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FOCAL REGION FIELD OF PEMC PARABOLOIDAL REFLECTOR PLACED IN HOMOGENOUS CHIRAL MEDIUM

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Abstract—Focal region fields of perfect electromagnetic conductor (PEMC) paraboloidal reflector placed in homogenous and reciprocal chiral medium are analyzed. Maslov's method is used to derive the expressions for the focal region fields because the geometrical optics (GO) fails at these points. The results obtained by this method are solved numerically, and line plots for the reflected field of the paraboloidal PEMC reflector are obtained for different values of admittance of the PEMC reflector and chirality parameter of the medium.

1. INTRODUCTION

Perfect electromagnetic conductor (PEMC) is a non-reciprocal generalization of both perfect electric conductor (PEC) and perfect magnetic conductor (PMC). Lindell and Sihvola suggested that the PEMC boundary can be realized in terms of a layer of certain nonreciprocal materials resting on a PEC plane [1, 2]. The layer of a bi-isotropic medium can act as a PEMC boundary by choosing the parameters of the medium properly. The boundary conditions for the

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perfect electric conductor and perfect magnetic conductor are given by following relations

$$\mathbf{n} \times \mathbf{E} = 0, \quad \mathbf{n} \cdot \mathbf{B} = 0 \quad \text{(PEC)} \qquad \mathbf{n} \times \mathbf{H} = 0, \quad \mathbf{n} \cdot \mathbf{D} = 0 \quad \text{(PMC)}$$

Boundary conditions for PEMC are more general and are given as

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = 0, \qquad \mathbf{n} \cdot (\mathbf{D} - M\mathbf{B}) = 0$$

where, $\mathbf{n}$ is the unit normal to the surface and M is the admittance of the PEMC boundary. PEMC reduces to PMC when $M = 0$, and PEC when $M \longrightarrow \pm\infty$. The above conditions have a basic theoretical interest because, in differential form formalism [3], they correspond to the simplest possible medium by a single scalar medium parameter M. PEMC can serve as boundary material, because it does not allow electromagnetic energy to enter. The most notable difference is the nonreciprocity of the PEMC boundary when M has a finite nonzero value. The nonreciprocity of the PEMC boundary can be demonstrated by showing that the polarization of plane wave reflected from its surface is rotated, the sense and angle of rotation depending on the admittance parameter M.

In present work, our interest is to find expressions for the high frequency fields around the focal region for the PEMC paraboloidal reflector when it is placed in homogenous and reciprocal chiral medium using geometrical optics (GO) and method proposed by Maslov [4]. GO or asymptotic ray theory (ART) is widely used in many areas of electromagnetics. However GO show singularities at the focal region. In various applications, such as cylindrical reflectors and other focusing systems, the field strength at these regions is of practical importance. Hence, an asymptotic method based on Maslov's theory is used to study the behavior of field pattern around the focal regions. Maslov's method is used by many authors to study the field behavior of various focusing systems [6–15]. This work is an extension of the previous work, in which field at the caustic of a PEC paraboloidal reflector placed in chiral medium was studied [5].

In Section 2, reflection of plane waves from PEMC plane boundary is studied. In Section 3, we have discussed the geometrical optics field of PEMC parabolic reflector placed in chiral medium and plots around focal point are discussed. Concluding remarks are presented in Section 4.

2. PLANE WAVE REFLECTION FROM PEMC PLANE PLACED IN CHIRAL MEDIUM

In this paper, we want to derive the high frequency field expressions for a PEMC paraboloidal reflector. To achieve this first we study the reflection of plane wave from the PEMC placed in chiral medium.

When the RCP wave having unit amplitude, phase velocity ω/kn_2 and making angle α with normal (z-axis) strike the PEMC plane placed in chiral medium. Two waves of opposite handedness (RCP and LCP) are reflected as shown in Figure 1. The RCP wave has amplitude $(\cos\alpha - \cos\alpha_1)/(\cos\alpha + \cos\alpha_1)$ and makes an angle α with z-axis and the LCP wave with amplitude

$$\left(\frac{M\eta - j}{M\eta + j}\right)\left(\frac{2\cos\alpha}{\cos\alpha + \cos\alpha_1}\right)$$

