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THE FIELD OF AN ELECTRIC DIPOLE AND THE POLARIZABILITY OF A CONDUCTING OBJECT EMBEDDED IN THE INTERFACE BETWEEN DIELECTRIC MATERIALS

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Abstract—In this paper, a study is made of the electrostatic potential and field of an electric dipole located *in* the interface between two dielectric regions. When the dipole is oriented perpendicular to the interface, the detailed position of the charges of the dipole relative to the location of the interface has a significant effect on the value of the field produced away from the dipole, unlike the case of a dipole parallel to the interface. It is shown that it is the total dipole moment (due to both free and bound charges), rather than simply the impressed (free) dipole moment that is important in determining the field in this case. Based on these results, the question of defining and determining the electric polarizability of a perfectly conducting object partially embedded in a dielectric interface is examined. The example of a conducting sphere embedded halfway in the interface is studied as a demonstration of our general formulation. The results of this paper are important for the proper modeling of arrays of scatterers embedded in an interface, such as frequency-selective surfaces (FSSs) and metafilms.

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1. INTRODUCTION

Periodically or randomly structured surfaces find many applications in electromagnetic engineering: frequency-selective surfaces (FSS), gratings and "smart" surfaces are some of the more prominent examples. A particular class of such surfaces is called a metafilm (a surface array of small discrete scatterers). In recent work [1, 2], Generalized Sheet Transition Conditions (GSTCs) for the average electromagnetic fields at a metafilm have been obtained, allowing modeling of the metafilm without a detailed solution of the rapidly varying fields in its near field. Using these boundary conditions, plane wave reflection and transmission coefficients from the metafilm have been derived and their dependencies on scatterer geometry and incidence angle investigated [3, 4]. The coefficients in the GSTCs depend on the density and polarizabilities of the scatterers that make up the metafilm. The analysis in [1, 3, 4] assumed that the metafilm is embedded in an infinite homogeneous medium, in which case the meaning of dipole moments and polarizabilities is well understood.

However, in many surface arrays the particles may be partially embedded in the interface between two different material regions [5–8]. When a metafilm is placed at an interface between two different media, the GSTCs must be modified to account for the influence of the interface. For example, if the scatterers have zero thickness in the direction perpendicular to the interface, so that only tangential electric currents can be induced in them, the metafilm can only produce tangential electric dipoles and normal magnetic dipoles at the interface. In such a case, it appears that a simple modification of the the GSTCs for a metafilm in a homogeneous medium will provide the correct result for the metafilm at an interface. This has been done for the case of an infinite strip grating in [9, 10]. For the case when the scatterers are not thin, currents can be induced in the direction normal to the interface, and it is not clear what modifications must be made to the GSTCs in this case. Attempts to treat this more general case were made in [2] and [11], with only limited success.

Indeed, when an electric dipole normal to the interface between two media is embedded in that interface, it is not even obvious what definition should be used for the dipole moment, since (as we will see) the fields produced by a dipole of a given moment can be different if the dipole is completely placed in one medium or the other, and different still if it is placed midway in the interface. Part of the issue is how the dipole moment is to be defined in the first place. This issue was not addressed in [11], and the boundary behavior of the fields found in that paper cannot be used without further clarification.

Later work on potentials produced by dipole monolayers at an air-liquid interface [12, 13] and on optical properties of thin films, island films and rough surfaces [14] has emphasized the importance of a clear understanding of the potential and fields of dipoles at an interface. However, the ambiguity of the definition of the normal component of surface excess dielectric polarization at an interface has remained unresolved.

In Sommerfeld's original 1909 paper [15] on the antenna over earth problem, he considered the antenna to be a dipole which is embedded half in the earth and half in the air. Later, in his 1926 paper [16], he took the antenna to be located completely in the air, though still at (just above) the interface. This fact was pointed out by Boella and Einaudi [17], where it was also noticed that this detail of position resulted in different fields in the two cases. Krasil'nikov [18] further investigated the potential, impedance and effective length of a vertical dipole antenna embedded halfway into the earth.

In this paper we present a detailed study of the problem of a static electric dipole placed in the interface between two media. The purpose of this study is to find a suitable unambiguous definition for the dipole moment and to determine correctly the resulting field. Based on this, we will determine a proper specification of the polarizability of a conducting object partially embedded in a dielectric interface. In Section 2, we obtain expressions for the potential and field of a certain model for an electric dipole located at a dielectric interface. In Section 3, the distributions of free and bound charge resulting from this dipole are carefully considered and various possible definitions for the dipole moment are obtained. In Section 5, the question of a proper definition for the electric polarizability is studied. For the special case of an arbitrary symmetric perfectly conducting object, whose electric field is known when placed in a uniform incident static field in free space, we find the field when the object is embedded symmetrically in the interface. From this, we are able to obtain the polarizability of the object in the interface in terms of its free space value. The case of a sphere is then presented as an illustrative example. We conclude with a discussion of the results and how they might apply to various electromagnetic modeling problems.

2. STATIC ELECTRIC DIPOLE AT THE INTERFACE BETWEEN TWO MEDIA

Consider two semi-infinite dielectrics with an interface at $z = 0$. The permittivity is ϵ_1 for the upper medium ($z > 0$), and ϵ_2 for the lower medium ($z < 0$). In this section we examine the problem of a static electric dipole placed at this interface.

