

## NUMERICAL SOLUTION OF SCATTERING FROM THIN DIELECTRIC-COATED CONDUCTORS BASED ON TDS APPROXIMATION AND EM BOUNDARY CONDITIONS

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**Abstract**—Thin dielectric sheet (TDS) approximation and electromagnetic (EM) boundary conditions are considered together to derive out a set of integral equations as an alternative to the impedance boundary condition (IBC) method to solve the electromagnetic scattering from thin dielectric-coated conductors. Only with discretizing the induce current on the conductor surfaces and solving an integral equation similar to that for a perfect electric conductor (PEC), the scattering fields from the whole coating system (electric or magnetic material coating) are computed. Both the electric field integral equation (EFIE), magnetic field integral equation (MFIE) and their combination form are presented. These equations are converted to a matrix equation by Galerkin's method and then solved with multilevel fast multipole algorithm (MLFMA) to obtain the far fields scattering from these coated objects.

### 1. INTRODUCTION

For the purpose of electrical stealth, heat insulation or protection, many metallic objects are coated by dielectrics which usually include both electric media and magnetic media. This necessitates an efficient numerical solution for scattering calculating from dielectric-coated conductors and the integral equation methods (IEM) are commonly used due to their high accuracy and generality for arbitrary shapes [1–8]. For a hybrid object, the conducting component is usually modeled

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as a perfect electric conductor (PEC) and can be easily solved with surface integral equation (SIE) method [1]. However, the dielectric component always occupies dominant computational resources both in memory storage and CPU time, no matter it is solved with volume integral equation (VIE) approach [1–3] or coupled integral equation (CIE) approach [4–8].

When the thickness of the dielectric is quite small, many simplifications can easily be done. The impedance boundary condition [9–11] has provided good numerical results while it has much physical restrictions to the practical applications. The hybrid PEC-dielectric formulation [12], which bases on the rigorous integral formulation, provides an alternative to the IBC for dielectric-coated metallic surfaces by combining the thin dielectric sheet approximation [13, 14] with explicit boundary conditions. However, all previous researches, including our recent improving research work [15–20], only can solve the electric dielectric-coated problems. While unfortunately, most dielectric coatings are consisted of not only electric materials but also magnetic materials.

In this paper, a method to simulate the scattering from the dielectric-coated metallic structures is presented by combining the thin dielectric sheet (TDS) approximation altogether with the explicit PEC boundary conditions. As an alternative to the IBC, this method removes the need to solve the field in dielectric layer as well while no physical restrictions are introduced due to its bases on rigorous integral formulation. With this method, we only need to discretize the induced current on the conductor surfaces and solve an integral equation similar to that for a PEC in order to calculate the scattering fields from the whole coating system. Different from the work in [12], the proposed hybrid PEC-dielectric formulation in this paper can deal with the coating with electric and magnetic materials without any increase of computational complexity. The electric field integral equation (EFIE), magnetic field integral equation (MFIE) and their combination form- combined field integral equation (CFIE) are presented. These equations are final converted to a matrix equation by Galerkin's method and then solved with multilevel fast multipole algorithm (MLFMA) [21, 22].

The paper is organized as follows. In Section 2 the hybrid PEC-dielectric formulation for EFIE is derived; The MFIE and their combination forms are introduced in Section 3. In Section 4, the MLFMA is used to solve the final integral equations and the disaggregation and aggregation terms of MLFMA are detailed represented in this section since they are quite different from those for only PEC situation. There are two numerical examples to show

the validation and accuracy of this method in Section 5 and some conclusions have been finally given in Section 6.

## 2. FORMULATIONS FOR ELECTRONIC FILED INTEGRAL EQUATION

For a dielectric-coated PEC system (as shown in Figure 1, a PEC enclosed by the surface  $S$  and a dielectric coating with volume  $V$ ), the electromagnetic field can be described by following two coupled equations. On the PEC surface, the tangential electric field, contributed from both the incident field and the scattered field, is zero, i.e.,

$$[\mathbf{E}^{inc}(\mathbf{r}) + \mathbf{E}^{scat}(\mathbf{r})]_t = 0, \quad \mathbf{r} \in S. \quad (1)$$

