Progress In Electromagnetics Research M, Vol. 7, 149–164, 2009

RANDOM ERRORS MODELLING AND THEIR EFFECTS UPON RCS FOR AN ARTIFICIAL OBJECT CONTAINING THIN LONG PEC NEEDLES

Z. A. Awan and A. A. Rizvi

Department of Electronics
Quaid-i-Azam University
Islamabad, Pakistan

Abstract—The scattering of electromagnetic plane wave from an artificial object containing thin long perfectly conducting needles embedded in a homogeneous background material is characterized by parameters like positioning, orientations and lengths of needles. Firstly, models of random errors in positioning and orientation of perfectly conducting needles are proposed. Secondly, their effects upon ensemble averaged RCS is analyzed. It is investigated theoretically that increasing error in positioning and orientation of conducting needles reduces ensemble averaged RCS.

1. INTRODUCTION

Scattering of electromagnetic waves from perfectly conducting thin wires has been the subject of several investigations. It has been of great interest in the study of artificial wire medium or called rodded medium, Epsilon-Negative (ENG) materials, sensing technology and radar engineering problems. Scattering characteristics of a single finite length perfectly conducting thin wire has been analyzed in [1–4] using numerical and approximate analytical techniques. Lin and Maston studied the backscattering from two identical finite length perfectly conducting parallel thin wires based upon integral equation method [5].

A surface composed of densely packed thin long wires is called super-dense dipole surface or gangbuster surface in the literature [6, 7]. For such types of surfaces, wire lengths may be greater than the lattice periodicity. These surfaces have applications in broad-band frequency-selective surfaces and polarization-selective surfaces. Experimental

Corresponding author: Z. A. Awan (zeeshan@qau.edu.pk).

study of scattering of EM waves from parallel wires is given in [8, 9]. In the papers listed above, thin wires or needles were assumed to be parallel to each other. The effects of randomly oriented wires upon scattering properties have been analyzed by Dedrick using Stokes parameters [10].

A little effort has been made to analyze the effects of random errors in positioning and orientation of needles. In [11], effects of random errors in positioning are analyzed with reference to frequency selective surfaces and linear antenna arrays. Manabe [12] has analyzed the effects of slight irregularity in grid position and grid rotation of wire grids consisting of conducting and infinitely long wires in millimeter and sub-millimeter wave regions. The T-matrix approach is used and mutual coupling effects are incorporated as wires are closely placed.

In this paper, we have taken a volume containing N thin long and finite length, perfectly conducting needles embedded in a homogeneous background material. The needle density is taken to be sparse and needles are arranged in periodic order along three orthogonal directions. Such an object is defined as an artificial object containing thin long PEC needles. The effects of random errors in positioning and orientation of needles upon far zone scattered field are analyzed for this artificial object. The study of random errors in thin long PEC needles is important because thin long needles are one of the basic components of double negative (DNG) material.

In Section 2, electromagnetic scattering from an arbitrarily located and oriented thin long PEC needle is analyzed. It is extended for N needles under sparse assumption in Section 3. Effects of random errors in positioning and orientation of needles are analyzed in Sections 4 and 5 respectively. Finally, conclusions are presented in Section 6.

2. EM SCATTERING FROM A THIN LONG PEC NEEDLE

Consider an incident plane wave which is propagating in the direction $\hat{k}_i$ and polarized along $\hat{e}_i$

$$\bar{E}^{inc}(\bar{r}) = \hat{e}_i E_o e^{i\bar{k}_i\cdot\bar{r}} \tag{1}$$

where

$$\bar{k}_i = k_o\hat{k}_i = k_{ix}\hat{x} + k_{iy}\hat{y} + k_{iz}\hat{z}$$

The time dependance is assumed to be $e^{-i\omega t}$ and has been suppressed throughout. Free space propagation constant is $k_o = \omega\sqrt{\mu_o\epsilon_o}$ with μ_o and ϵ_o as permeability and permittivity of free space respectively. When this incident wave impinges upon a PEC scatterer of volume V'

in free space, it induces current in it. If induced current distribution $\bar{J}^{ind}(\bar{r}')$ of a scatterer is known then the scattered field is given by,

$$\bar{E}^{sc}(\bar{r}) = i\omega\mu_o \int_{V'} \bar{G}_e(\bar{r}, \bar{r}') \cdot \bar{J}^{ind}(\bar{r}')dV' \tag{2}$$

where $\bar{r}'$ and $\bar{r}$ are position vectors for scatterer and observation point respectively. $\bar{G}_e(\bar{r}, \bar{r}')$ is a free space dyadic green's function, i.e.,

$$\bar{G}_e(\bar{r}, \bar{r}') = \left[\bar{I} + \frac{\nabla'\nabla'}{k_o^2}\right] \left[\frac{e^{ik_o|\bar{r}-\bar{r}'|}}{4\pi|\bar{r} - \bar{r}'|}\right] \tag{3}$$

Taking far field assumption, i.e., $r \gg r'$, free space dyadic green's function can be approximated as [13],

$$\bar{G}_e(\bar{r}, \bar{r}') \simeq (\bar{I} - \hat{k}_s\hat{k}_s)\frac{e^{ik_or}}{4\pi r} e^{-i\bar{k}_s\cdot\bar{r}'} \tag{4}$$

where $\bar{I}$ is a unit dyadic and $\hat{k}_s$ is a unit vector along direction of scattering or observation and $\bar{k}_s = k_o\hat{k}_s$. Thus, the far zone scattered field from a PEC scatterer with background medium as free space is,

$$\bar{E}^{sc}(r) = -i\omega\mu_o\frac{e^{ik_or}}{4\pi r}\hat{k}_s \times \left[\hat{k}_s \times \int_{V'} e^{-i\bar{k}_s\cdot\bar{r}'}\bar{J}^{ind}(\bar{r}')\right] dV' \tag{5}$$

