that the EOC performance can be ameliorated by a better exploitation of the area of each tile. In order to confirm this expectation, a second situation has been considered, wherein two uniformly excited circular apertures, with diameters 3.1λ and 1.9λ , respectively, are put in each tile, see Fig. 9. The total occupancy of the single tile is now 64%, which corresponds to an aperture efficiency of -1.9 dB. The corresponding architecture (reported in Fig. 10) allows a full satisfaction of constraints in [25], see Fig. 11 for a cut of the essentially circularly symmetric pattern. An EOC directivity of 44.7 dBi is achieved, thus gaining 1.3 dB with respect to the previous solution, which, again, checks with the increase in the tile aperture efficiency. Again, performances achieve the theoretical limitations (see above) but for the aperture efficiency of the single tiles.

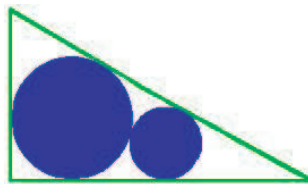


Figure 9. Tile with two elementary circular sources, of diameters 3.1λ and 1.9λ , respectively.

Notably, the insertion of a second circular source into each tile, while increasing the occupation of the tile from 46% to 64%, does not

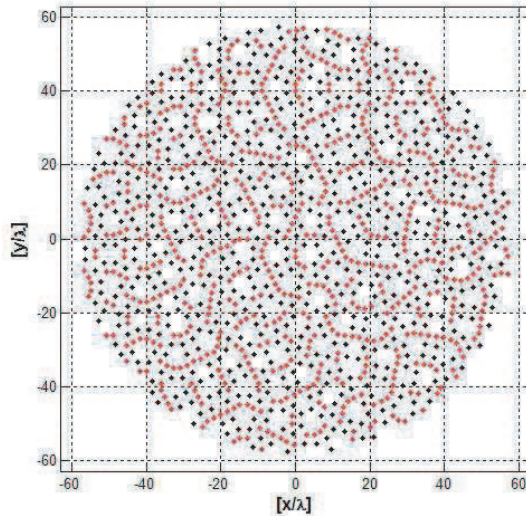


Figure 10. Direct Radiating array with two circular apertures for each tile.

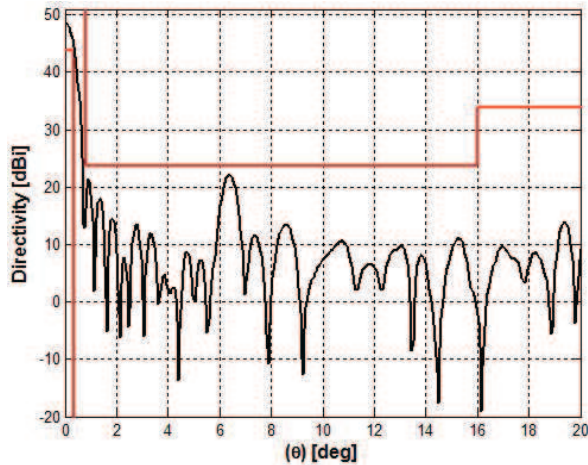


Figure 11. Directivity pattern corresponding to the DRA of Fig. 10 and Fig. 9.

increase neither the overall dimensions of the array nor the number of active control points.

Such a result has stimulated an interesting search to find out the way to exploit at best the geometry of the *pinwheel* tiles, i.e., to cover the whole area of the tiles.

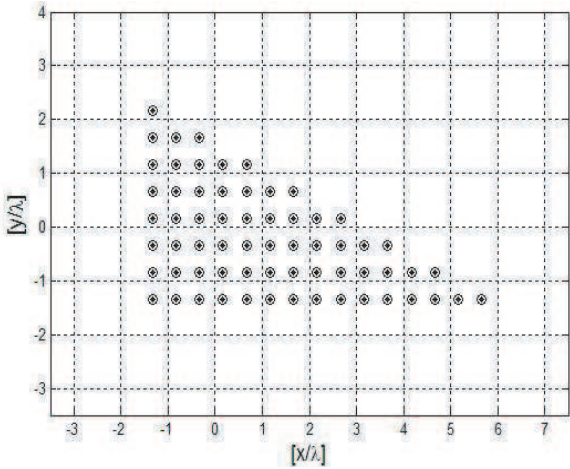


Figure 12. Sub-array covering the whole tile with 64, $\lambda/2$ spaced, radiating elements.