A traditional way to model an ideal dipole in a homogeneous dielectric is as follows. A specified excess positive charge Q is placed at a position $\mathbf{r} + \mathbf{h}/2$, and a corresponding excess negative charge $-Q$ at $\mathbf{r} - \mathbf{h}/2$, where $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the position vector of the dipole location, and $\mathbf{h}$ is the vector "length" of the dipole (here, $\mathbf{a}_c$ denotes a unit vector in the direction of the c-coordinate). By "excess", we mean that these charges are introduced from an outside source, in order to distinguish them from the bound charges that are induced in the surrounding dielectric. They may be either free or bound charges, brought in on conductors or some other dielectrics into the homogeneous "background" dielectric medium. We now take the limit as $Q \to \infty$ and $\mathbf{h} \to 0$, such that the product $Q\mathbf{h} = \mathbf{p}$ remains constant. We will call this vector the *excess* dipole moment $\mathbf{p}_e$.

When the position of the dipole is a point on an interface ($\mathbf{r} = 0$, say), it may be that during the limiting process one of the point charges is in the upper medium while the other charge is in the lower medium, or that both charges are in the same medium. It is not immediately obvious whether this will have any impact on the field produced by the dipole, and we investigate the question separately for dipoles oriented normal to, or tangential to the interface. The general case is then recovered by superposition.

2.1. Electric Potential of a Normally Oriented Dipole

We consider first the case of a dipole oriented perpendicular to the interface. The point charges constituting the dipole are taken to be a charge Q located at $(0, 0, h_1)$ above the interface, and $-Q$ located at

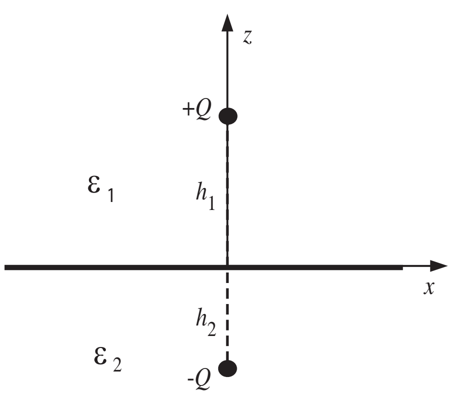

Figure 1. A pair of point charges near an interface.

$(0, 0, -h_2)$ below the interface (see Fig. 1). If desired, we could think of this case as the superposition of two separate dipoles created by locating additional point charges $-Q$ at $(0, 0, 0^+)$ and Q at $(0, 0, 0^-)$; the only difference in the resulting field would occur in an infinitesimal region at the interface. We will find the static electric field in the limit when h_1 and $h_2 \to 0$ and $Q \to \infty$ in such a way that Qh_1 and Qh_2 approach finite values, at least one of them nonzero. To help define a polarizability properly later on, we will specifically examine the behavior of the field at $z = 0^{\pm}$.

We first find the electric scalar potential of single excess charges near the interface of different media. This can be done using image theory [19, pp. 219–220]. For a charge $+Q$ placed at the point $(0, 0, h_1)$, the potential can be written as:

$$\Phi_1 = \frac{Q}{4\pi\epsilon_1}\left\{\frac{1}{\sqrt{\rho^2+(z-h_1)^2}} + \left(\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z+h_1)^2}}\right\} \quad \text{for} \quad z>0 \tag{1}$$

$$\Phi_1 = \frac{Q}{4\pi\epsilon_2}\left(\frac{2\epsilon_2}{\epsilon_1+\epsilon_2}\right)\left\{\frac{1}{\sqrt{\rho^2+(z-h_1)^2}}\right\} \quad \text{for} \quad z<0 \tag{2}$$

where $\rho^2 = x^2 + y^2$. In a similar manner, a charge $-Q$ at $(0, 0, -h_2)$ has the potential:

$$\Phi_2 = \frac{-Q}{4\pi\epsilon_1}\left\{\left(\frac{2\epsilon_1}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z+h_2)^2}}\right\} \quad \text{for} \quad z>0 \tag{3}$$

$$\Phi_2 = \frac{-Q}{4\pi\epsilon_2}\left\{\frac{1}{\sqrt{\rho^2+(z+h_2)^2}} - \left(\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z-h_2)^2}}\right\} \quad \text{for} \quad z<0 \tag{4}$$

By superposition, the potential $\Phi = \Phi_1 + \Phi_2$ of a dipole made up of both charges can be written as:

$$\Phi = \frac{Q}{4\pi\epsilon_1}\left\{\frac{1}{\sqrt{\rho^2+(z-h_1)^2}} - \left(\frac{2\epsilon_1}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z+h_2)^2}} + \left(\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z+h_1)^2}}\right\} \quad \text{for} \quad z>0 \tag{5}$$

$$\Phi = \frac{-Q}{4\pi\epsilon_2}\left\{\frac{1}{\sqrt{\rho^2+(z+h_2)^2}} - \left(\frac{2\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z-h_1)^2}} - \left(\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{1}{\sqrt{\rho^2+(z-h_2)^2}}\right\} \quad \text{for} \quad z<0 \tag{6}$$

The same expressions for the potential were obtained in [2] using a Fourier transform approach.