The exponential term $e^{-i\bar{k}_s\cdot\bar{r}'}$ can be expanded in a series of $-i\bar{k}_s \cdot \bar{r}'$. In the far zone, the dominant contribution comes only from the first term of the series, i.e., induced electric dipole moment. The strengths of higher order terms, i.e., induced multipole moments fall off faster than the first term. Retaining first term of series expansion, it is found that the far zone scattered field in terms of an induced electric dipole moment $\bar{p}$ of a scatterer at origin is given in [14], as,

$$\bar{E}^{sc}(r) = k_o^2\frac{e^{ik_or}}{4\pi\epsilon_or}\hat{k}_s \times (\bar{p} \times \hat{k}_s) \tag{6}$$

with

$$\bar{p} = \frac{i}{\omega}\int_{V'} \bar{J}^{ind}(\bar{r}')dV' \tag{7}$$

In order to calculate the scattered field from a scatterer, it is necessary to first calculate the induced dipole moment. Here a scatterer is taken to be a perfectly conducting (PEC) needle with length l and radius a. Its length is taken to be greater than free space wavelength

$(l > \lambda_o)$. For thin wire approximation, it is assumed that $a \ll \lambda_o$ where transverse and circumferential variation of current can be neglected with respect to axial current on the needle. The end effects can be ignored for thin wire because for large radii there exist additional charge accumulation on the end caps of the wire and would tend to increase the polarizability [15]. Therefore, for thick needles higher order moments can not be neglected. If a needle is aligned along z-axis with its center at origin, then the induced current density $\bar{J}^{ind}(\bar{r}')$ can be modelled as in [16],

$$\bar{J}^{ind}(\bar{r}') = \hat{e}_z I^{ind} \delta(x')\delta(y') \left[\frac{\cos(k_o z') - \cos(k_o l/2)}{1 - \cos(k_o l/2)}\right] \tag{8}$$

where $\delta(.)$ is a dirac delta function. The induced current in a short circuited antenna by an incident wave is given by [17] as,

$$I^{ind} = \frac{1}{V_t} \int_{V'} \bar{E}^{inc}(\bar{r}') \cdot \bar{J}_t(\bar{r}') dV' \tag{9}$$

where $\bar{J}_t(\bar{r}')$ is the current density in the transmitting regime along a thin wire excited by a point voltage source V_t at its center and can be written using [18],

$$\bar{J}_t(\bar{r}') = \hat{e}_z I_t \delta(x')\delta(y') \left[\frac{\sin[k_o\{l/2 - |z'|\}]}{\sin(k_o l/2)}\right] \tag{10}$$

and I_t is constant terminal current amplitude measured in amperes. If a needle is displaced from an origin to a point whose position vector is $\bar{r}_j$ and orientation of thin needle is characterized by a unit vector $\hat{e}_j$, then the induced current from Eq. (9) is,

$$\begin{aligned} I_j^{ind} &= \frac{Y_{in}(\omega)(\hat{e}_i \cdot \hat{e}_j) e^{i\bar{k}_i \cdot \bar{r}_j} E_o}{\sin(k_o l/2)} \int_{-l/2}^{l/2} e^{ik_{ih}h} \sin[k_o\{l/2 - |h|\}] dh \\ &= \frac{2k_o Y_{in}(\omega)(\hat{e}_i \cdot \hat{e}_j) e^{i\bar{k}_i \cdot \bar{r}_j} E_o}{(k_o^2 - k_{ih}^2)} \left[\frac{\cos(k_{ih} l/2) - \cos(k_o l/2)}{\sin(k_o l/2)}\right] \end{aligned} \tag{11}$$

where $Y_{in}(\omega) = I_t/V_t$ is the input admittance of a needle and is discussed in Section 3. The factor $k_{ih} = \bar{k}_i \cdot \hat{e}_j$ is the component of $\bar{k}_i$ along $\hat{e}_j$ and h is a dummy coordinate of integration along the axial direction of a needle. Thus, the induced electric dipole moment of a perfectly conducting thin long needle oriented along a unit vector $\hat{e}_j$ and having position vector $\bar{r}_j$ can be computed by using Eqs. (8)–(11)

in Eq. (7) and is given by,

$$\bar{p}_j = \hat{e}_j(\hat{e}_i \cdot \hat{e}_j)\frac{i}{\omega}\frac{2Y_{in}(\omega)e^{i\bar{k}_i\cdot\bar{r}_j}E_o}{(k_o^2 - k_{ih}^2)}\left[\frac{2\sin(k_ol/2) - k_ol\cos(k_ol/2)}{1-\cos(k_ol/2)}\right] \left[\frac{\cos(k_{ih}l/2) - \cos(k_ol/2)}{\sin(k_ol/2)}\right] \tag{12}$$