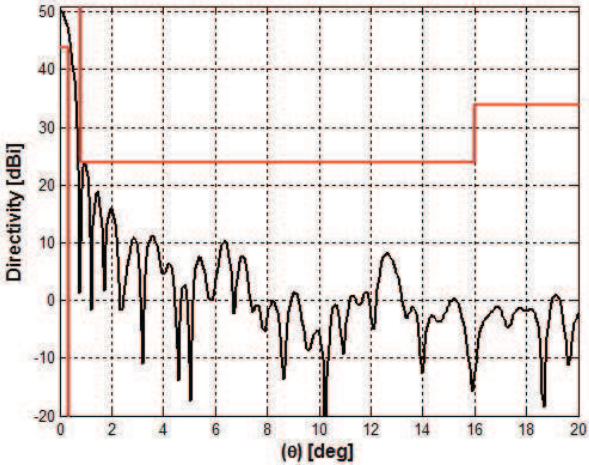


Figure 13. Broadside directivity pattern corresponding to the pinwheel tile of Fig. 12.

A rather obvious way to achieve this goal is to use a sufficiently dense sub-array covering the whole tile, as for example by adopting the triangular array shown in Fig. 12, made by 64, $\lambda/2$ spaced, elementary sources.

By so doing, as shown in Fig. 13, an EOC directivity of 46.6 dBi

(very near to the best possible performance) is achieved, with a 3.2 dB improvement with respect to the first solution, while still fulfilling the sidelobes constraints (sidelobes 21.6 dB lower than the EOC gain on the worst cut). This likely makes such a kind of solution compliant with the design constraints of [25] even when taking into account the losses of the single elements and of the sub-array Beam Forming Network (BFN).

Therefore, the presented DRA architecture deserves a particular interest, as it allows to sample at best in a very flexible way a given aperture distribution, while being as much as possible insensitive to grating lobes problems. Anyway, the analysis clearly shows the capability of the proposed architecture to fulfil the given constraints for the central spot with a reduced number of control points.

A further reduction of the number of control points is presumably possible for the central beam. However, the actual goal is to synthesize a multi-beam antenna, so that performances have to be checked also for beams pointing towards directions other than bore-sight. When pointing the beam towards a different direction by means of a suitable linear phase across the array control points, three inter-related problems affecting performances come into play, i.e.:

- i high lobes originally outside the visible region (we could refer to them as to *pseudo* grating lobes) may enter the observation space, thus impairing directivity and/or separation;
- ii every cluster of tiles has a single entry point with a unique phase value, so that the ideal linear phase across the individual radiating elements is approximated by means of a step-like behavior (the dimension of each step will be possibly large in case of large tiles and/or clusters). As a consequence, the corresponding array factor is not simply a translation of the original one;
- iii large individual elements may imply a degradation due to a decaying of the corresponding element factor.

Luckily, all these causes of degradation are mitigated by the fact that in the application at hand the different required beams are quite near in terms of angles. For instance, a maximum scanning angle of 1.12° is required in [25]. For such a scanning angle the performances one achieves by using the DRA array architecture achieved by using as elementary brick the triangular array of Fig. 12 has a directivity at the EOC equal to 46.35 dBi, and is fully compliant with all design specifications (sidelobes 21.5 dB lower than the EOC gain on the worst cut), see Fig. 14. Again, the available margin amongst the realized directivity and the required gain at EOC implies that constraints could be satisfied even when taking into account typical losses of BFN and single elements.