traveling with phase velocity ω/kn_1 and makes an angle $\alpha_1 = \sin^{-1}\{(n_2/n_1)\sin\alpha\}$ with z-axis. In this paper, $n_1 = 1/(1 - k\beta)$ and $n_2 = 1/(1 + k\beta)$ are the equivalent refractive indices of the chiral medium seen by LCP and RCP waves respectively. β represents the chirality parameter and has the unit of length. If we take $k\beta > 0$ then $n_1 > n_2$ and $\alpha_1 < \alpha$, LCP wave bends towards normal, because it is traveling slower than RCP wave. Similarly when LCP wave with unit amplitude and angle α with z-axis, is incident on perfect electromagnetic conducting (PEMC) plane we get two reflected waves, the RCP wave with amplitude

$$\left\{-\left(\frac{1 - jM\eta}{1 + jM\eta}\right)\left(\frac{2\cos\alpha}{\cos\alpha + \cos\alpha_2}\right)\right\}$$

traveling with phase velocity ω/kn_2 and makes an angle $\alpha_2 = \sin^{-1}\{(n_1/n_2)\sin\alpha\}$ with z-axis and the LCP wave with amplitude

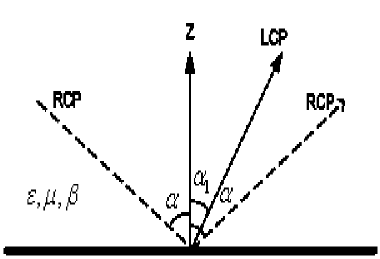

Figure 1. RCP wave reflection from PEMC plane.

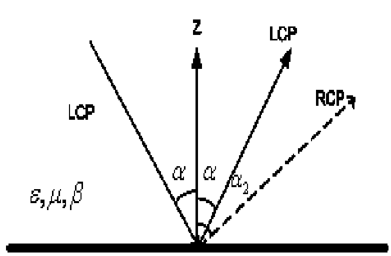

Figure 2. LCP wave reflection from PEMC plane.

$(\cos\alpha - \cos\alpha_2)/(\cos\alpha + \cos\alpha_2)$ traveling with phase velocity ω/kn_1 and makes an angle α with z-axis. If we take $k\beta > 0$ then $n_1 > n_2$ and $\alpha_2 > \alpha$. If $k\beta = 0$ then only normal reflection take place, and if $k\beta$ increases the difference between the angle α and α_1, α_2 increases.

3. GO FIELD OF PEMC PARABOLOIDAL REFLECTOR PLACED IN CHIRAL MEDIUM

Consider a PEMC paraboloidal reflector as shown in Figure 3.

$$\zeta = g(\xi,\eta) = f - \frac{\rho^2}{4f} = f - \frac{\xi^2+\eta^2}{4f} \tag{1}$$

where, (ξ,η,ζ) are the initial values of (x,y,z), f is the focal length of the paraboloidal reflector and $\rho^2 = \xi^2+\eta^2$. The reflector is placed in homogenous and reciprocal chiral medium which can be define by Drude-Born-Fadorov (DBF) constitutive relations [16]. Let there be two incident plane waves of opposite handedness traveling in chiral medium along positive z-axis, which satisfy the general wave are given as

$$\mathbf{Q}_L = (\mathbf{a}_x + j\mathbf{a}_y)\exp(-jkn_1z) \tag{2}$$
$$\mathbf{Q}_R = (\mathbf{a}_x - j\mathbf{a}_y)\exp(-jkn_2z) \tag{3}$$

In the above relations $\mathbf{Q}_L$, $\mathbf{Q}_R$ are LCP and RCP wave respectively. Where, $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vectors along x-axis and y-axis respectively. We suppress the polarization, and henceforth it will remain suppressed, and take the incident fields to be of unit amplitude as follows

$$Q_L = \exp(-jkn_1z),\ Q_R = \exp(-jkn_2z)$$

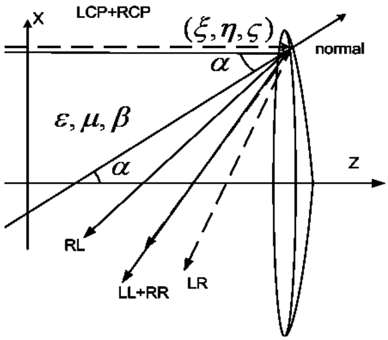

Figure 3. paraboloidal reflector placed in chiral medium.