The scattered field due to an induced diploe moment at origin is given by Eq. (6). As a needle is displaced from origin to a position vector $\bar{r}_j$ then an associated phase factor $e^{-i\bar{k}_s\cdot\bar{r}_j}$ is multiplied with the given scattered field at origin. Likewise, the induced dipole moment given by Eq. (12) is used in Eq. (6) instead of $\bar{p}$. Thus, the scattered electric field due to a thin needle at position $\bar{r}_j$ can be written in general form as,

$$\bar{E}_j^{sc}(r) = \bar{F}_j(\hat{k}_i, \hat{k}_s)\frac{E_oe^{ik_or}}{r} \tag{13}$$

with

$$\bar{F}_j(\hat{k}_i, \hat{k}_s) = \frac{i\omega\mu_oY_{in}(\omega)e^{i\bar{k}_d\cdot\bar{r}_j}}{2\pi(1-\cos(k_ol/2))}\left[\frac{\cos(k_{ih}l/2) - \cos(k_ol/2)}{(k_o^2 - k_{ih}^2)}\right] \left[2 - \frac{k_ol}{\tan(k_ol/2)}\right]\left[\hat{k}_s \times \left(\hat{e}_j(\hat{e}_i \cdot \hat{e}_j) \times \hat{k}_s\right)\right] \tag{14}$$

and $\bar{k}_d = \bar{k}_i - \bar{k}_s$ is the vectorial change in wave vector during the scattering. Using Maxwell's equation, the scattered magnetic field in the far zone is,

$$\begin{aligned}\bar{H}^{sc}(r) &= \frac{1}{i\omega\mu_o}\nabla \times \bar{E}^{sc}(r) \\ &= \frac{E_oe^{ik_or}}{r}\left[\frac{ik_oY_{in}(\omega)e^{i\bar{k}_d\cdot\bar{r}_j}}{2\pi(1-\cos(k_ol/2))}\left\{\frac{\cos(k_{ih}l/2) - \cos(k_ol/2)}{(k_o^2 - k_{ih}^2)}\right\}\right. \\ &\quad \left.\left\{2 - \frac{k_ol}{\tan(k_ol/2)}\right\}\left\{\hat{k}_s \times \hat{e}_j(\hat{e}_i \cdot \hat{e}_j)\right\}\right]\end{aligned} \tag{15}$$

3. EM SCATTERING FROM A COLLECTION OF PEC NEEDLES

Consider a volume V which encloses N conducting needles having random positions and orientations. The volume under consideration is assumed to be sparse where multiple scattering is negligible i.e., separation between needles approaches to a wavelength or greater than it [19]. Taking incident wave to be a TEM wave i.e., $\hat{e}_i \cdot \hat{k}_i = 0$. If $\hat{e}_i$ and $\hat{k}_i$ are characterized by the angles θ_i and ϕ_i in spherical coordinates, then

$$\hat{e}_i = \cos\phi_i \cos\theta_i \hat{x} + \sin\phi_i \cos\theta_i \hat{y} - \sin\theta_i \hat{z} \tag{16}$$

$$\hat{k}_i = \cos\phi_i \sin\theta_i \hat{x} + \sin\phi_i \sin\theta_i \hat{y} + \cos\theta_i \hat{z} \tag{17}$$

In general, this volume scatters wave in all directions but $\hat{k}_s$ has been taken as a direction of far-zone scattered field. It can be expressed in spherical angles θ_s and ϕ_s as,

$$\hat{k}_s = \cos\phi_s \sin\theta_s \hat{x} + \sin\phi_s \sin\theta_s \hat{y} + \cos\theta_s \hat{z} \tag{18}$$

The randomness in positioning of a jth needle can be described by a random vector $\tilde{\bar{r}}_j$ and its random orientation $\tilde{e}_j = \cos\tilde{\phi}_j \sin\tilde{\theta}_j \hat{x} + \sin\tilde{\phi}_j \sin\tilde{\theta}_j \hat{y} + \cos\tilde{\theta}_j \hat{z}$ by random variables $(\tilde{\theta}_j,\ \tilde{\phi}_j)$. As random orientation of a needle has no effect upon its random position. So, we can take them as independent. Likewise, it is assumed that $\tilde{\theta}_j$ and $\tilde{\phi}_j$ are independent of each other. Thus, joint probability density function becomes $p(\tilde{\bar{r}}_j, \tilde{\theta}_j, \tilde{\phi}_j) = p(\tilde{\bar{r}}_j)p(\tilde{\theta}_j)p(\tilde{\phi}_j)$. It is further assumed that random variables $\tilde{\theta}_j$ and $\tilde{\phi}_j$ are identically distributed, i.e., $\tilde{\theta}_j = \tilde{\theta}$, $\tilde{\phi}_j = \tilde{\phi}$ and $\tilde{e}_j = \tilde{e}$. The random position vector of a jth needle can be taken as a sum of jth mean vector $\bar{m}_j$ and a random error vector $\tilde{\bar{n}}_j$ i.e., $\tilde{\bar{r}}_j = \bar{m}_j + \tilde{\bar{n}}_j$. The components of random error vector n_{xj}, n_{yj} and n_{zj} are taken to be independent and identically distributed along three orthogonal directions, i.e., $\tilde{\bar{n}}_j = \tilde{\bar{n}}$. The mean vector $\bar{m}_j$ is periodic in x, y and z directions with periods d_x, d_y and d_z respectively. By taking ensemble average over random orientations $(\tilde{\theta}_j, \tilde{\phi}_j)$ and random positions $\tilde{\bar{r}}_j$ of a jth needle, the ensemble averaged scattered field from a collection of N PEC needles can be written as,

$$\left\langle \bar{E}^{sc}(r) \right\rangle = \sum_{j=1}^{N} \left\langle \bar{F}_j(\hat{k}_i, \hat{k}_s) \right\rangle \frac{E_o e^{ik_o r}}{r} \tag{19}$$

