A SIMPLE FOUR-ORDER CROSS-COUPLED FILTER WITH THREE TRANSMISSION ZEROS

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Abstract—Generalized cross-coupled filters require implementation of both positive and negative cross-coupled elements. A positive element frequently uses inductive coupling, while a negative one uses capacitive coupling. Traditional methods for realizing capacitive couplings which are difficult to adjust in practice have included the use of capacitive probes in coaxial cavity. And this kind of n-order cross-coupled filters without the coupling between input and output ports can only produce $n-2$ transmission zeros at most. In this paper, we present a convenient method for capacitive coupling. Based on the method a four-order cross-coupled filter is realized, and the measured results match well with the theoretical prediction. Especially, there are three transmission zeros near the pass band.

1. INTRODUCTION

Quasi-elliptic filters with finite frequency transmission zeros are now finding ever-increasing applications in wide range of wireless and mobile communication systems due to their high performance and compact size. These filters provide multiple coupling paths through which a signal may pass, so that a signal cancellation can take place at given finite frequencies for enhancing a skirt selectivity, or at given imaginary frequencies for achieving a flat group delay, or even both simultaneously, depending on the phase and magnitude conditions of signals [1, 2].

It is well known that both positive and negative coupling are needed to generate transmission zeros at finite frequencies in a cross-coupled filter [3]. Positive coupling can be obtained conveniently by a magnetic coupling structure, e.g., it can be realized by a square

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aperture between the waveguide cavity narrow walls. However, in many cases, it is difficult to produce a negative coupling. Usually, in a canonical folded waveguide filter, a square aperture between the top and bottom cavities can be used to generate the required negative coupling [4, 5]. In coaxial cavity filters, in order to obtain the negative coupling, the electric length of metal rod in coaxial cavity has to be greater than $\pi/4$, and the capacitive probe is necessary. All of them would make the structure more complex, and it is so hard to machine for achieving a nonadjacent electric coupling coefficient [6, 7]. In this paper, we present a simple structure for achieving negative coupling without the capacitive probes in coaxial cavity.

The paper is organized as follows. In Section 2, derived from coupled-line, the theoretical solutions of positive and negative coupling in coaxial cavity are investigated. Synthesis, realization, and the experiments of filter will be presented in Section 3. Here, we manufacture a cross-coupled filter which the signal is fed by magnetic coupling at input/output. Finally, conclusions are given in Section 4.

2. COUPLING ANALYSIS

The filters of coaxial cavity mainly use the quarter-wave resonators. There are two important characteristics in these resonators. It is obvious that at resonance of the fundamental mode, each of the quarter-wave resonators has the maximum electric field density at the open-end and the maximum magnetic field density at the opposite side [8]. It follows that the electric coupling can be obtained if the open-ends of two coupled resonators are proximately placed, and the magnetic coupling can be obtained if the sides with the maximum magnetic field are proximately placed. It seems easy to realize both positive and negative coupling by adjusting the position and size of the square aperture between two coaxial cavities. However, even if the position of the square aperture between two coaxial cavities is near the open sides, the coupling would be mostly magnetic coupling, because the magnetic and electric coupling always exists at the same time. Furthermore, the electric coupling is weaker than magnetic coupling in case when the value of equivalent capacitance at the open end is large enough. The capacitance would reduce the electric length of metal rod in coaxial cavity and deposit the main electric field. To generate electric coupling we should make use of another characteristic of the quarter-wave resonator. We know that the electric coupling decays faster than the magnetic coupling against the spacing [3]. On the other hand, the electric coupling is dominant for the small coupling spacing, whereas the magnetic coupling becomes dominant when the

spacing is larger. Thereby if we make the spacing of metal rods closer, the electric coupling would be stronger than magnetic coupling. Using these two characteristics, it is very easy to obtain electric coupling without capacitive probes and realize a cross-coupled filter.

We know that two quarter-wave resonators of coaxial cavity have the same coupled characteristics as the quarter-wave resonators of the coupled transmission lines. It is convenient to create circuit model and analyze the phase and magnitude responses by the theory of coupled transmission lines. According to the condition of square aperture between resonators and the coupling modes at input/output ports, there are four cases should be discussed as follows.