Now we consider the limit as $Q \to \infty$ and $h_1 \to 0$ and $h_2 \to 0$ such that the "partial" dipole moments $h_1 Q = p_{e1z}$ and $h_2 Q = p_{e2z}$ remain constant. The quantities p_{e1z} and p_{e2z} are the portions of excess dipole moment (that is, dipole moment due only to the charges $\pm Q$) lying in the upper and lower half-space respectively. Using the approximation

$$\frac{1}{\sqrt{\rho^2+(z-h)^2}} \simeq \frac{1}{r}\left(1+h\frac{z}{r^2}\right) \tag{7}$$

for $|h| \ll r = \sqrt{\rho^2+z^2}$, with $h = \pm h_1$ or $\pm h_2$, the foregoing expressions for Φ reduce to

$$\Phi = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}z}{4\pi\epsilon_1 r^3} \qquad \text{for } z>0 \tag{8}$$

and

$$\Phi = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}z}{4\pi\epsilon_2 r^3} \qquad \text{for } z<0 \tag{9}$$

Here

$$p_{nz} = \epsilon_0\left(\frac{p_{e1z}}{\epsilon_1}+\frac{p_{e2z}}{\epsilon_2}\right) \tag{10}$$

is a weighted sum of the partial dipole moments whose physical meaning will be clarified in the next section, and

$$\epsilon_\perp = \frac{2\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2} \tag{11}$$

is a kind of "series" connection of the two half-space permittivities (as in a capacitor with its electric field perpendicular to a dielectric interface). We will next use Equations (8) and (9) to find the electric field.

2.2. Electric Field of a Normally Oriented Dipole

In a cylindrical coordinate system (ρ, ϕ, z) the electric field can be calculated from the potential using

$$\mathbf{E} \;=\; -\nabla\Phi = -\mathbf{a}_\rho\frac{\partial V}{\partial \rho} - \mathbf{a}_z\frac{\partial V}{\partial z} \tag{12}$$

where due to symmetry, there is no field in the direction of $\mathbf{a}_\phi$. In $z > 0$, the electric field is obtained from (8) and (12) as:

$$E_\rho|_{z>0} = \frac{\epsilon_\perp}{\epsilon_0}\frac{3p_{nz}\rho z}{4\pi\epsilon_1 r^5} = \frac{\epsilon_\perp}{\epsilon_0}\frac{3p_{nz}\sin\theta\cos\theta}{4\pi\epsilon_1 r^3} \tag{13}$$

$$E_z|_{z>0} = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}(3z^2 - r^2)}{4\pi\epsilon_1 r^5} = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}(3\cos^2\theta - 1)}{4\pi\epsilon_1 r^3} \tag{14}$$

In a similar manner, for $z < 0$:

$$E_\rho|_{z>0} = \frac{\epsilon_\perp}{\epsilon_0}\frac{3p_{nz}\rho z}{4\pi\epsilon_2 r^5} = \frac{\epsilon_\perp}{\epsilon_0}\frac{3p_{nz}\sin\theta\cos\theta}{4\pi\epsilon_2 r^3} \tag{15}$$

$$E_z|_{z>0} = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}(3z^2 - r^2)}{4\pi\epsilon_2 r^5} = \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}(3\cos^2\theta - 1)}{4\pi\epsilon_2 r^3} \tag{16}$$

The electric displacement field $\mathbf{D}$ is given by a single expression in both regions:

$$\begin{aligned}\mathbf{D} &= \frac{\epsilon_\perp}{\epsilon_0}\frac{p_{nz}}{4\pi r^3}\left[\mathbf{a}_\rho 3\sin\theta\cos\theta + \mathbf{a}_z(3\cos^2\theta - 1)\right] \\ &= \frac{\epsilon_\perp}{\epsilon_0}\frac{3\left(\mathbf{p}_n\cdot\mathbf{a}_r\right)\mathbf{a}_r - \mathbf{p}_n}{4\pi r^3}\end{aligned} \tag{17}$$

where $\mathbf{p}_n = \mathbf{a}_z p_{nz}$.

From these expressions we see that a normally oriented dipole at an interface produces fields and potentials that depend only on the quantity p_{nz} rather than the excess dipole moment $p_{ez} = p_{e1z} + p_{e2z}$. In particular, its $\mathbf{D}$ field depends only on $\epsilon_\perp$ rather than ϵ_1 and ϵ_2 separately, has no tangential components at $z = 0$ and is continuous everywhere except the origin. The same cannot be said of $\mathbf{E}$. The potential and field strengths are dependent on where the dipole is located "microscopically" — i.e., how much of the dipole is located in the upper medium, and how much in the lower one. We conclude that a normal dipole at an interface is most naturally characterized by the weighted dipole moment p_{nz} and its $\mathbf{D}$ field.