The angular bracket $\langle\ \rangle$ show ensemble average. After some manipulation, it can shown that the ensemble averaged scattered field is,

$$\left\langle \bar{E}^{sc}(r) \right\rangle = \frac{E_o e^{ik_o r}}{r} \left[i\alpha_1 S \left\{ \hat{k}_s \times \left(\bar{F} \times \hat{k}_s \right) \right\} \right] \tag{20}$$

with

$$\alpha_1 = \frac{\eta_o Y_{in}(\omega)}{2\pi k_o}\left[2 - \frac{k_o l}{\tan(k_o l/2)}\right] \tag{21}$$

$$\begin{aligned} S = & N\Phi_{n_x}(k_{dx})\Phi_{n_y}(k_{dy})\Phi_{n_z}(k_{dz}) \\ & \exp\left[\frac{i}{2}\left\{k_{dx}(N_x+1)d_x + k_{dy}(N_y+1)d_y + k_{dz}(N_z+1)d_z\right\}\right] \\ & \frac{\sin\mathrm{c}(k_{dx}N_x d_x/2\pi)\sin\mathrm{c}(k_{dy}N_y d_y/2\pi)\sin\mathrm{c}(k_{dz}N_z d_z/2\pi)}{\sin\mathrm{c}(k_{dx}d_x/2\pi)\sin\mathrm{c}(k_{dy}d_y/2\pi)\sin\mathrm{c}(k_{dz}d_z/2\pi)} \end{aligned} \tag{22}$$

$$\begin{aligned} \bar{F} &= \left\langle \widetilde{e}(\hat{e}_i \cdot \widetilde{e})\Psi(\theta_i, \phi_i, \widetilde{\theta}, \widetilde{\phi}, l/\lambda_o) \right\rangle \\ &= \int_{\widetilde{\phi}}\int_{\widetilde{\theta}} R\Psi(\theta_i, \phi_i, \widetilde{\theta}, \widetilde{\phi}, l/\lambda_o) p(\widetilde{\theta}) p(\widetilde{\phi}) d\widetilde{\theta} d\widetilde{\phi} \cdot \hat{e}_i \\ &= \left\langle R\Psi(\theta_i, \phi_i, \widetilde{\theta}, \widetilde{\phi}, l/\lambda_o) \right\rangle \cdot \hat{e}_i \end{aligned} \tag{23}$$

and

$$\Psi(\theta_i, \phi_i, \widetilde{\theta}, \widetilde{\phi}, l/\lambda_o) = \left[\frac{\cos(\widetilde{k}_{ih} l/2) - \cos(k_o l/2)}{(1 - \cos(k_o l/2))(1 - \widetilde{k}_{ih}^2/k_o^2)}\right] \tag{24}$$

$$R = \begin{bmatrix} \sin^2\widetilde{\theta}\cos^2\widetilde{\phi} & \sin^2\widetilde{\theta}\sin\widetilde{\phi}\cos\widetilde{\phi} & \sin\widetilde{\theta}\cos\widetilde{\theta}\cos\widetilde{\phi} \\ \sin^2\widetilde{\theta}\sin\widetilde{\phi}\cos\widetilde{\phi} & \sin^2\widetilde{\theta}\sin^2\widetilde{\phi} & \sin\widetilde{\theta}\cos\widetilde{\theta}\sin\widetilde{\phi} \\ \sin\widetilde{\theta}\cos\widetilde{\theta}\cos\widetilde{\phi} & \sin\widetilde{\theta}\cos\widetilde{\theta}\sin\widetilde{\phi} & \cos^2\widetilde{\theta} \end{bmatrix} \tag{25}$$

where $\Phi_{n_x}(k_{dx})$, $\Phi_{n_y}(k_{dy})$ and $\Phi_{n_z}(k_{dz})$ are characteristic functions of random errors n_x, n_y and n_z respectively with $\bar{k}_d = k_o(\hat{k}_i - \hat{k}_s) = k_{dx}\hat{x} + k_{dy}\hat{y} + k_{dz}\hat{z}$. The total number of PEC needles is $N = N_x N_y N_z$ and η_o is the intrinsic impedance of free space. The factor $\widetilde{k}_{ih} = \bar{k}_i \cdot \widetilde{e} = k_o[\cos\phi_i \sin\theta_i \cos\widetilde{\phi}\sin\widetilde{\theta} + \sin\phi_i \sin\theta_i \sin\widetilde{\phi}\sin\widetilde{\theta} + \cos\theta_i \cos\widetilde{\theta}]$ is dependent upon incident wave spherical angles (θ_i, ϕ_i) and random variables $(\widetilde{\theta}, \widetilde{\phi})$. It can be observed from Eq. (20) that the ensemble averaged scattered field is mainly dependent upon three factors α_1, S and $\bar{F}$.