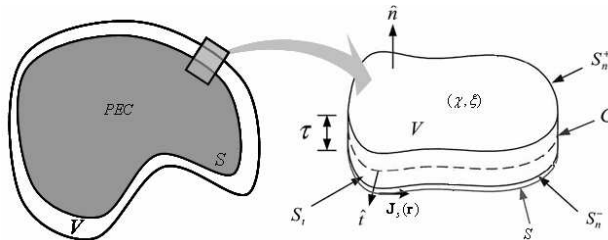
The subscript “ $t$ ” stands for taking the tangential component of the vector. Inside the medium, the incident field, scattered field and total field satisfy

$$\mathbf{E}^{inc}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) - \mathbf{E}^{scat}(\mathbf{r}), \quad \mathbf{r} \in V \quad (2)$$

where  $\mathbf{r}$  denotes the field point, and  $\mathbf{E}^{inc}$ ,  $\mathbf{E}^{scat}$  stands for the incident field and scattered field, respectively. The scattering fields include contributions from both conductor components and dielectric components ( $\varepsilon_r \neq 1$  or  $\mu_r \neq 1$ ), that is

$$\mathbf{E}^{scat}(\mathbf{r}) = \mathbf{E}_{pec}^{scat}(\mathbf{r}) + \mathbf{E}_{die}^{scat}(\mathbf{r}) \quad (3)$$

The scattering field from the PEC is established by (time



**Figure 1.** Geometry of a PEC coated by thin dielectric with associated quantities.

dependence  $e^{-i\omega t}$  is assumed)

$$\begin{aligned}\mathbf{E}_{pec}^{scat}(\mathbf{r}) &= i\omega\mu_0 \int_S \bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \bullet \mathbf{J}_s(\mathbf{r}') dS' \\ &= i\omega\mu_0 \int_S \mathbf{J}_s(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') + \frac{1}{k_0^2} \nabla g(\mathbf{r}, \mathbf{r}') \nabla' \bullet \mathbf{J}_s(\mathbf{r}') dS' \quad (4)\end{aligned}$$

and that from the dielectric is given as

$$\begin{aligned}\mathbf{E}_{die}^{scat}(\mathbf{r}) &= i\omega\mu_0 \int_V \bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \bullet \mathbf{J}_v(\mathbf{r}') dV' - \int_V \nabla g(\mathbf{r}, \mathbf{r}') \times \mathbf{M}_v(\mathbf{r}') dV' \\ &= i\omega\mu_0 \int_V \mathbf{J}_v(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') + \frac{1}{k_0^2} \nabla' \bullet \mathbf{J}_v(\mathbf{r}') \nabla g(\mathbf{r}, \mathbf{r}') dV' \\ &\quad - \int_V \nabla g(\mathbf{r}, \mathbf{r}') \times \mathbf{M}_v(\mathbf{r}') dV' \quad (5)\end{aligned}$$

Here the primes refer to the source coordinates.  $\mathbf{J}_s$  is the conducted surface current density on  $S$ ,  $\mathbf{J}_v/\mathbf{M}_v$  is the polarization volume electric/magnetic current density in  $V$ .  $\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$  is the dyadic Green function defined as

$$\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left[ \bar{\mathbf{I}} - \frac{1}{k^2} \nabla \nabla' \right] g(\mathbf{r}, \mathbf{r}') \quad (6)$$

and  $g(\mathbf{r}, \mathbf{r}')$  is the scalar Green's function in free space which is defined by  $g(\mathbf{r}, \mathbf{r}') = \frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$ . All other quantities are defined according to convention.

The polarization volume electric/magnetic current density in the dielectric is defined by

$$\mathbf{J}_v(\mathbf{r}') = i\omega(\varepsilon_0 - \varepsilon)\mathbf{E}(\mathbf{r}') = i\omega\chi(\mathbf{r}')\mathbf{D}(\mathbf{r}') \quad (7)$$

$$\mathbf{M}_v(\mathbf{r}') = i\omega(\mu_0 - \mu)\mathbf{H}(\mathbf{r}') = i\omega\xi(\mathbf{r}')\mathbf{B}(\mathbf{r}') \quad (8)$$

where

$$\chi(\mathbf{r}') = \frac{1}{\varepsilon_r(\mathbf{r}')} - 1 \quad (9)$$

$$\xi(\mathbf{r}') = \frac{1}{\mu_r(\mathbf{r}')} - 1 \quad (10)$$

$\mathbf{E}$ ,  $\mathbf{H}$  and  $\mathbf{D}$ ,  $\mathbf{B}$  are the field and flux vectors in the dielectric region.