2.3. Tangentially Oriented Dipoles

A similar derivation for an x-directed dipole located in the upper medium at $(0, 0, 0^+)$, whose excess dipole moment is p_{e1x}, gives

$$\Phi_1 = \frac{p_{e1x}x}{4\pi\epsilon_\parallel r^3} \tag{18}$$

for any z, where

$$\epsilon_{\|} = \frac{\epsilon_1 + \epsilon_2}{2} \tag{19}$$

is a kind of "parallel" connection of the two half-space permittivities (as in a capacitor with its electric field parallel to a dielectric interface). If a dipole of moment p_{e2x} is placed just below the interface at $(0, 0, 0^-)$, the resulting potential is given by the same expression (with p_{e1x} replaced by p_{e2x}). The total potential is thus

$$\Phi = \frac{(p_{e1x} + p_{e2x})x}{4\pi\epsilon_{\|} r^3} \tag{20}$$

Similar conclusions can be obtained for any horizontally oriented dipole, and we have in general

$$\Phi = \frac{\mathbf{p}_{et} \cdot \mathbf{r}}{4\pi\epsilon_{\|} r^3} \tag{21}$$

where the subscript t denotes a vector with only tangential (x and y) components, and

$$\mathbf{p}_{et} = \mathbf{a}_x (p_{e1x} + p_{e2x}) + \mathbf{a}_y (p_{e1y} + p_{e2y}) \equiv \mathbf{a}_x p_{ex} + \mathbf{a}_y p_{ey} \tag{22}$$

The electric field can be found by computing $\mathbf{E} = -\nabla\Phi$ as in the case of the normally oriented dipole above:

$$\mathbf{E} = \frac{3\,(\mathbf{p}_{et} \cdot \mathbf{a}_r)\,\mathbf{a}_r - \mathbf{p}_{et}}{4\pi\epsilon_{\|} r^3} \tag{23}$$

It is observed that the $\mathbf{E}$ field has no z-component at $z = 0$, is continuous everywhere except the origin and depends only on the excess dipole moment p_{et} and $\epsilon_{\|}$. The same cannot be said of $\mathbf{D}$. We conclude that a tangential dipole at an interface is most naturally characterized by its excess dipole moment $\mathbf{p}_{et}$ and its $\mathbf{E}$ field.

3. FREE AND BOUND CHARGES AND DIPOLE MOMENTS

The dipole moments $\mathbf{p}_e$ used in the previous section are excess dipole moments, computed from the charges $\pm Q$ only. If bound charges due to polarization of the dielectric half-spaces are also taken into account, we obtain somewhat different dipole moments that in some cases more naturally describe the potentials and fields. There will be a bound charge distribution at the interface $z = 0$ between the two media as well as at the locations of the charges $\pm Q$, so we first obtain a complete expression for the bound charge density.

Because the normal electric field E_z at the interface of two media is not continuous, but the displacement D_z is (in the absence of free surface charge), the normal component of the polarization density $\mathbf{P}$ will be written as a function of the electric displacement rather than as a function of the electric field. We have

$$\begin{aligned}\mathbf{P} &= (\epsilon-\epsilon_0)\,\mathbf{E}\\ &= (\epsilon-\epsilon_0)\,\mathbf{E}_t+\left(1-\frac{\epsilon_0}{\epsilon}\right)\mathbf{a}_z D_z\\ &= \{\epsilon_1\vartheta(z)+\epsilon_2\,[1-\vartheta(z)]-\epsilon_0\}\,\mathbf{E}_t\\ &\quad+\left\{1-\frac{\epsilon_0}{\epsilon_1}\vartheta(z)-\frac{\epsilon_0}{\epsilon_2}\,[1-\vartheta(z)]\right\}\mathbf{a}_z D_z \qquad (24)\end{aligned}$$

where ϑ is the unit step function

$$\begin{aligned}\vartheta(z) &= 1 \qquad z>0\\ &= 0 \qquad z<0 \qquad (25)\end{aligned}$$

The bound charge density ρ_b is

$$\rho_b = -\nabla\cdot\mathbf{P} \qquad (26)$$

By substituting from (24) into (26), we obtain

$$\begin{aligned}\rho_b &= -\{\epsilon_1\vartheta(z)+\epsilon_2\,[1-\vartheta(z)]-\epsilon_0\}\,\nabla\cdot\mathbf{E}_t\\ &\quad-\left\{1-\frac{\epsilon_0}{\epsilon_1}\vartheta(z)-\frac{\epsilon_0}{\epsilon_2}\,[1-\vartheta(z)]\right\}\nabla\cdot(\mathbf{a}_z D_z)+D_z\left(\frac{\epsilon_0}{\epsilon_1}-\frac{\epsilon_0}{\epsilon_2}\right)\delta(z)\\ &= -\rho_e+\left\{\frac{\epsilon_0}{\epsilon_1}\vartheta(z)+\frac{\epsilon_0}{\epsilon_2}\,[1-\vartheta(z)]\right\}\rho_e+D_z\left(\frac{\epsilon_0}{\epsilon_1}-\frac{\epsilon_0}{\epsilon_2}\right)\delta(z) \qquad (27)\end{aligned}$$

where we have used the fact that the excess charge density is $\rho_e = \nabla\cdot\mathbf{D} = \nabla\cdot(\epsilon\mathbf{E})$, and have assumed that $\rho_e = 0$ at $z = 0$ (i.e., that we carry out these calculations before taking the limit as the positions of the charges $\pm Q$ approach the interface).