The first factor α_1 can be taken as a shape factor because it is dependent upon the shape of the needle, i.e., length l and radius a of the needle. The input impedance of an infinitesimal small PEC needle ($l \ll \lambda_o$) is capacitive [20]. For a thin long PEC needle (i.e., $l > \lambda_o$), its input impedance can have inductance as well. This is due to a reason that by increasing its length beyond λ_o, we are incorporating its higher multipole moments. In the far zone, only electric and magnetic dipole moments will contribute for a localized source [14]. As there exist no magnetic dipole moment for a thin long PEC needle so only electric dipole moment will contribute. Thus, for the scattered field from the volume containing N PEC needles in the far zone, only electric dipole moments of all needles will give dominant contributions. For a long thin perfectly conducting needle, its input admittance given by [20],

$$Y_{in}(\omega) = \left[\frac{R_m}{\sin^2(k_o l/2)} - i\frac{X_m}{\sin^2(k_o l/2)}\right]^{-1} \tag{26}$$

where R_m and X_m are the real and imaginary parts of the input impedance referred to at the current maximum and are given by,

$$\begin{aligned} R_m &= \frac{\eta_o}{2\pi}\left[C+\ln(k_o l)-\mathrm{C}i(k_o l)+\frac{1}{2}\sin(k_o l)\left\{\mathrm{S}i(2k_o l)-2\mathrm{S}i(k_o l)\right\}\right. \\ &\quad \left.+\frac{1}{2}\cos(k_o l)\left\{C+\ln(k_o l/2)+\mathrm{C}i(2k_o l)-2\mathrm{C}i(k_o l)\right\}\right] \qquad (27) \\ X_m &= \frac{\eta_o}{4\pi}\left[2\mathrm{S}i(k_o l)+\cos(k_o l)\{2\mathrm{S}i(k_o l)-\mathrm{S}i(2k_o l)\}\right. \\ &\quad \left.-\sin(k_o l)\left\{2\mathrm{C}i(k_o l)-\mathrm{C}i(2k_o l)-\mathrm{C}i\left(\frac{2k_o a^2}{l}\right)\right\}\right] \qquad (28) \end{aligned}$$

where $\mathrm{C}i(.)$ and $\mathrm{S}i(.)$ are the cosine and sine integrals respectively. Likewise, $C = 0.5772$ is Euler's constant.

The second factor is needle positioning error factor and described by a scaler value S. It completely specifies the effects of random errors in positioning of needles. If there exist no errors in position coordinates and all needles are present at their mean positions then characteristic functions correspond to unity. In this case, $|S|^2$ correspond to a factor which is well known as a structure factor and given by Jackson [14, p-462]. It is further observed that for scattering in forward direction $\bar{k}_d = 0$ and S equals N, no matter how randomly needles are placed in volume V. Therefore errors in location alone of the needles do not effect the ensemble averaged scattered field in forward direction.

The third factor is a needle orientation error factor, i.e., $\bar{F}$. It is a vector and completely specify the effects of random errors in orientations of needles. As a simple case, if all needles are aligned along z-axis with no errors in orientations of needles and incident wave is also polarized along z-axis, then $\bar{F}$ corresponds to $\hat{z}$ with $|\bar{F}| = 1$. Likewise, similar effects are observed for needles and incident polarization aligned along x and y-axes respectively. In order to analyze the scattering characteristics of a volume containing PEC needles, it is desired to find the ensemble average radar cross section. It can be calculated using the well known definition i.e.,

$$\sigma_{av} = \lim_{r\to\infty} 4\pi r^2 \frac{\left|\left\langle \bar{E}^{sc}\right\rangle\right|^2}{\left|\bar{E}^{inc}\right|^2} \tag{29}$$

where $\left\langle \bar{E}^{sc}\right\rangle$ and $\bar{E}^{inc}$ are the far field ensemble averaged scattered and incident electric field intensities, respectively. Thus, using Eq. (1) and Eqs. (20)–(23) in Eq. (29) and after some simplification, the normalized averaged radar cross section σ_{av}/σ_n from a collection of N needles can be written as,

$$\begin{aligned}\frac{\sigma_{av}}{\sigma_n} &= \left|\Phi_{n_x}(k_{dx})\Phi_{n_y}(k_{dy})\Phi_{n_z}(k_{dz})\right|^2\left|\overline{F}\right|^2\left(\frac{\sin\gamma}{\sin\theta_s}\right)^2\\ &= \left|L\right|^2\left|O\right|^2\end{aligned} \tag{30}$$

where σ_n is a RCS of N PEC needles enclosed by volume V with all needles aligned along some reference direction and are present at their mean positions i.e., no error in orientation and positioning. It can be written as,

$$\begin{aligned}\sigma_n = 4\pi N^2\left|\alpha_1\right|^2&\left|\frac{\mathrm{sinc}\left(k_{dx}N_xd_x/2\pi\right)\mathrm{sinc}\left(k_{dy}N_yd_y/2\pi\right)\mathrm{sinc}\left(k_{dz}N_zd_z/2\pi\right)}{\mathrm{sinc}\left(k_{dx}d_x/2\pi\right)\mathrm{sinc}\left(k_{dy}d_y/2\pi\right)\mathrm{sinc}\left(k_{dz}d_z/2\pi\right)}\right|^2\\ &\cdot\sin^2\theta_s\end{aligned} \tag{31}$$

The factor γ is an angle between an averaged vector $\bar{F}$ and a unit vector along direction of scattering $\hat{k}_s$. It is obvious from Eq. (30) that the normalized averaged RCS is a product of two factors, i.e., $L = \Phi_{n_x}(k_{dx})\Phi_{n_y}(k_{dy})\Phi_{n_z}(k_{dz})$ and $O = |\bar{F}|(\sin\gamma/\sin\theta_s)$. The factor L is a representative of positioning errors in needles and dependent upon vectorial change in wave vector during scattering $\bar{k}_i - \bar{k}_s$, wavelength λ_o and positioning error variances σ_q^2 with $q = x, y, z$. The dependency of L upon wavelength shows frequency dispersion. It is independent of the length l of a needle. Likewise, orientational errors in needles is described by O and dependent upon incident wave vector $\bar{k}_i$, incident polarization $\hat{e}_i$, direction of scattering $\hat{k}_s$, l/λ_o and orientational error variances σ_R^2 with $R = \widetilde{\theta}, \widetilde{\phi}$. The dependency of orientational errors O upon wavelength and incident wave vector leads to frequency and spatial dispersion respectively.