Substitute formulas (7) and (8) into expression (5) to achieve

$$\begin{aligned} \mathbf{E}_{die}^{scat}(\mathbf{r}) = & i\omega\mu_0 \left[ \int_V \chi(\mathbf{r}')g(\mathbf{r}, \mathbf{r}')i\omega\mathbf{D}(\mathbf{r}') \right. \\ & + \frac{1}{k_0^2} \chi(\mathbf{r}') \nabla g(\mathbf{r}, \mathbf{r}') i\omega \nabla' \bullet \mathbf{D}(\mathbf{r}') dV' - \frac{1}{k_0^2} \int_{SA} \chi(\mathbf{r}') \nabla g(\mathbf{r}, \mathbf{r}') i\omega \hat{n}' \bullet \mathbf{D}(\mathbf{r}') dS' \left. \right] \\ & - \int_V \nabla g(\mathbf{r}, \mathbf{r}') \times i\omega \xi(\mathbf{r}') \mathbf{B}(\mathbf{r}') dV' \end{aligned} \quad (11)$$

In the above formula,  $SA$ , which is consist of  $S_n^+$ ,  $S_n^-$  and  $S_t$  (top, bottom and side surfaces respectively), denotes the interfaces where the dielectric constant is discontinuous at each side and  $\hat{n}'$  is the unit normal vector directing out of the dielectric at these interfaces.

Taking into account that

$$\nabla \bullet \mathbf{D}(\mathbf{r}') = 0 \quad (12)$$

finally we obtain contributions from the dielectric component when it is completely coated on the conductor surfaces:

$$\begin{aligned} \mathbf{E}_{die}^{scat}(\mathbf{r}) = & i\omega\mu_0 \left[ \int_V \chi(\mathbf{r}')g(\mathbf{r}, \mathbf{r}')i\omega\mathbf{D}(\mathbf{r}')dV' \right. \\ & - \frac{1}{k_0^2} \int_{S_n^+} \chi \nabla g(\mathbf{r}, \mathbf{r}') i\omega \hat{n}' \bullet \mathbf{D}(\mathbf{r}') dS' + \frac{1}{k_0^2} \int_{S_n^-} \chi \nabla g(\mathbf{r}, \mathbf{r}') i\omega \hat{n}' \bullet \mathbf{D}(\mathbf{r}') dS' \left. \right] \\ & - \int_V \nabla g(\mathbf{r}, \mathbf{r}') \times i\omega \xi(\mathbf{r}') \mathbf{B}(\mathbf{r}') dV' \end{aligned} \quad (13)$$

here,  $\hat{n}'$  is the unit vector normal to the upper surface.

When the thickness of the dielectric coating is very small compared to the wavelength, the field varies little in the normal direction. A straightforward way is to approximate the fields to no variation with respect to the normal direction and this procedure is so named thin dielectric sheet (TDS) approximation [12–20]. Decompose the  $\mathbf{D}$  and  $\mathbf{B}$  fluxes into tangential and normal components, namely

$$\mathbf{D}(\mathbf{r}') = \mathbf{D}_t(\mathbf{r}') + \hat{n}' D_n(\mathbf{r}') \quad \mathbf{B}(\mathbf{r}') = \mathbf{B}_t(\mathbf{r}') + \hat{n}' B_n(\mathbf{r}') \quad (14)$$

then approximate  $\mathbf{D}_t(\mathbf{r}') \approx 0$ ,  $B_n(\mathbf{r}') \approx 0$  in the whole coating layer because they are continuous across the PEC surface and almost no variation. Further consider the current continuous boundary conditions

$$\begin{aligned} i\omega\hat{n}' \bullet \mathbf{D}(\mathbf{r}') &= i\omega D_n = \nabla' \bullet \mathbf{J}_s(\mathbf{r}') \\ \hat{n}' \times \mathbf{B}_t(\mathbf{r}') &= \mu\hat{n}' \times \mathbf{H}_t(\mathbf{r}') = \mu\mathbf{J}_s(\mathbf{r}'), \end{aligned} \quad r' \in S_n^- \quad (15)$$

and finally obtain these two approximate expressions in the whole dielectric layer:

$$\begin{aligned} D_n(\mathbf{r}') &\approx \nabla' \bullet \mathbf{J}_s(\mathbf{r}')/i\omega \\ \mathbf{B}_t(\mathbf{r}') &= -\hat{n}' \times \hat{n}' \times \mathbf{B}_t(\mathbf{r}') \approx -\mu\hat{n}' \times \mathbf{J}_s(\mathbf{r}'), \end{aligned} \quad r' \in V \quad (16)$$