We now calculate the dipole moment $\mathbf{p}_b$ due only to the bound charge density using (27):

$$\begin{aligned}\mathbf{p}_b &= \int \rho_b\mathbf{r}\,dV\\ &= -\mathbf{p}_e+\mathbf{p}_n+\left(\frac{\epsilon_0}{\epsilon_1}-\frac{\epsilon_0}{\epsilon_2}\right)\int_{z=0} D_z(x,y)\,(x\mathbf{a}_x+y\mathbf{a}_y)\,dxdy \qquad (28)\end{aligned}$$

where $\mathbf{r}$ is the position vector, dV is the volume element, $\mathbf{p}_e = \int \rho_e \mathbf{r}\, dV$ is the excess dipole moment and

$$\mathbf{p}_n = \epsilon_0 \left(\frac{\mathbf{p}_{e1}}{\epsilon_1} + \frac{\mathbf{p}_{e2}}{\epsilon_2} \right) \tag{29}$$

is the weighted dipole moment used in Section 2. The last term of (28) is equal to zero, because for a normally oriented dipole $D_z(x, y)$ is an even function of x and y, while for a tangentially oriented dipole $D_z = 0$ at $z = 0$. Equation (28) then reduces to

$$\mathbf{p_b} = -\mathbf{p}_e + \mathbf{p}_n \tag{30}$$

In other words, the weighted dipole moment $\mathbf{p}_n$ defined above is in fact the *net* (excess plus bound) dipole moment:

$$\mathbf{p}_n = \mathbf{p}_b + \mathbf{p}_e \tag{31}$$

We can summarize the results of this section and the previous one as follows:

1. The potential and field produced by a dipole normal to the interface is proportional not to the excess dipole moment but to the net dipole moment.
2. The potential and field produced by a dipole parallel to the interface is proportional to the excess dipole moment, regardless of how those dipole moments are split microscopically on either side of the interface.
3. If $\epsilon_1 > \epsilon_0$ and $\epsilon_2 > \epsilon_0$, then $|\mathbf{p}_n| < |\mathbf{p}_e|$ (a screening effect occurs for the normal dipole).
4. The excess dipole moment does not depend on ϵ_1 or ϵ_2, and only depends on h_1 and h_2 in the combination $h_1 + h_2$.
5. The net dipole moment does depend on ϵ_1, ϵ_2, h_1 and h_2; i.e., on the detailed position of the dipole relative to interface, as well as the permittivities of the two media.

We should note that these conclusions would be not be the same if a different microscopic model were chosen for the dipole. For instance, in [20], a vacuum layer of small thickness on either side of the plane $z = 0$ is postulated, the dipole is placed at $z = 0$ and the thickness of the vacuum layer is allowed to approach zero. In this case, no additional bound charge will be associated with that of the dipole. However, this model does not as accurately reflect what is happening microscopically in our intended application to scatterers partially embedded in an interface, so we have not employed it in this work.

4. JUMP CONDITIONS AT SURFACE DIPOLE LAYERS (POLARIZATION SHEETS)

We next consider a surface distribution of dipoles at a dielectric interface. Previous authors [14, 20–24] have considered the problem of obtaining jump conditions for the fields across such a polarization sheet, but the distinction between excess and bound dipole moments in these treatments has not been made clear. We follow the method of [24], adapted for our situation.

Suppose an excess surface polarization density $\mathbf{P}_{Se1}$ (density of dipole moment per unit area) is located in medium 1 at the plane $z = z_1 > 0$. Similarly, let there be a surface polarization density $\mathbf{P}_{Se2}$ (density of dipole moment per unit area) located in medium 2 at the plane $z = z_2 < 0$. The total polarization density is then

$$\mathbf{P} = \mathbf{P}_{\text{bulk}} + \mathbf{P}_{Se1}\delta(z - z_1) + \mathbf{P}_{Se2}\delta(z - z_2) \tag{32}$$

where $\mathbf{P}_{\text{bulk}}$ contains no delta-function terms. As in [24], we postulate representations for $\mathbf{E}$ and $\mathbf{D}$ as the sums of terms with delta functions, step functions and continuous functions. These are then substituted into Maxwell's equations for the electrostatic field, whence terms with the same order of singularity are equated to each other. This yields the jumps in the z-component of $\mathbf{D}$ and the tangential components of $\mathbf{E}$ across $z = z_1$ and $z = z_2$. After this, we use the continuity conditions at $z = 0$ and take the limit as $z_1 \to 0^+$ and $z_2 \to 0^-$. The final jump conditions are:

$$D_z\big|_{z=0^-}^{0^+} = -\nabla_t \cdot \mathbf{P}_{Se} \tag{33}$$

$$\mathbf{E}_t\big|_{z=0^-}^{0^+} = -\nabla_t\left(\frac{P_{Se1z}}{\epsilon_1} + \frac{P_{Se2z}}{\epsilon_2}\right) = -\frac{1}{\epsilon_0}\nabla_t P_{Snz} \tag{34}$$

where (in analogy with the definitions used in the previous sections)

$$\mathbf{P}_{Se} = \mathbf{P}_{Se1} + \mathbf{P}_{Se2} \tag{35}$$

$$\mathbf{P}_{Sn} = \epsilon_0\left(\frac{\mathbf{P}_{Se1}}{\epsilon_1} + \frac{\mathbf{P}_{Se2}}{\epsilon_2}\right) \tag{36}$$

We see that the discontinuity in the normal component of $\mathbf{D}$ is proportional to the excess tangential surface polarization density, while the discontinuity in tangential $\mathbf{E}$ is proportional to the net normal surface polarization density. Once again, the excess polarization affects one part of the field, and the net polarization the other. Our results are compatible with those of [12], where the discontinuity in potential across a surface dipole sheet was given.