4. EFFECTS OF RANDOM ERRORS IN POSITIONING OF NEEDLES

The effects of random errors in positioning of needles are characterized by the characteristic functions $\Phi_{n_x}(k_{dx})$, $\Phi_{n_y}(k_{dy})$ and $\Phi_{n_z}(k_{dz})$. In order to fulfill sparse condition, it is required that variances $\sigma_q^2 \ll d_q$ where $q = x, y, z$ showing that random errors in x, y and z-directions of a needle are very small. Two types of pdfs associated with random errors in positioning of needles are assumed, i.e., uniform and normal. It is desired to analyze the effects of maximum possible errors in positioning of needles without violating sparse condition. It is well known [21] that for a random error $\widetilde{n}_q$ with given variance σ_q^2, the maximum entropy, i.e., maximum error occurs if $\widetilde{n}_q$ is gaussian or normally distributed. Likewise, if maximum value of $\widetilde{n}_q$ is given then maximum entropy can obtained if $\widetilde{n}_q$ is uniformly distributed. That is the reason why uniform and normal pdfs are assumed here. Mean values of random errors in positioning of needles can be taken as systematic errors. Such type of errors can easily be removed and without loss of generality, their pdfs can be assumed to have zero means. Moreover this mean value effects only the phase of the averaged scattered field and does not effect the radar cross section.

To analyze the effects of random errors in positioning of needles upon the averaged radar cross section (RCS) σ_{av}/σ_n from a collection of N needles, it is assumed that there exist no error in orientations of needles i.e., $\widetilde{\hat{e}}_j = \hat{z}$. Also length l of each needle is 0.8 m and the operating wavelength is taken to be λ_o=0.3 m. In this case, $|\bar{F}|^2 \sin^2\gamma$ becomes $\sin^2\theta_s$ and averaged radar cross section (RCS) σ_{av}/σ_n is,

$$\frac{\sigma_{av}}{\sigma_n} = |L|^2 \tag{32}$$

Likewise, it is further assumed that incident wave unit vector $\hat{k}_i$ lies in the xy-plane and incident polarization $\hat{e}_i$ is aligned along z-direction, i.e., $\hat{e}_i = \hat{z}$ and $\hat{k}_i = -\cos\phi_i\hat{x} - \sin\phi_i\hat{y}$.

As a first case, it is assumed that random errors in positioning of a needle along x, y and z directions are normally distributed with means $m_q = 0$ and variances $\sigma_q^2, q = x, y, z$. Then, the factor L is,

$$L = \exp[-(k_{dx}^2\sigma_x^2 + k_{dy}^2\sigma_y^2 + k_{dz}^2\sigma_z^2)/2] \tag{33}$$

The effects of random errors in positioning of needles upon normalized average RCS assuming gaussian distribution is shown in Figure 1. It is observed that when $\bar{k}_i$ is parallel to $\bar{k}_s$ then RCS remains unaffected regardless of any value of σ_q^2 with $q = x, y, z$. In backward direction, effects of positioning errors are dominant. As positioning error is increased along three orthogonal directions, the normalized averaged RCS decreases for all values of θ_s instead of forward direction where $\theta_s = -\pi/2$.

In the second case, random errors in x, y and z directions are taken to be uniformly distributed between ζ_q and $-\zeta_q$ with means $m_q = 0$ and variances $\zeta_q^2/3$, $q = x, y, z$. Then, L is given by,

$$L = \left[\frac{\sin(\sqrt{3}k_{dx}\sigma_x)}{\sqrt{3}k_{dx}\sigma_x}\right]\left[\frac{\sin(\sqrt{3}k_{dy}\sigma_y)}{\sqrt{3}k_{dy}\sigma_y}\right]\left[\frac{\sin(\sqrt{3}k_{dz}\sigma_z)}{\sqrt{3}k_{dz}\sigma_z}\right] \tag{34}$$

The effects of random errors in positioning of needles upon normalized average RCS assuming uniform distribution is shown in Figure 2. Similar effects are observed as for normal distribution but in backward direction, error effects are more dominant. Effects of positioning errors are analyzed for error variances upto $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 0.015\lambda$ where sparse condition is satisfied.

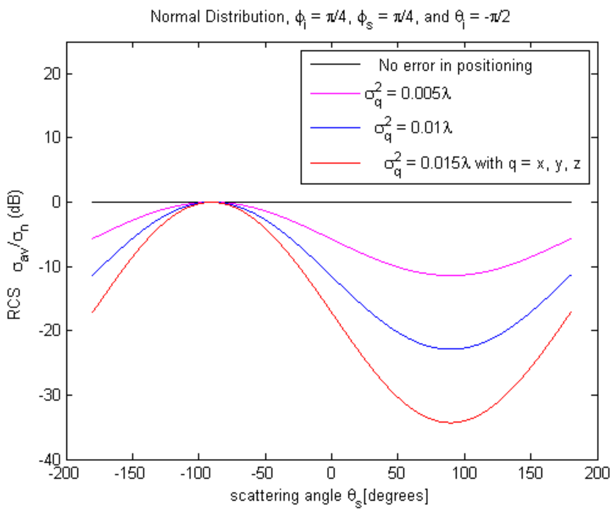

Figure 1. Effects of random errors in positioning of needles upon normalized averaged RCS using gaussian or normal distribution.