The volume integral can be further approximated to surface integral through conversion  $dV \approx \tau dS$ . Here,  $\tau$  is the thickness of the dielectric coatings. Therefore, the scattering field from the coatings can be expressed as

$$\begin{aligned} \mathbf{E}_{die}^{scat}(\mathbf{r}) &= i\omega\mu_0 \left[ \int_S \hat{n}' \tau \chi g(\mathbf{r}, \mathbf{r}') \nabla' \bullet \mathbf{J}_s(\mathbf{r}') dS' \right. \\ &\quad + \frac{\nabla}{k_0^2} \int_S \chi [g(\mathbf{r}, \mathbf{r}') - g_\tau(\mathbf{r}, \mathbf{r}')] \nabla' \bullet \mathbf{J}_s(\mathbf{r}') dS' \\ &\quad \left. + (1 - \mu_r) \int_S \tau \nabla g(\mathbf{r}, \mathbf{r}') \times \hat{n}' \times \mathbf{J}_s(\mathbf{r}') dS' \right] \quad (17) \end{aligned}$$

here  $g_\tau(\mathbf{r}, \mathbf{r}') = g(\mathbf{r}, \mathbf{r}' + \tau\hat{n}')$ , representing the contribution from the sources on the interface  $S_n^+$ .

Up to this point, the total scattering field can be represented by combining (4) and (17)

$$\begin{aligned} \mathbf{E}^{scat}(\mathbf{r}) &= i\omega\mu_0 \left[ \frac{(1 - \mu_r)}{2} \mathbf{J}_s + \int_S g(\mathbf{J}_s + \hat{n}' \tau \chi \nabla' \bullet \mathbf{J}_s) dS' \right. \\ &\quad + \frac{\nabla}{k_0^2} \int_S (g + \chi g - \chi g_\tau) \nabla' \bullet \mathbf{J}_s dS' \\ &\quad \left. + (1 - \mu_r) \int_S \tau \nabla g \times \hat{n}' \times \mathbf{J}_s dS' \right] \quad (18) \end{aligned}$$

In (18), the first term in the right-hand side is the principle value of the last integral and there is only one unknown quantity  $\mathbf{J}_s$ . So only Equation (1) is needed to establish a linear equation and solve this problem. The final integral equation is established as

$$\begin{aligned} \mathbf{E}^{inc}(\mathbf{r})|_t = & -i\omega\mu_0 \left[ \frac{(1-\mu_r)}{2} \mathbf{J}_s + \int_S g(\mathbf{J}_s + \hat{n}'\tau\chi\nabla' \bullet \mathbf{J}_s) dS' \right. \\ & + \frac{\nabla}{k_0^2} \int_S (g + \chi g - \chi g_\tau) \nabla' \bullet \mathbf{J}_s dS' \\ & \left. + (1-\mu_r) \int_S \tau \nabla g \times \hat{n}' \times \mathbf{J}_s dS' \right] \quad \mathbf{r} \in S. \end{aligned} \quad (19)$$

This is the electric field integral equation (EFIE) form resulting from electric field boundary condition. In this equation, only the induced current density on the PEC surfaces and surface integral are involved. By using the curvilinear triangular patches to discretize the PEC surfaces and selecting the curvilinear RWG as the basis functions, we can solve this integral equation with multilevel fast multipole algorithm (MLFMA) [21] as we do for pure electric conductors.

### 3. MAGNETIC FIELD AND COMBINED FIELD INTEGRAL EQUATIONS

As is known, the electric field integral equation converges slowly for iteration solution and suffers from inner resonance problems. While the combined field integral equation (CFIE), which is a combination of electric field integral equation and magnetic field integral equation (MFIE) with a certain combine coefficient, can overcome the inner resonance problem and reduce the matrix condition number, improving the accuracy and accelerating the iteration speed. This section will focus on the derivation of the magnetic field integral equation starting from magnetic field boundary condition:

$$\hat{n} \times [\mathbf{H}^{inc}(\mathbf{r}) + \mathbf{H}^{scat}(\mathbf{r})] = \mathbf{J}_s(\mathbf{r}), \quad \mathbf{r} \in S. \quad (20)$$

where  $\hat{n}$  denotes the unit vector normal to the PEC surfaces. Similarly, the scattering fields include contributions from both conductor

components and dielectric components

$$\mathbf{H}^{scat}(\mathbf{r}) = \mathbf{H}_{pec}^{scat}(\mathbf{r}) + \mathbf{H}_{die}^{scat}(\mathbf{r}) \quad (21)$$

$$\mathbf{H}_{pec}^{scat}(\mathbf{r}) = \int_S \nabla g(\mathbf{r}, \mathbf{r}') \times \mathbf{J}_s(\mathbf{r}') dS' \quad (22)$$

$$\begin{aligned} \mathbf{H}_{die}^{scat}(\mathbf{r}) = & i\omega\epsilon_0 \int_V \mathbf{M}_v(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') + \frac{1}{k_0^2} \nabla' \bullet \mathbf{M}_v(\mathbf{r}') \nabla g(\mathbf{r}, \mathbf{r}') dV' \\ & + \int_V \nabla g(\mathbf{r}, \mathbf{r}') \times \mathbf{J}_v(\mathbf{r}') dV' \end{aligned} \quad (23)$$