CASE A: If the size of the square aperture between quarter-wave resonators of coaxial cavity is large enough, coupling was taken place at the open-end and short-end of two coupled resonators. In addition, the signal is fed by the electric coupling at input and output. As shown in Figure 1, the M2CLIN is the model of two quarter-wave resonators of coaxial cavity.
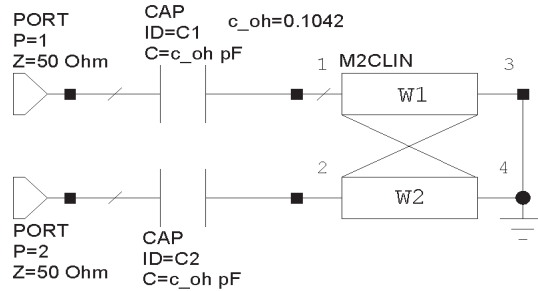
Figure 1. The model of CASE A.

Because the network is symmetrical, the network analysis will be simplified by even- and odd-mode networks parameters [9]. For simplicity, we assume that the coupled lines have the same phase velocity for the even- and odd-modes, so the input even- and odd-mode impedance can be given by

$$Z_{ine} = Z_{0e} \frac{Z_L + jZ_{0e}\text{tg}\frac{\pi w}{4w_0}}{Z_{0e} + jZ_L\text{tg}\frac{\pi w}{4w_0}} + \frac{1}{jwc} \tag{1}$$

$$Z_{ino} = Z_{0o} \frac{Z_L + jZ_{0o}\text{tg}\frac{\pi w}{4w_0}}{Z_{0o} + jZ_L\text{tg}\frac{\pi w}{4w_0}} + \frac{1}{jwc} \tag{2}$$

where the Z_{0e} is the even-mode characteristic impedance and Z_{0o} is the

odd-mode characteristic impedance. The w_0 is the resonance frequency of one resonator. Because the 3rd and 4th ports are connected to ground, the value of Z_L should be 0. Referring to reflection coefficient, we have the even- and odd- mode S parameters.
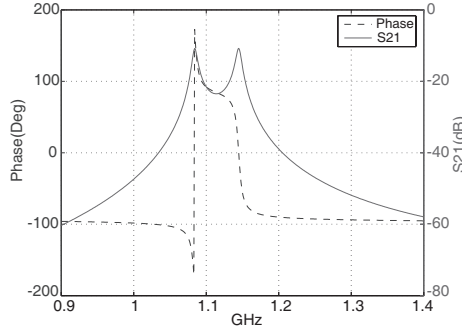
$$S_{11e} = \frac{Z_{ine} - Z_0}{Z_{ine} + Z_0} = \frac{jZ_{0e}\text{tg}\frac{\pi w}{4w_0} + \frac{1}{jwc} - Z_0}{jZ_{0e}\text{tg}\frac{\pi w}{4w_0} + \frac{1}{jwc} + Z_0} \tag{3}$$

$$S_{11o} = \frac{Z_{ino} - Z_0}{Z_{ino} + Z_0} = \frac{jZ_{0o}\text{tg}\frac{\pi w}{4w_0} + \frac{1}{jwc} - Z_0}{jZ_{0o}\text{tg}\frac{\pi w}{4w_0} + \frac{1}{jwc} + Z_0} \tag{4}$$

The S parameters of network is then given by [9]

$$S_{11} = \frac{1}{2}(S_{11e} + S_{11o}) \tag{5}$$

$$S_{21} = \frac{1}{2}(S_{11e} - S_{11o}) \tag{6}$$

Adjusting the Z_{oe}, Z_{oo}, we can get the coupling coefficient and phase characteristic which we need (See Figure 2).
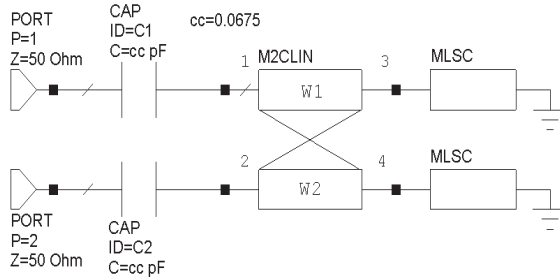

Figure 2. The phase and magnitude of S_{21} in CASE A.

CASE B: If the square aperture between quarter-wave resonators of coaxial cavity is small and near the open-end of the resonators, there is no coupling between short-ends. In addition, the signal is fed by the electric coupling at input and output. The circuit model is shown in Figure 3. The resonators are made up of M2CLIN and MLSC.
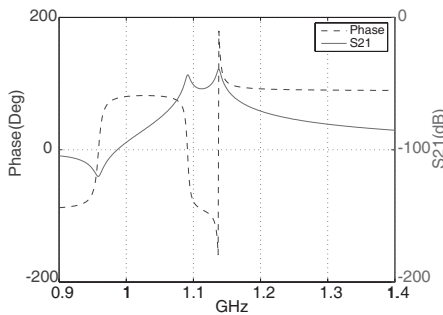
Similarly, we have:

$$Z_{ine} = Z_{0e}\frac{Z_L + jZ_{0e}\text{tg}\theta_{M2CLIN}}{Z_{0e} + jZ_L\text{tg}\theta_{M2CLIN}} + \frac{1}{jwc} \tag{7}$$
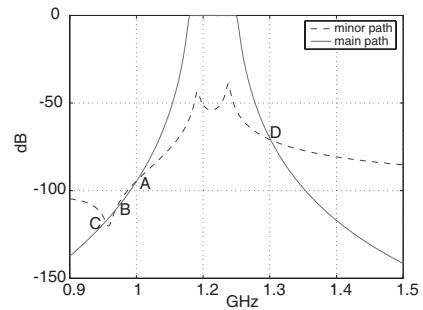
$$Z_{ino} = Z_{0o}\frac{Z_L + jZ_{0o}\text{tg}\theta_{M2CLIN}}{Z_{0o} + jZ_L\text{tg}\theta_{M2CLIN}} + \frac{1}{jwc} \tag{8}$$

Figure 3. The model of CASE B.

Figure 4. The phase and magnitude of S_{21} in CASE B.

Figure 5. The magnitude relation of minor and main path.

Because of the MLSC connecting to the coupled line, Z_L should be $jZ_0 \mathrm{tg}\theta_{MLSC}$. Where θ_{MLSC} is the electric length of MLSC. In addition, θ_{M2CLIN} is the electric length of M2CLIN. When resonating, they satisfy:

$$\theta_{M2CLIN} + \theta_{MLSC} = \frac{\pi}{4} + n\frac{\pi}{2} \quad n = 1, 2, \ldots \tag{9}$$

Referring to the functions (3) to (6), we can get S parameters of the network. The response is shown in Figure 4.

By comparing the phase responses in Figure 2 and Figure 4, we can observe that both are out of phase near the resonant frequencies. The simple structures of positive and negative coupling in coaxial cavity are what we search for. From the magnitude of S_{21} in Figure 4, we find out that there is one transmission zero locating at low frequency. In fact, the magnetic and electric coupling all exist between resonators in Figure 3. They would cancel each other [9] and then produce

one transmission zero. The location of transmission zero is mainly depended on two conditions. One is the ratio of θ_{M2CLIN} and θ_{MLSC}. The other is the strength of the electric coupling at input/output ports. These characteristics are very useful. For example, when we design a four-order cross-coupled filter, the main coupled path can be made up of the structure shown in Figure 1, and the minor coupled path can be made up of the structure shown in Figure 2. Because one transmission locates at low frequency in Figure 4, it would easily lead to four points where the main and minor paths have the same magnitude of S_{21}, as shown in Figure 5.

We know that when the signal which passes through the main coupled path has the same power and opposite phase as the one which passes though the minor coupled path, a transmission zero would occur. In Figure 5, according to the phase relation in Figure 2 and Figure 4, the points A, B and D satisfy the conditions, and the point C is in phase. It means that there are two transmission zeros at low stop band and one at high stop band, resulting in higher selectivity than normal cross-coupled filter on the side of low pass band.

CASE C: If the size of the square aperture between the quarter-wave resonators of coaxial cavity is large enough, coupling would take place at the open-end and short-end. In addition, the signal is fed by the magnetic coupling at input and output. The circuit model is shown in Figure 6.

Here, the W_1 of M2CLIN and W_2 of M4CLIN compose the metal rod of one resonator. It is the same as the W_2 of M2CLIN with W_3 of M4CLIN. The W_1 and W_4 of M4CLIN act as a magneto coupler.
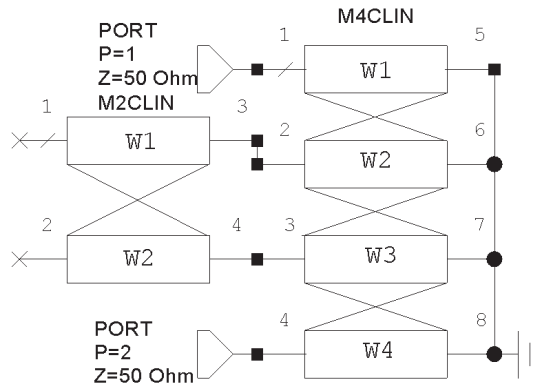
Figure 6. The model of CASE C.

In this case, it is better to analyze this model by the ABCD matrix of coupled line. The ABCD matrix parameters for the coupled line may be computed by [10]. The responses are shown in Figure 7.