These waves are making an angle α with the normal to the surface of a PEMC paraboloidal reflector. The unit normal vector to the surface can be written as

$$\mathbf{a}_n = \sin\alpha\cos\gamma\mathbf{a}_x + \sin\alpha\sin\gamma\mathbf{a}_y + \cos\alpha\mathbf{a}_z \tag{4}$$

where, $\mathbf{a}_z$ is the unit vector along z-axis and α, γ are given as

$$\sin\alpha = \frac{\rho}{\sqrt{\rho^2+4f^2}}, \quad \cos\alpha = \frac{2f}{\sqrt{\rho^2+4f^2}}, \quad \tan\gamma = \frac{\eta}{\xi}$$

Four waves are reflected when both LCP and RCP waves strike the PEMC paraboloidal surface. These waves are represented by RR, RL, LL and LR waves. RR and RL waves are RCP and LCP reflected waves respectively, when RCP wave is incident. LL and LR waves are LCP and RCP reflected waves respectively, when LCP wave is incident. The expressions for the fields around the focal region has been calculated for PEC paraboloidal reflector in [5]. For the case of PEMC case LL and RR waves have the same initial amplitudes, while RL and LR waves have different initial amplitude as compered with the corresponding case of PEC case. The angle of reflection for all the reflected waves are same as in case of PEC. The field expressions for the LL and RR rays are the same in both PEC and PEMC case, but the field expressions for RL and LR rays are different for both cases. The field expressions around the focal points for the PEMC paraboloidal reflector are given by

$$\begin{aligned} u_{LL}(r) &= \frac{j2kn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left(\frac{\cos\alpha-\cos\alpha_2}{\cos\alpha+\cos\alpha_2}\right)\tan\alpha \\ &\quad \times\exp\{-jkn_1 s_{LL}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{5}$$

$$\begin{aligned} u_{RR}(r) &= \frac{j2kn_2 f}{\pi}\int_0^H\int_0^{2\pi}\left(\frac{\cos\alpha-\cos\alpha_1}{\cos\alpha+\cos\alpha_1}\right)\tan\alpha \\ &\quad \times\exp\{-jkn_1 s_{RR}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{6}$$

$$\begin{aligned} u_{RL}(r) &= \frac{jkn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left\{\left(\frac{M\eta-j}{M\eta+j}\right)\frac{2\cos\alpha}{\cos\alpha+\cos\alpha_1}\right\} \\ &\quad \times\sec^{3/2}\alpha\sqrt{X_1}\{\sin\alpha\sin(\alpha+\alpha_1)(\tan\alpha\sin(\alpha+\alpha_1) \\ &\quad +\cos(\alpha+\alpha_1))\}^{1/2}\exp\{-jkn_1 s_{RL}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{7}$$

$$\begin{aligned} u_{LR}(r) &= \frac{jkn_2 f}{\pi}\int_0^H\int_0^{2\pi}\left\{-\left(\frac{1-jM\eta}{1+jM\eta}\right)\frac{2\cos\alpha}{\cos\alpha+\cos\alpha_2}\right\} \\ &\quad \times\sec^{3/2}\alpha\sqrt{X_2}\,\{\sin\alpha\sin(\alpha+\alpha_2)(\tan\alpha\sin(\alpha+\alpha_2) \\ &\quad +\cos(\alpha+\alpha_2))\}^{1/2}\exp\{-jkn_2 s_{LR}(p_x,p_y)\}d\alpha d\gamma \end{aligned} \tag{8}$$

In the above equations the term H is calculated as

$$H = \tan^{-1}(D/2f)$$

where, D is the height of the paraboloidal reflector from the horizontal axis. The phase functions in the above equations are given by

$$\begin{aligned}
s_{LL}(p_x, p_y) &= n_1\left(2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right)\\
s_{RR}(p_x, p_y) &= n_2\left(2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right)\\
s_{RL}(p_x, p_y) &= n_1\left\{\frac{n_2}{n_1}f\frac{\cos 2\alpha}{\cos^2\alpha} - (x\cos\gamma + y\sin\gamma - 2f\tan\alpha)\right.\\
&\qquad \left.\times\sin(\alpha+\alpha_1) - \left(z - f\frac{\cos 2\alpha}{\cos^2\alpha}\right)\cos(\alpha+\alpha_1)\right\}\\
s_{LR}(p_x, p_y) &= n_2\left\{\frac{n_1}{n_2}f\frac{\cos 2\alpha}{\cos^2\alpha} - (x\cos\gamma + y\sin\gamma - 2f\tan\alpha)\right.\\
&\qquad \left.\times\sin(\alpha+\alpha_2) - \left(z - f\frac{\cos 2\alpha}{\cos^2\alpha}\right)\cos(\alpha+\alpha_2)\right\}
\end{aligned}$$