Through the thin dielectric sheet approximation and boundary conditions mentioned in last section,

$$\begin{aligned} \mathbf{H}_{die}^{scat}(\mathbf{r}) = & i\omega\epsilon_0 \int_S -i\omega\mu_0(1 - \mu_r)\tau g(\mathbf{r}, \mathbf{r}') \hat{n}' \times \mathbf{J}_s(\mathbf{r}') dS' \\ & + \int_S \tau \chi \nabla' \bullet \mathbf{J}_s \nabla g(\mathbf{r}, \mathbf{r}') \times \hat{n}' dS' \end{aligned} \quad (24)$$

$$\begin{aligned} \mathbf{H}^{scat}(\mathbf{r}) = & -\frac{1}{2} \hat{n} \times \mathbf{J}_s + \int_S \nabla g \times \mathbf{J}_s dS' + \int_S k^2(1 - \mu_r)\tau g \hat{n}' \times \mathbf{J}_s dS' \\ & + \int_S \tau \chi \nabla' \bullet \mathbf{J}_s \nabla g \times \hat{n}' dS' \end{aligned} \quad (25)$$

The final magnetic field integral equation is established as

$$\begin{aligned} \hat{n} \times \mathbf{H}^{inc}(\mathbf{r}) = & \frac{1}{2} \mathbf{J}_s - \hat{n} \times \left[ \int_S \nabla g \times \mathbf{J}_s dS' + \int_S k^2(1 - \mu_r)\tau g \hat{n}' \times \mathbf{J}_s dS' \right. \\ & \left. + \int_S \tau \chi \nabla' \bullet \mathbf{J}_s \nabla g \times \hat{n}' dS' \right] \end{aligned} \quad (26)$$

Combining the EFIE (19) with MFIE (26), the CFIE is defined by

$$\text{CFIE} = \alpha \text{EFIE} + \eta(1 - \alpha) \text{MFIE} \quad (27)$$

where,  $\alpha$  is the combine coefficient and  $\eta$  is the wave impedance in free space.

#### 4. APPLICATION OF MULTILEVEL FAST MULTIPOLE ALGORITHM

The integral Equations (19), (26) or (27) is readily to be converted to a matrix equation  $\bar{\mathbf{Z}} \bullet \mathbf{I} = \mathbf{V}$  with Galerkin's method. In this section, multilevel fast multipole algorithm (MLFMA) is used to accelerate the calculating of the final matrix equation.

The multilevel fast multipole algorithm is a most robust electromagnetic analysis approach for accelerating the matrix-vector multiplication in an iterative solver, which would reduce the computational complexity (in terms of the memory requirement and CPU time) to  $O(N \log N)$  for an  $N$ -unknown problem. For a given testing function, all the basis functions are classified into two categories based on the distance between the testing and basis functions. The matrix-vector multiplication is represented as the summation of the contribution from the near- and far-region

$$\sum_i Z_{ji} I_i = \sum_{i \in NR} Z_{ji} I_i + \sum_{i \in FR} Z_{ji} I_i \quad (28)$$

where  $NR$  and  $FR$  represent the near-region and far-region, respectively. The near-region matrix elements are calculated directly in the way as in the conventional MoM [23], and the far-region coupling is dealt with the MLFMA.

For far-region matrix element calculation, the dyadic Green's function can be written into a multipole expression when the addition theorem is applied

$$\bar{\mathbf{G}}(\mathbf{r}_j, \mathbf{r}_i) = \frac{ik}{(4\pi)^2} \int d^2 \hat{k} (\bar{\mathbf{I}} - \hat{k} \hat{k}) e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \alpha_{mm'}(\hat{\mathbf{r}}_{mm'} \bullet \hat{\mathbf{k}}), \quad |\mathbf{r}_{mm'}| > |\mathbf{r}_{jm} - \mathbf{r}_{im'}| \quad (29)$$