CASE D: If the square aperture between quarter-wave resonators of coaxial cavity is small and near the open-end of the resonators, coupling would only take place at the open-ends. In addition, the signal is fed by the magnetic coupling at input and output. The circuit model is shown in Figure 8. Similarly, this model can be analyzed by ABCD matrix of coupled line. The responses are shown in Figure 9.

By comparing the phase responses in Figure 7 and Figure 9, we can observe that both are out of phase near the resonant frequencies, and there is one transmission zero located at high frequency in Figure 9.
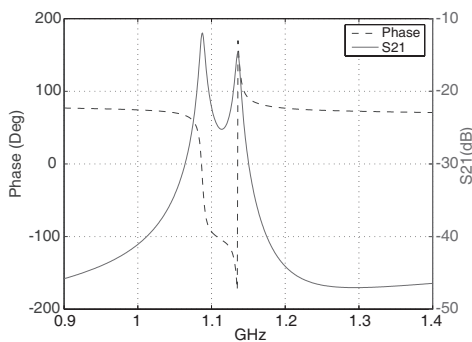

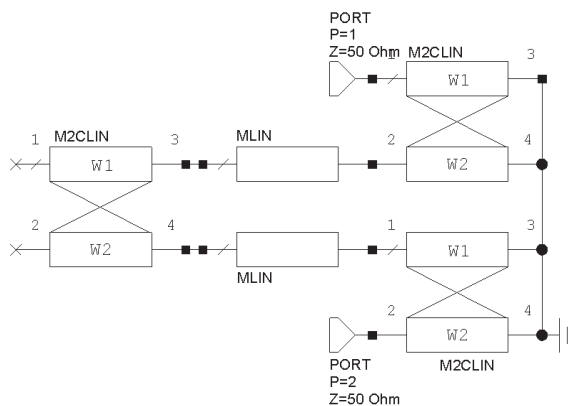**Figure 7.** The phase and magnitude of S_{21} in CASE C.
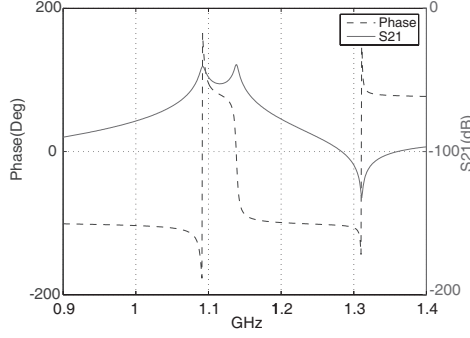

Figure 8. The model of CASE D.

Figure 9. The phase and magnitude of S_{21} in CASE D.

The location of the transmission zero mainly depends on the coupled part's electric length of the resonators and the strength of the magnetic coupling at input and output ports. It is similar to the case introduced previously. We can use these two structures which are described in CASE C and CASE D to conveniently implement a four-order cross-coupled filter. The main coupled path is made up of the structure shown in Figure 6, and the minor coupled path is made up of the structure shown in Figure 8. Because of one transmission locates at high frequency in Figure 9, it would easily lead to four points where the main and minor paths have the same magnitude of S_{21}, as shown in Figure 10. In Figure 10, according to the phase relation in Figure 7 and Figure 9, the points A, B and C is out of phase, and the point D is in phase. So there are two transmission zeros at high stop band and only one at low stop band. That would result in higher selectivity than the normal cross-coupled filter on the side of high pass band.

3. DESIGN EXAMPLES

To verify the theory, we design a four-order cross-coupled filter of coaxial cavity without capacitive probes and manufacture this one. At input/output ports, the signal is fed by the magnetic coupling. For our demonstration, the filter is employed for a power amplification system and designed to meet the following specifications:

   Center frequency $= 1115$ MHz
   Bandwidth of pass band $= 69$ MHz
   Return loss in the pass band < -18 dB
   Rejection > 35 dB for frequency < 1038.1 MHz
   Rejection > 43 dB for frequency > 1187.5 MHz