5. POLARIZABILITY OF A CONDUCTING SCATTERER IN AN INTERFACE

We finally turn our attention to the question of how to define the electric polarizability dyadic for an object partially embedded in a dielectric interface. This problem has previously been considered in [25], but there the scatterer was considered to be completely on one side or the other of the interface, and to have the same permittivity as one of the halfspaces. In particular, the distinction between free and total dipole moments did not arise there in the way it does here. In other works where the polarizability of an object at an interface was considered, either "free" polarization is assumed, or the question of whether excess or net dipole moments were used in the definition was not made clear [11, 14, 21, 22]. Other treatments of scattering by *dielectric* spheres partially embedded at an interface are given in [26–29].

5.1. Polarizability of a Scatterer in an Infinite Homogeneous Medium

For simplicity, we limit our consideration to a perfectly conducting scatterer. First, let it be located in free space, subjected to a constant incident electric field given by $\mathbf{E}^i = \mathbf{D}^i/\epsilon_0$. The incident field induces a rearrangement of the free charges on the scatterer. Considering these free charges as excess charges, the excess dipole moment induced on the scatterer is

$$\mathbf{p}_e = \int \mathbf{r}\rho_{Se}\, dS = \int \mathbf{r}\mathbf{D} \cdot \mathbf{a}_n\, dS \tag{37}$$

where $\mathbf{E}$ and $\mathbf{D}$ are the *total* (incident plus induced) fields at the scatterer. The dyadic electric polarizability of the scatterer $\overleftrightarrow{\alpha}_{E0}$ in free space is then conventionally defined by the relation

$$\mathbf{p}_e = \epsilon_0 \overleftrightarrow{\alpha}_{E0} \cdot \mathbf{E}^i \tag{38}$$

In free space, the excess dipole moment $\mathbf{p}_e$ and the net dipole moment $\mathbf{p}_n$ are the same.

If the same scatterer were placed in an infinite homogeneous dielectric of permittivity ϵ and subjected again to the same incident field $\mathbf{D}^i$ (but a generally different incident $\mathbf{E}^i = \mathbf{D}/\epsilon$), the same induced free surface charge density on the scatterer and the same excess dipole moment $\mathbf{p}_e$ will result, while net dipole moment is different:

$$\mathbf{p}_n = \frac{\epsilon_0}{\epsilon}\mathbf{p}_e \tag{39}$$

In terms of the excess dipole moment given by (37), we have

$$\mathbf{p}_e = \overleftrightarrow{\alpha}_{E0} \cdot \mathbf{D}^i \tag{40}$$

while for the net dipole moment

$$\mathbf{p}_n = \epsilon_0 \overleftrightarrow{\alpha}_{E0} \cdot \mathbf{E}^i \tag{41}$$

It would seem that either (40) or (41) could serve as the natural generalization of polarizability to this case, and would have the advantage that the polarizability would be the same as in free space. But we could also write

$$\mathbf{p}_e = \epsilon \overleftrightarrow{\alpha}_{E0} \cdot \mathbf{E}^i \tag{42}$$

or

$$\mathbf{p}_n = \frac{\epsilon_0}{\epsilon} \overleftrightarrow{\alpha}_{E0} \cdot \mathbf{D}^i \tag{43}$$

As we will see below, there are advantages to these other definitions, especially when a dielectric interface must be taken into account.

5.2. Polarizability of a Scatterer at or Near a Dielectric Interface

If this same scatterer is now placed near (or partially embedded in) the interface $z = 0$ between the half-spaces with permittivities ϵ_1 and ϵ_2 considered earlier, we must expect in general that the surface charge density induced on the scatterer will not be simply related to what is induced when it is in free space. Moreover, the tangential incident field will be most naturally expressed by $\mathbf{E}_t^i$, while the normal incident field is most naturally D_z^i, since these components are the continuous ones when the scatterer is not present [14]. On the other hand, as we have shown, the most natural dipole moments by means of which to compute the induced field of the scatterer are the excess tangential and net normal dipole moments. It therefore appears most reasonable to define the polarizability of a conducting scatterer in an interface by the equation

$$\mathbf{r} = \overleftrightarrow{\alpha}_E \cdot \mathbf{N}^i \tag{44}$$

where the generalized incident field vector $\mathbf{N}$ is defined by

$$\mathbf{N}^i = \begin{bmatrix} \epsilon_0 E_x^i \\ \epsilon_0 E_y^i \\ D_z^i \end{bmatrix} \tag{45}$$

and the generalized dipole vector is defined by

$$\mathbf{r} = \begin{bmatrix} p_{ex} \\ p_{ey} \\ p_{nz} \end{bmatrix} \tag{46}$$

(note that the normalizing factors ϵ_0 that we have included in (45) to make all elements of the polarizability dyadic have units of volume were absent in [21]).

In general, no simple relation exists between the polarizability of a scatterer in a homogeneous medium and that of the same scatterer embedded in an interface. If a certain degree of symmetry exists, however, some results in this direction can be obtained, as will be explained in the following subsection. Otherwise, numerical methods will have to be used in general, either specialized ones such as those in [14, 26–29] or general ones such as FDTD or finite-elements ([25], for example).

5.3. Polarizability of a Symmetric Conducting Scatterer

Consider a perfectly conducting scatterer that has reflection symmetry with respect to the plane $z = 0$. If it is placed in free space, we may consider two cases of excitation by an electrostatic field.