Similarly, the gradient of Green's function

$$\nabla g \approx i\mathbf{k}g = i\mathbf{k} \left[ \frac{ik}{(4\pi)^2} \int d^2 \hat{k} e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \alpha_{mm'}(\hat{\mathbf{r}}_{mm'} \bullet \hat{\mathbf{k}}) \right], \quad |\mathbf{r}_{mm'}| > |\mathbf{r}_{jm} - \mathbf{r}_{im'}| \quad (30)$$

where  $\mathbf{r}_j$  and  $\mathbf{r}_i$  are the vectors of the field and source points, respectively,  $\mathbf{r}_m$  and  $\mathbf{r}'_m$  respectively present the center of the field and source group.  $\alpha_{mm'}$  denotes the translation term. Therefore, the second term in the right-hand side of (28), which represents the far-field

matrix elements, can be written as

$$\begin{aligned} & \sum_{i \in FR} Z_{ji} I_i \\ &= \frac{ik}{(4\pi)^2} \int d^2 \hat{k} \mathbf{V}_{fmj}(\hat{k}) \sum_{m' \in FR} \alpha_{mm'}(\hat{r}_{mm'} \bullet \hat{k}) \sum_{i \in G_{m'}} \mathbf{V}_{sm'i}(\hat{k}) I_i \quad (31) \end{aligned}$$

where  $\mathbf{V}_{fmj}(\hat{k})$  and  $\mathbf{V}_{sm'i}(\hat{k})$  represent the disaggregation and aggregation terms, respectively.

For the electric field integral equation, we define

$$\langle \mathbf{t}_j, -i\mathbf{E}^{scat}(\mathbf{r})/k\eta \rangle = \sum_i Z_{ji}^E I_i \quad (32)$$

According to all the concerning formulas, the impedance element for far-region is rewritten as

$$\begin{aligned} Z_{ji}^E = \int_S dS \mathbf{t}_j \bullet \left\{ \int_{S'} dS' \frac{ik}{(4\pi)^2} \int d^2 \hat{k} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \right. \\ \left. \alpha_{mm'}(\hat{r}_{mm'} \bullet \hat{k})(\mathbf{j}_i + \hat{n}' \tau \chi \nabla' \bullet \mathbf{j}_i) + \int_{S'} dS' i\mathbf{k} \times \frac{ik}{(4\pi)^2} \right. \\ \left. \int d^2 \hat{k} e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \alpha_{mm'}(\hat{r}_{mm'} \bullet \hat{k}) [\tau(1 - \mu_r) \hat{n}' \times \mathbf{j}_i] \right\} \quad i \in FR \quad (33) \end{aligned}$$

Through comparing (33) with (31), we can obtain the disaggregation and aggregation terms for the electric field integral equation:

$$\mathbf{V}_{fmj}^E(\hat{k}) = \int_S dS e^{i\mathbf{k} \bullet \mathbf{r}_{jm}} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet \mathbf{t}_j(\mathbf{r}_{jm}) \quad (34)$$

$$\begin{aligned} \mathbf{V}_{sm'i}^E(\hat{k}) = \int_{S'} dS' e^{-i\mathbf{k} \bullet \mathbf{r}_{im'}} \left\{ (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet (\mathbf{j}_i + \hat{n}' \tau \chi \nabla' \bullet \mathbf{j}_i) \right. \\ \left. + i\mathbf{k} \times [\tau(1 - \mu_r) \hat{n}' \times \mathbf{j}_i] \right\} \quad (35) \end{aligned}$$

For the magnetic field integral equation, we define

$$\langle \mathbf{t}_j, -i\hat{n} \times \mathbf{H}^{scat}(\mathbf{r})/k \rangle = \sum_i Z_{ji}^M I_i \quad (36)$$

$$\begin{aligned} Z_{ji}^M = & -\hat{n} \times \frac{i}{k} \int_S dS \mathbf{t}_j \bullet \left\{ \int_{S'} dS' \frac{ik}{(4\pi)^2} \int d^2\hat{k} i\mathbf{k} \times e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \right. \\ & \alpha_{mm'}(\hat{r}_{mm'} \bullet \hat{k})(\mathbf{j}_i + \hat{n}'\tau\chi\nabla' \bullet \mathbf{j}_i) + \int_{S'} dS' \frac{ik}{(4\pi)^2} \int d^2\hat{k} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \\ & \left. \bullet e^{i\mathbf{k} \bullet (\mathbf{r}_{jm} - \mathbf{r}_{im'})} \alpha_{mm'}(\hat{r}_{mm'} \bullet \hat{k}) [k^2\tau(1-\mu_r)\hat{n}' \times \mathbf{j}_i] \right\} \quad i \in FR \quad (37) \end{aligned}$$