For simplicity, we can change the specification to the condition that rejection is higher than $35\,\mathrm{dB}$ for frequency $> 1187.5\,\mathrm{MHz}$ and synthesize it according to the synthesis method for normal cross-coupled filter. Choosing the structure in Figure 8 as the minor path to produce the 3rd transmission zeros, it would be easy to optimize $8\,\mathrm{dB}$ for frequency $> 1187.5\,\mathrm{MHz}$. The normal synthesis method for the cross-coupled filters has been presented by Rhodes [11], Atia et al. [1], Wenzel [12], and Levy [2]. For this design, the normalized coupling coefficient matrix and the normalized external quality factor are:

$$m = \begin{bmatrix} 0 & 0.890 & 0 & -0.12 \\ 0.890 & 0 & 0.738 & 0 \\ 0 & 0.738 & 0 & 0.890 \\ -0.12 & 0 & 0.890 & 0 \end{bmatrix}$$

$$q_{ei} = q_{eo} = 0.92$$

The coupling coefficient can then be extracted by using the following relation [9]:

$$k = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2} \tag{10}$$

where f_1 and f_2 stand for the high and low resonant frequencies respectively, and k represents the coupling coefficient between two coaxial cavity resonators. The external quality factor Q_e is calculated by [9]

$$Q_e = \frac{2f_0}{\Delta f_{3\,\mathrm{dB}}} \tag{11}$$

where f_0 is the frequency at which S_{21} reaches its maximum value, and $\Delta f_{3\,\mathrm{dB}}$ is the $3\,\mathrm{dB}$ bandwidth for which S_{21} is reduced by $3\,\mathrm{dB}$ from its maximum value.

Based on those parameters, the filter generally has only two transmission zeros at finite frequencies. The key point is the realization of the normalized coupling coefficient whose value is -0.12. If we get it by the capacitive probe, there would be only electric coupling and only one transmission zero at high stop band. When we get the coupling coefficient by the structure in Figure 8, there would be two transmission zeros at high stop band and one at low stop band. It is easy to achieve in practice. By adjusting the square aperture and the distance between the metal rods of resonator, we can obtain a minor path in which the normalized coupling coefficient is -0.12, and there is one transmission zero in high frequency. According to the coupling coefficient and external quality factor, we create the simulated model and manufacture it. The finished product is shown in Figure 11. The

filter is made of aluminum plating with silver. The simulated and measured responses are given in Figure 12.

In Figure 12, there are two transmission zeros at high stop band and only one at low stop band. It agrees with the case C and case D.

Additionally, for validating our theory, in full-wave electromagnetic (EM) simulator, we have created another simulated model of four-order cross-coupled filter in which the signal is fed by the electric coupling at input/output ports. The simulated report is shown in Figure 13. We find that there are two transmission zeros at low stop band and one at high stop band. It agrees with the case A and case B.
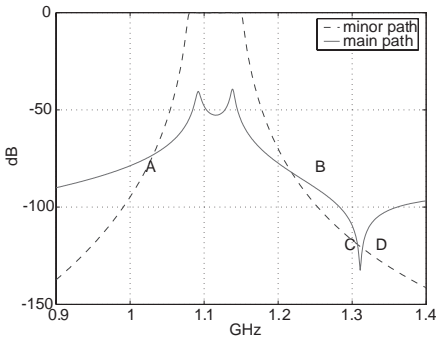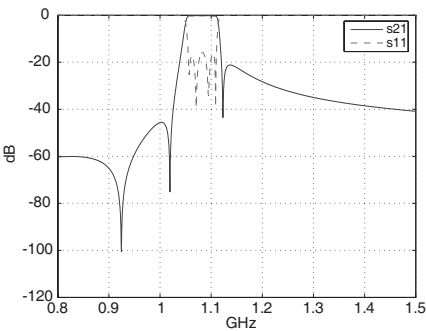

Figure 10. The magnitude relation of minor and main path.
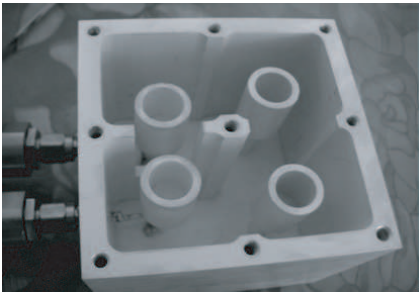
Figure 11. Cross-coupled filter without capacitive probes
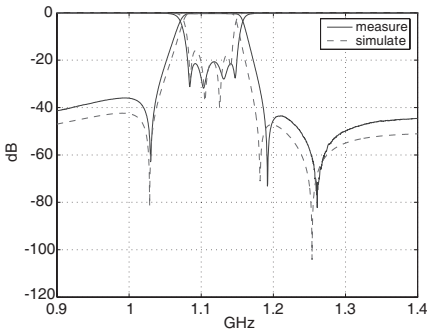
Figure 12. The responses simulated for another filter.

Figure 13. The responses simulated for another filter.

4. CONCLUSIONS

We have presented a simple electric coupling structure to realize the cross-coupled filter in coaxial cavity and taken two examples to verify our analysis. Obviously, the structure is convenient to machine but bring in high performance of filters. Based on this method the designer has higher degree of freedom to design and machine a cross-coupled filter.

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