where

$$X_1 = \frac{\sqrt{n_1^2 - n_2^2\sin^2\alpha} + n_2\cos\alpha}{\sqrt{n_1^2 - n_2^2\sin^2\alpha}}, \quad X_2 = \frac{\sqrt{n_2^2 - n_1^2\sin^2\alpha} + n_1\cos\alpha}{\sqrt{n_2^2 - n_1^2\sin^2\alpha}}$$

4. RESULTS AND SIMULATIONS

To study the field behavior around the focal region of the PEMC paraboloidal reflector the (5)–(8) were solved numerically. The field of LL and RR waves are same as for PEC case, and line plots for LL and RR waves are the same as in the case of PEC case. Only plots of $|u_{LR}|$ and $|u_{RL}|$ along x-axis and z-axis are given in this paper. As the paraboloidal reflector is symmetric therefore, the line plots along x-axis and y-axis are same. In Figures 4–7 plots for the reflected fields are given for different values of $k\beta$ and $M\eta$ along x-axis and z-axis. These figures show that the trend of the plots are same as in the case PEC reflector, i.e., increase in the value of $k\beta$ results in the shifting of focal point point of LR ray to right and that of RL to the left [5].

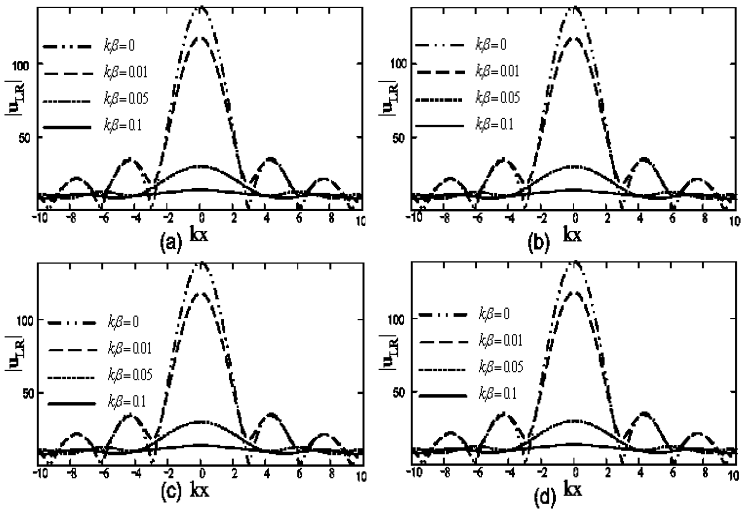

Figure 4. Plots for $|u_{RL}|$ along x-axis, of PEMC paraboloidal reflector for $k\beta = 0, 0.01, 0.05, 0.1$ and (a) $M\eta = 0, \infty$, (b) $M\eta = 0.1$, (c) $M\eta = 1$, (d) $M\eta = 10$.

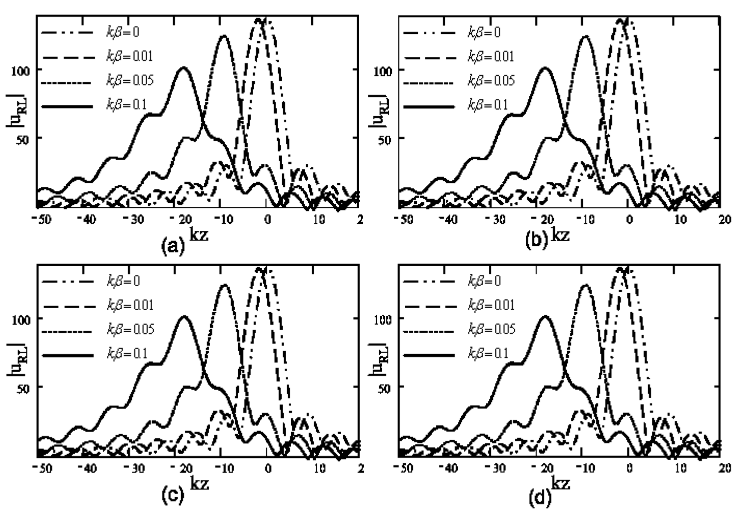

Figure 5. Plots for $|u_{RL}|$ along z-axis, of PEMC paraboloidal reflector for $k\beta = 0, 0.01, 0.05, 0.1$ and (a) $M\eta = 0, \infty$, (b) $M\eta = 0.1$, (c) $M\eta = 1$, (d) $M\eta = 10$.