5. EFFECTS OF RANDOM ERRORS IN ORIENTATIONS OF NEEDLES

The effects of random errors in orientation of a needle can be characterized by ensemble averaged vector $\bar{F}$. In previous section,

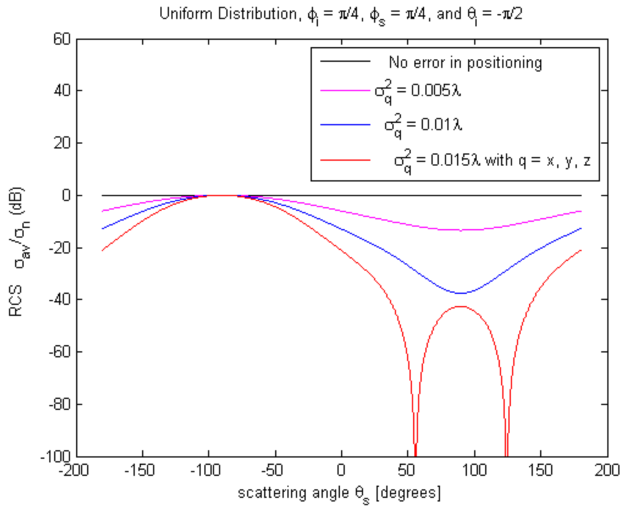

Figure 2. Effects of random errors in positioning of needles upon normalized averaged RCS using uniform distribution.

no error in orientation is assumed i.e., all needles are aligned along z-axis. Likewise, needles may also be aligned along either x or y-axis and these two cases can also be considered as aligned cases with no error in orientation. If needles are not exactly aligned then random errors in orientation are completely specified by random variables $\widetilde{\theta}$ and $\widetilde{\phi}$ and their associated pdfs. In order to simplify the analysis, it is assumed that all needles have no error in positioning, i.e., all needles are at their mean positions. The normalized averaged radar cross section from a collection of N needles is,

$$\frac{\sigma_{av}}{\sigma_n} = |O|^2 = |\bar{F}|^2 \left(\frac{\sin\gamma}{\sin\theta_s}\right)^2 \tag{35}$$

This result is general and can be applied for all types of random errors in orientations of needles in a given volume for any incident plane wave having arbitrary incidence and polarization. This result can also be used to analyze the effects of random errors for planer finite length sparse wire grids of PEC needles. Depending upon random error variables $\widetilde{\theta}$ and $\widetilde{\phi}$, three cases can be considered for the planer finite length sparse wire grids.

Case 1: Consider all needles in the xy-plane i.e., $\widetilde{\theta} = \pi/2$. In this case, the needle orientation error vector $\bar{F}$ is only described by a random variable $\widetilde{\phi}$ and its associated pdf and the factor $\widetilde{k}_{ih}$ simplifies to,

$$\widetilde{k}_{ih} = k_o \left[\cos\phi_i \sin\theta_i \cos\widetilde{\phi} + \sin\phi_i \sin\theta_i \sin\widetilde{\phi}\right] \tag{36}$$

This result can further be reduced to two special cases of no error in orientations of needles. The first case can be considered with $\widetilde{\phi} = 0$ showing that $\widetilde{\hat{e}} = \hat{x}$ and all needles are aligned along +ve x-axis. Likewise, for second case, $\widetilde{\phi} = \pi/2$, i.e., $\widetilde{\hat{e}} = \hat{y}$ and all needles are aligned along +ve y-axis.

Case 2: In this case, all needles lie in the xz-plane. The effects of random errors in orientations are described by a random variable $\widetilde{\theta}$ and its pdf. The factor $\widetilde{k}_{ih}$ becomes,

$$\widetilde{k}_{ih} = k_o \left[\cos\phi_i \sin\theta_i \sin\widetilde{\theta} + \cos\theta_i \cos\widetilde{\theta}\right] \tag{37}$$

Case 3: All needles are in the yz-plane then effects of random errors in orientations are only dependent upon a random variable $\widetilde{\theta}$ and $\widetilde{k}_{ih}$ becomes,

$$\widetilde{k}_{ih} = k_o \left[\sin\phi_i \sin\theta_i \sin\widetilde{\theta} + \cos\theta_i \cos\widetilde{\theta}\right] \tag{38}$$

A special case of it can be considered with $\widetilde{\theta} = 0$ and $\widetilde{\phi} = \pi/2$, i.e., $\widetilde{\hat{e}} = \hat{z}$, where all needles are aligned along +ve z-axis having no error in orientations. To analyze the effects of errors in orientation of needles, incident E-field is taken to be aligned along z-axis and orientation of needles are described by $\widetilde{\theta}$ and $\widetilde{\phi}$. Random variables $\widetilde{\theta}$ and $\widetilde{\phi}$ are taken to be uniformly distributed with means 0 and $\pi/2$ respectively. Integrals given by Eq. (23) are difficult to compute analytically. They are solved numerically using sample mean of the generated sample space for a particular fixed value of $l/\lambda_o \approx 2.667$. This length is assumed due to the reason that a finite length but long needle can have a length in the range of $2\lambda_o$ to $3\lambda_o$ where λ_o is the free space wavelength. The normalized averaged RCS with no error in orientation is compared with error in orientation of needles in Figure 3. It is observed that as error in orientation of needles increases, the normalized averaged RCS reduces. Thus, the above analysis can be used to analyze the effects of random errors in orientations of perfectly conducting needles enclosed by a particular volume and planer sparse wire grids of finite length.