According to the identity:  $-\hat{k} \times \hat{k} \times \mathbf{A} = \mathbf{A} - \hat{k}(\hat{k} \bullet \mathbf{A}) = (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet \mathbf{A}$ , there exists that

$$(\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet k^2\tau(1-\mu_r)\hat{n}' \times \mathbf{j}_i = i\mathbf{k} \times i\mathbf{k} \times [\tau(1-\mu_r)\hat{n}' \times \mathbf{j}_i] \quad (38)$$

Substitute (38) into (37) and then the disaggregation and aggregation terms for the magnetic field integral equation are derived out as follows:

$$\mathbf{V}_{fmj}^M(\hat{k}) = -\hat{k} \times \int_S dS e^{i\mathbf{k} \bullet \mathbf{r}_{jm}} \mathbf{t}_j(\mathbf{r}_{jm}) \times \hat{n} \quad (39)$$

$$\begin{aligned} \mathbf{V}_{sm'i}^M(\hat{k}) = \mathbf{V}_{sm'i}^E(\hat{k}) = & \int_{S'} dS' e^{-i\mathbf{k} \bullet \mathbf{r}_{im'}} \left\{ (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet (\mathbf{j}_i + \hat{n}'\tau\chi\nabla' \bullet \mathbf{j}_i) \right. \\ & \left. + i\mathbf{k} \times [\tau(1-\mu_r)\hat{n}' \times \mathbf{j}_i] \right\} \quad (40) \end{aligned}$$

Finally the disaggregation and aggregation terms for the combined field integral equation can be expressed as:

$$\begin{aligned} \mathbf{V}_{fmj}^C(\hat{k}) = & \alpha \mathbf{V}_{fmj}^E(\hat{k}) + (1-\alpha) \mathbf{V}_{fmj}^M(\hat{k}) \\ = & \alpha \int_S dS e^{i\mathbf{k} \bullet \mathbf{r}_{jm}} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \bullet \mathbf{t}_j(\mathbf{r}_{jm}) \\ & - (1-\alpha) \hat{k} \times \int_S dS e^{i\mathbf{k} \bullet \mathbf{r}_{jm}} \mathbf{t}_j(\mathbf{r}_{jm}) \times \hat{n} \quad (41) \end{aligned}$$

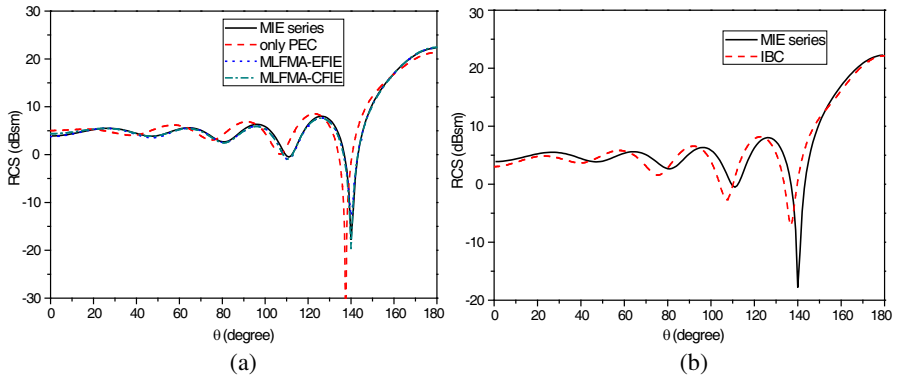
$$\mathbf{V}_{sm'i}^C(\hat{k}) = \int_{S'} dS' e^{-i\mathbf{k} \cdot \mathbf{r}_{im'}} \left\{ (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot (\mathbf{j}_i + \hat{n}'\tau\chi\nabla' \cdot \mathbf{j}_i) + i\mathbf{k} \times [\tau(1 - \mu_r)\hat{n}' \times \mathbf{j}_i] \right\} \quad (42)$$

The expression of translation term is the same as references and will not be given here.

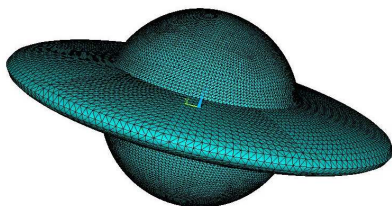
## 5. NUMERICAL EXPERIMENTS

The resulting linear system has the same number of unknowns as a pure PEC problem and the same computational complexity in solving the final matrix equation. In this section, two numerical applications are considered.