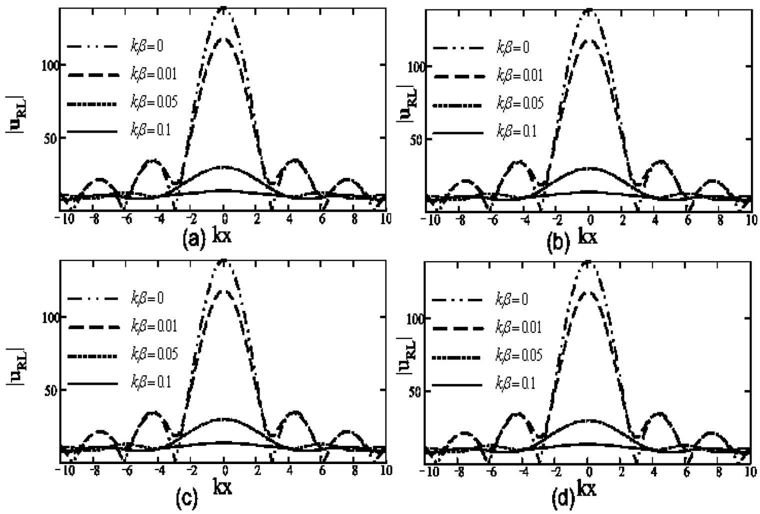

Figure 6. Plots for $|u_{LR}|$ along x-axis, of PEMC paraboloidal reflector for $k\beta = 0,\ 0.01,\ 0.05,\ 0.1$ and (a) $M\eta = 0,\ \infty$, (b) $M\eta = 0.1$, (c) $M\eta = 1$, (d) $M\eta = 10$.

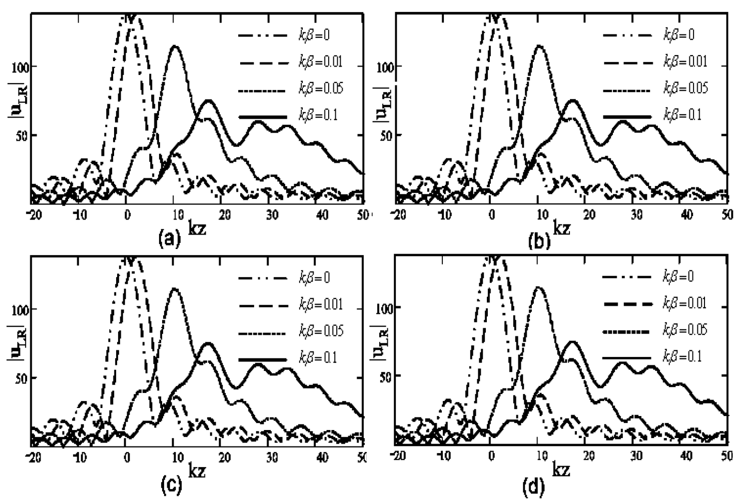

Figure 7. Plots for $|u_{LR}|$ along x-axis, of PEMC paraboloidal reflector for $k\beta = 0,\ 0.01,\ 0.05,\ 0.1$ and (a) $M\eta = 0,\ \infty$, (b) $M\eta = 0.1$, (c) $M\eta = 1$, (d) $M\eta = 10$.

5. CONCLUSIONS

The field around the focal region of PEMC paraboloidal reflector is analyzed, which is placed in reciprocal and homogenous chiral medium using geometrical optics and Maslov's method. It has been seen that fields of LL and RR rays are exactly the same as that of PEC paraboloidal reflector. The field behaviors of LR and RL are similar, but not exactly the same to that of PEC paraboloidal reflector case. As the chirality parameter $k\beta$ increases the focal point of LR wave moves to the right, and that of RL wave moves to the left.

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