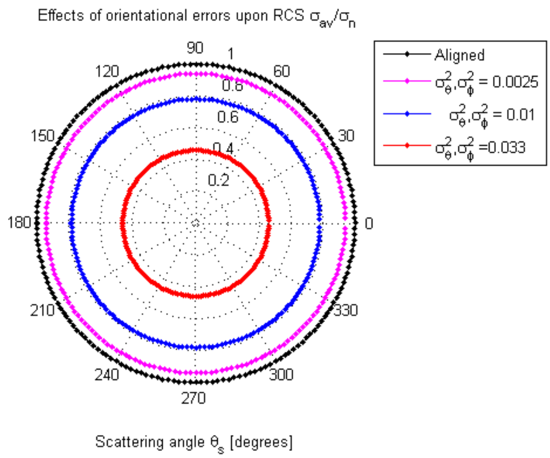

Figure 3. Effects of random errors in orientations of needles upon normalized averaged RCS.

6. CONCLUSIONS

The effects of random errors in orientations and positioning of needles upon normalized average RCS is analyzed using sparse assumption. It is observed that in forward direction scattering, normalized average RCS is independent of positioning errors and in backward direction scattering it plays significant rule, i.e., increasing error causes reduction in normalized average RCS. Likewise, reduction in normalized averaged RCS is observed with increased random errors in orientations of needles.

REFERENCES

1. Miller, E. K., G. J. Burke, B. J. Maxum, G. M. Pjerrou, and A. R. Neureuther, "Radar cross section of a long wire," *IEEE Trans. Antennas and Propagation*, Vol. 17, 381–384, May 1969.
2. Bowman, J. J., T. B. A. Senior, and P. L. E. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes*, Chap. 12, North-Holland Pub. Comp., Amsterdam, 1969.
3. Medgyesi-Mitschang, L. N. and C. Eftimiu, "Scattering from wires and open circular cylinders of finite length using entire domain galerkin expansions," *IEEE Trans. Antennas and Propagation*, Vol. 30, No. 4, 628–636, July 1982.

4. Mack, R. B., "Phase properties of backscattered fields from thin rods," *IEEE Trans. Antennas and Propagation*, Vol. 19, 450–451, May 1971.
5. Lin, L. J. and D. J. Maston, "On the backscattering from two arbitrarily located identical parallel wires," *IEEE Trans. Antennas and Propagation*, Vol. 19, 697–700, Sep. 1971.
6. Larson, C. J. and B. A. Munk, "The broad-band scattering response of periodic arrays," *IEEE Trans. Antennas and Propagation*, Vol. 31, No. 2, 261–267, Mar. 1983.
7. Schneider, S. W. and B. A. Munk, "The scattering properties of super dense arrays of dipoles," *IEEE Trans. Antennas and Propagation*, Vol. 42, No. 2, 463–472, Apr. 1994.
8. Rembold, B., "Radar cross section of long wires," *IEEE Trans. Antennas and Propagation*, Vol. 32, No. 10, 1124–1126, Oct. 1984.
9. Lewis, G., J. Fortuny-Guasch, and A. Sieber, "Bistatic radar scattering experiments of parrallel wire grids," *IEEE Geoscience and Remote Sensing Symposium*, Vol. 1, 444–446, 2002.
10. Dedrick, K. G., A. R. Hessing, and G. L. Johnson, "Bistatic radar scattering by randomly oriented wires," *IEEE Trans. Antennas and Propagation*, Vol. 26, No. 3, 420–426, May 1978.
11. Sjöberg, D., "Coherent effects in single scattering and random errors in antenna technology," *Progress In Electromagnetics Research*, PIER 50, 13–39, 2005.
12. Manabe, T. and A. Murk, "Transmission and reflection charactersitics of slightly irregular wire-grids with finite conductivitry for arbitrary angles of incidence and grid rotation," *IEEE Trans. Antennas and Propagation*, Vol. 53, No. 1, 250–259, Jan. 2005.
13. Papas, C. H., *Theory of Electromagnetic Wave propagation*, Chap. 2, McGraw-Hill Company, 1965.
14. Jackson, J. D., *Classical Electrodynamics*, 3rd Edition, John Wiley and Sons Inc., New York, 1999.
15. Heldring, A., E. Ubeda, and J. M. Rius, "Effecient computation of the effect of wire ends in thin wire analysis," *IEEE Trans. Antennas and Propagation*, Vol. 54, No. 10, 250–259, Oct. 2006.
16. Tretyakov, S. A., S. Maslovski, and P. A. Belov, "An analytical model of metamaterials based on loaded wire dipoles," *IEEE Trans. Antennas and Propagation*, Vol. 51, No. 10, 2652–2658, Oct. 2003.
17. Jordan, E. C. and K. G. Balman, *Electromagnetic Waves and Radiating Systems*, 483–485, 2nd Edition, Prentice Hall, Englewood Cliffs, NJ, 1968.

18. Schelkunoff, S. A. and H. T. Friis, *Antennas: Theory and Practice*, Chap. 8, John Wiley and Sons Inc., New York, 1952.
19. Yatsenko, V. V., S. A. Tretyakov, S. I. Maslovski, and A. A. Sochava, "Higher order impedance boundry conditions for sparse wire grids," *IEEE Trans. Antennas and Propagation*, Vol. 48, No. 5, 720–727, May 2000.
20. Balanis, C. A., *Antenna Theory, Analysis and Design*, 2nd edition, John Wiley and Sons Inc., New York, 1997.
21. Lathi, B. P., *Modern Digital and Analog Communication Systems*, 703–705, 3rd edition, Oxford University Press, Mar. 1998.