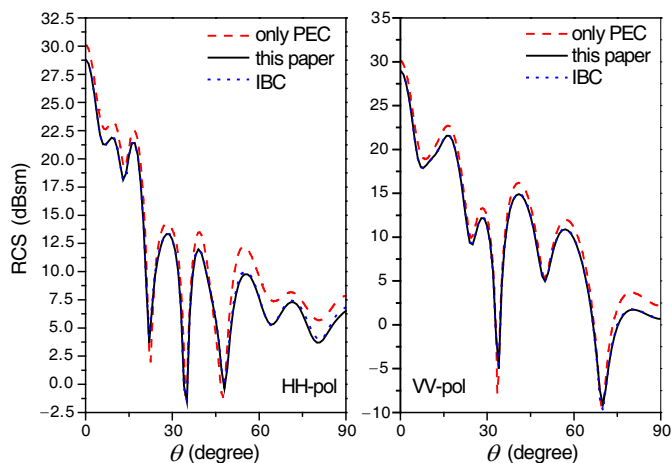
The first example is the scattering from a PEC sphere coated by thermal protective materials with relative permittivity  $4.5 + i0.15$  and relative permeability  $0.16 + i0.09$ . The radius of the sphere is 1 m and the coating thickness is 0.06 m. The plane wave is incident from  $(0^\circ, 0^\circ)$  at 0.3 GHz. There are 2,352 patches used in this simulation, which results in an unknown count of 3,528 unknowns and a 4-level MLFMA using both the EFIE and CFIE formulations. Figure 2 compares the bistatic radar cross section (RCS) using IBC and the formulations presented in this paper accelerated with MLFMA to the MIE series. The scattering result of the PEC sphere without dielectric coating is also give as a reference. Figure 2(a) shows that the results from TDS



**Figure 2.** Bistatic RCS at 0.3 GHz for a PEC sphere coated with thermal protective materials. (a) Calculated with TDS approximation. (b) Calculated with IBC.



**Figure 3.** The geometry of a UFO like objects.



**Figure 4.** Backscattering cross sections at 0.3 GHz for a UFO coated with wave absorbing materials.

approximation have good agreement with the analytical ones. The relative RMS error, which is defined as

$$\text{RMS} = \sqrt{\frac{\sum |f_c - f_r|^2}{\sum |f_r|^2}} \times 100\% \quad (43)$$

is less than 3%. Where  $f_c$  and  $f_r$  are the computational and reference values. However, because the dielectric constants do not stratify the validation criteria mentioned in reference [9], the IBC approximation brings great numerical errors to the final results as shown in Figure 2(b). Compared with the MIE series, the relative RMS for IBC is as large as 23%.

In the second example, a simple unidentified flying object (UFO) as shown in Figure 3 is calculated. It consists of a sphere with diameter

3 m and an ellipsoid with axial lengths 6 m, 6 m and 1 m, respectively. We presume wave-absorbing materials coated on the PEC object and the coating with electromagnetic parameters: relative permittivity  $29.78 + i2.31$ , relative permeability  $1.87 + i1.96$  and coating thickness 5 mm. 25,840 triangular patches, which give rise to 38,760 unknowns, are used in this simulation. CFIE formulation with combine coefficient 0.8 and a 5-level MLFMA are chosen to solve this problem. The backscattering cross sections (HH-polarization and VV-polarization) at the frequency 0.3 G are calculated and compared with the results from IBC method. The final results, including a reference without dielectric coating are shown in Figure 4. We can see that the result from the proposed method in this paper almost fully coincide with that from IBC (RMS error is no more than 2%) and the RCS has an obvious reduction due to the influence of wave-absorbing materials.

## 6. CONCLUSION

In this paper, an alternative method to the impedance boundary condition (IBC) is presented to simulate the scattering from the dielectric-coated metallic structures by combining the thin dielectric sheet (TDS) approximation altogether with the explicit PEC boundary conditions. This method removes the need to solve the field in dielectric layer in a manner like IBC while no physical restrictions is introduced by basing on the rigorous integral formulation. The proposed hybrid PEC-dielectric formulations in this paper can deal with the coating with electric and magnetic materials with the same computational complexity as surface integral equations for PEC objects. The electric field integral equation (EFIE), magnetic field integral equation (MFIE) and combination field integral equation (CFIE) are presented. These equations are final converted to a set of matrix equations and then efficiently solved with multilevel fast multipole algorithm (MLFMA). Numerical results from this method agree well with the analytical ones or those from the IBC method, demonstrating its validation and accuracy. The detailed comparison and analysis on the accuracy and application scopes between this method and IBC will be presented shortly as our further research work.

## ACKNOWLEDGMENT

This work was supported partially by National Natural Science Foundation of China under Contract No. 60431010, and 973 Program under Contract No. 6136001, partially by the 111 Project under Contract No. B07046.

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