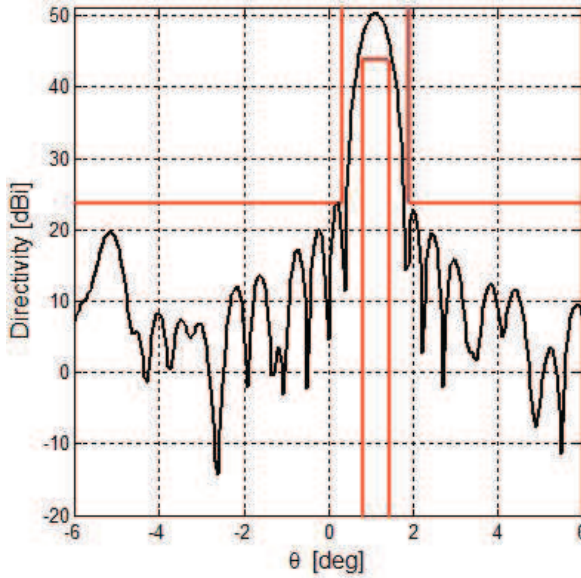


Figure 14. Scanning performances of the DRA with sub-array tiles.

4. CONCLUSIONS, AND A FURTHER INTERESTING CHANCE

Above analysis clearly shows the capability of the proposed architecture to fulfil typical requirements of antennas dedicated to multi-beam Tx from satellites using a reduced number of control points.

By virtue of the achieved robustness with respect to grating lobes, performances of the architecture are somehow related to the degree of filling of the single tile.

Moreover, some margin is available in terms of performances (at least insofar directivity is concerned) in case of fully populated tiles.

These latter two circumstances suggest a possible architectural solution for another problem of interest in satellite communications, i.e., the design of arrays able to transmit and receive at the same time, wherein the Tx and Rx chains operate at different frequencies. For instance, multibeam Rx capabilities at frequencies around 30 GHz are also looked for in [25]. Then, on the basis of all the above, an obvious possibility amounts to exploit a part of each tile for the Tx mode, and another part for the receiving one.

For instance, the two circular apertures of Fig. 9 could be respectively dedicated to Tx and Rx in the two frequency bands (see

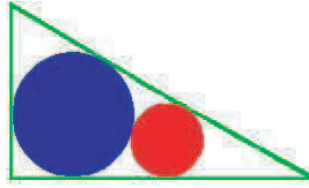


Figure 15. The second circular aperture, the one with diameter 1.9λ , is now devoted to Rx functions at 30 GHz, according to [25].

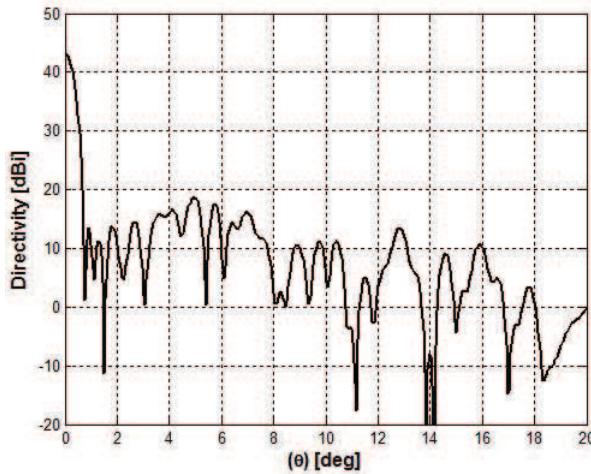


Figure 16. Directivity pattern in Rx modality at 30 GHz.

Fig. 15). Although some unavoidable degradation in performances must be accepted, the Rx directivity pattern of Fig. 16 fully confirms the viability and the interest of the proposed Tx/Rx architecture, and suggests to develop the idea in further depth.

As a final comment, note that the overall approach can be obviously exploited in other design problems by establishing a suitable continuous source in a first step, which is then discretized (according to a best-fitting criterion, see Fig. 4) onto the structure constituted by the pinwheel tiles. Once the pinwheel tiling has been chosen according to the criteria in the second part of Sect. 2, the only thing which still remains to be fixed is the density of the tiling. In fact, opposite to the methods in [6] and in [18], which lead to a random partition of the plane, the present partition, summarized by means of Fig. 1, leads to a deterministic partition of the plane. As far as the choice of the density of the grid is concerned, this comes out from a trade off amongst the

will to reduce the number of tiles, and the need to keep under control the pseudo grating lobes which however come into play (see [24]) for very large average spacings.

ACKNOWLEDGMENT

Main part of the work has been performed when M. G. Labate and M. D'Urso were with DIMET, Università Mediterranea di Reggio Calabria.

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