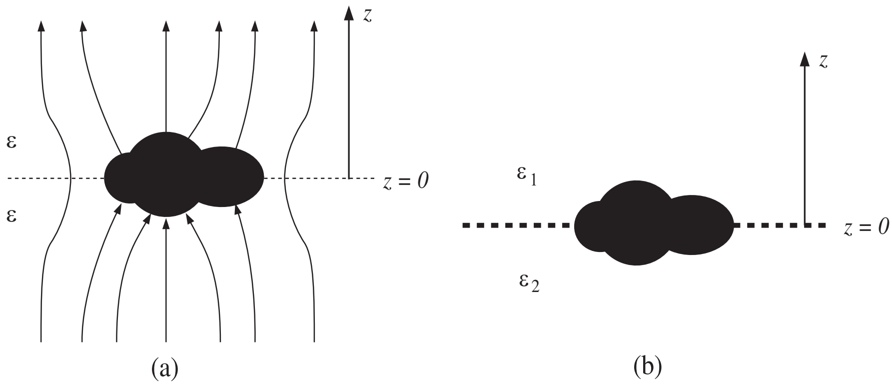

Figure 2. Symmetric conducting scatterer (a) in an infinite uniform medium, and (b) embedded in an interface.

Case 1: An incident z-directed electric displacement field is imposed [see Fig. 2(a)], given by

$$\mathbf{D}^i = \mathbf{a}_z D_z^i \tag{47}$$

then the total field will have a symmetry about $z = 0$: $E_z(x, y, -z) = E_z(x, y, z)$ and $\mathbf{E}_t(x, y, -z) = -\mathbf{E}_t(x, y, z)$, and similarly for $\mathbf{D}$. In particular,

$$\begin{aligned} \mathbf{E}(x, y, 0) &= \mathbf{a}_z E_z(x, y, 0) \\ \mathbf{D}(x, y, 0) &= \mathbf{a}_z D_z(x, y, 0) \end{aligned} \tag{48}$$

i.e., the total field is normal to the plane $z = 0$. The resulting induced dipole moment (either excess or net) of the scatterer will have only a z-component: $\mathbf{p} = \mathbf{a}_z p_z$.

Case 2: If the incident field is oriented in the tangential (xy) plane,

$$\mathbf{E}^i = \mathbf{E}^i_t \tag{49}$$

then the total field obeys the symmetry relations $E_z(x, y, -z) = -E_z(x, y, z)$ and $\mathbf{E}_t(x, y, -z) = \mathbf{E}_t(x, y, z)$, and similarly for $\mathbf{D}$. In particular,

$$\begin{aligned} \mathbf{E}(x, y, 0) &= \mathbf{E}_t(x, y, 0) \\ \mathbf{D}(x, y, 0) &= \mathbf{D}_t(x, y, 0) \end{aligned} \tag{50}$$

i.e., the total field is tangential to the plane $z = 0$. The resulting induced dipole moment of the scatterer will in this case have only transverse components: $\mathbf{p} = \mathbf{p}_t$.

We conclude from these symmetry relations that the free-space polarizability dyadic for this scatterer has the form

$$\overleftrightarrow{\alpha}_{E0} = \overleftrightarrow{\alpha}_{E0t} + \mathbf{a}_z \mathbf{a}_z \alpha_{E0zz} \tag{51}$$

where $\overleftrightarrow{\alpha}_{E0t}$ has only x and y components. Similar consequences of scatterer reflection symmetry have been obtained for the magnetic polarizability by Baum [30]. If the scatterer is placed in an infinite homogeneous dielectric of permittivity ϵ, we have from (42)–(46) that

$$\overleftrightarrow{\alpha}_{E} = \frac{\epsilon}{\epsilon_0} \overleftrightarrow{\alpha}_{E0t} + \frac{\epsilon_0}{\epsilon} \mathbf{a}_z \mathbf{a}_z \alpha_{E0zz} \tag{52}$$

5.4. Polarizability of a Symmetric Conducting Scatterer Half Embedded in an Interface

If this symmetric scatterer is placed in an environment of two half-spaces of permittivity ϵ_1 and ϵ_2, we can expect that the components of $\overleftrightarrow{\alpha}_{Et}$ as defined above will vary continuously from those of $(\epsilon_1/\epsilon_0) \overleftrightarrow{\alpha}_{E0t}$ (when the scatterer is located far above the interface in medium 1) to those of $(\epsilon_2/\epsilon_0) \overleftrightarrow{\alpha}_{E0t}$ (when the scatterer is located far below the interface in medium 2). In the same way, we expect that α_{Ezz} will vary from $(\epsilon_0/\epsilon_1)\alpha_{E0zz}$ to $(\epsilon_0/\epsilon_2)\alpha_{E0zz}$. Let the scatterer now be placed symmetrically in the interface [see Fig. 2(b)]. Consider first the case when the incident $\mathbf{D}^i$ field is normal to the interface and the same as in the free space case. Then the total $\mathbf{D}$ field must also be the same everywhere in space as it was when placed in the homogeneous medium, due to its continuity at $z = 0$. This implies that the excess charge density ρ_e (here equivalent to the free charge density ρ_f) and thus the excess dipole moment $\mathbf{p}_e$ on the scatterer will also be the same. The net charge density ρ_n will be either $\frac{\epsilon_0}{\epsilon_1}\rho_e$ on the top portion of the scatterer in $z > 0$, or $\frac{\epsilon_0}{\epsilon_2}\rho_e$ on the bottom portion of the scatterer in $z < 0$. Adding the contributions from top and bottom, one can then show that the resulting total dipole moment for the scatterer in the interface can be written in terms of the dipole moment in free space:

$$\mathbf{p}_n = \frac{1}{2}\left(\frac{\epsilon_0}{\epsilon_1} + \frac{\epsilon_0}{\epsilon_2}\right)\mathbf{p}_e = \frac{\epsilon_0}{\epsilon_\perp}\mathbf{p}_e \tag{53}$$

On the other hand, if the incident field is tangential to the interface, assume that it is now $\mathbf{E}^i$ that is the same as in the free space case. The total $\mathbf{E}$ field will be unchanged from its value when the scatterer is in free space, because it is tangential to and continuous at $z = 0$. The $\mathbf{D}$ field, however, is either ϵ_1/ϵ_0 (in the upper half-space) or ϵ_2/ϵ_0 (in the lower half-space) larger than its value in free space. Therefore, the excess charge density on the conductor also increases by those factors, while the net charge density is unchanged. As a result, $\mathbf{p}_n$ is unchanged in this case, while combining the top and bottom contributions to the excess dipole moment gives

$$\mathbf{p}_e = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_0} + \frac{\epsilon_2}{\epsilon_0}\right)\mathbf{p}_n = \frac{\epsilon_\parallel}{\epsilon_0}\mathbf{p}_n \tag{54}$$

From these results and (44), we conclude that the polarizabilities for the halfway-embedded symmetric scatterer are expressible in terms of the free-space polarizabilities as

$$\overset{\leftrightarrow}{\alpha}_{Et} = \frac{\epsilon_\parallel}{\epsilon_0}\overset{\leftrightarrow}{\alpha}_{E0t} \tag{55}$$

$$\alpha_{Ezz} = \frac{\epsilon_0}{\epsilon_\perp}\alpha_{E0zz} \tag{56}$$

5.5. Polarizability of a Sphere Symmetrically Embedded in an Interface

We now apply these results to the case of a conducting sphere embedded halfway in the interface between the media. This will be accomplished by using the well-known solution for a sphere located in infinite free space [31, pp. 205–207], and applying the results (55)–(56). If the sphere is placed into a uniform incident field $\mathbf{a}_z D_z^i = (\mathbf{a}_r \cos\theta - \mathbf{a}_\theta \sin\theta) D_z^i$, the scattered field $\mathbf{D}^s$ is [31]:

$$\mathbf{D}^s = \frac{a^3 D_z^{inc}}{r^3} \{2\mathbf{a}_r \cos\theta + \mathbf{a}_\theta \sin\theta\} \tag{57}$$

The total electric flux density at the surface of the sphere is then the sum of the incident and the scattered flux densities:

$$\mathbf{D}^{\text{tot}}\Big|_{r=a} = 3\mathbf{a}_r D_z^{inc} \cos\theta \tag{58}$$

Similarly, for incident fields E_x^i or E_y^i in the x or y direction, the total resulting field at the surface of the sphere is found to be:

$$\begin{aligned} \mathbf{E}^{\text{tot}}\Big|_{r=a} &= 3E_x^i \mathbf{a}_r \sin\theta \cos\phi \\ \mathbf{E}^{\text{tot}}\Big|_{r=a} &= 3E_y^i \mathbf{a}_r \sin\theta \sin\phi \end{aligned} \tag{59}$$

respectively. The dipole moment can then be written as

$$\mathbf{p}_e = \mathbf{p}_n = \int_s \mathbf{r}\rho ds = \mathbf{a}_z 4\pi a^3 D^i \tag{60}$$

From (40), the zz component of the free-space polarizability of the sphere is

$$\alpha_{E0}^{zz} = 4\pi a^3 \tag{61}$$

and from symmetry,

$$\alpha_{E0}^{xx} = \alpha_{E0}^{yy} = 4\pi a^3 \tag{62}$$

as well.

From (44)–(46) and (55)–(56), the polarizability of a sphere embedded halfway in the interface can now be written as:

$$\overleftrightarrow{\alpha}_E = \begin{bmatrix} \frac{\epsilon_\|}{\epsilon_0} & 0 & 0 \\ 0 & \frac{\epsilon_\|}{\epsilon_0} & 0 \\ 0 & 0 & \frac{\epsilon_0}{\epsilon_\perp} \end{bmatrix} 4\pi a^3 \tag{63}$$

6. CONCLUSION

In this paper, we have carefully considered the effects of a material interface on the fields produced by an electric dipole partially embedded in that interface. It was found that either the excess or the net dipole moment may be the most important characteristic of an embedded dipole, depending on its orientation. The concepts learned from the study of the dipole moments were then used to provide a clear definition for the electric polarizability of a scatterer embedded in an interface. Our general result was illustrated for the case of a perfectly conducting spherical scatterer. Other authors [14, 25] have treated very similar problems, but have reached somewhat different conclusions. We believe that our approach provides an unambiguous way of accounting for the scattering by small particles at an interface, and will enable the proper treatment of arrays of such particles at an interface, extending our previous work in [1–4]. A study of the magnetic moment of a dipole, along with the magnetic polarizability of a superconducting sphere, in the interface between two magnetic media is currently underway, as is the derivation of sheet transition conditions for a metafilm embedded in a material interface. The results of the work in this paper and corresponding ones for magnetic dipoles in an interface will have important implications for the modeling of nanostructures in multilayered circuit technology.

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