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EFFICIENT ANALYSIS TECHNIQUE OF MICROSTRIP STRUCTURES BASED ON METHOD OF MOMENTS AND USING GHOST FUNCTIONS IN MODELLING TEM ZONES

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Abstract—In this paper, we present a new electromagnetic modelling analysis based on the Analytical Spatial Method of Moments (AS-MoM) using roof-top basis functions and prove advantages of integrals' resolution in conjunction with technique that permit to reduce the number of unknowns in the MoM matrix and to minimize time and memory consumption. This technique takes into account the existence of TEM zones inside the structure and far from discontinuities. There, we represent the current under an exponential form to analyze wave's propagation inside the structure. The advantages of this contribution will be illustrated by numerical results.

1. INTRODUCTION

In the design of microwave monolithic integrated circuits (MMIC'S) and millimetre-wave integrated circuits, electromagnetic modelling of microstrip elements (filters, antennas and circuits) becomes important as the operating frequency becomes higher. Full wave analysis includes the effects of EM occurrence, surface wave, and the radiation loss while traditional quasi-static methods and equivalent waveguide models fail to yield sufficiently accurate results. The time factor in analysis of any technique becomes of great importance. Many works have treated amelioration of analytical integrals approaches [1, 2]. Thus, this article presents a new technique of an electromagnetic modelling contribution based on AS-MoM [1, 2] in conjunction with a technique that takes into account the existence of TEM zones inside the structure and far from discontinuities.

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Moreover, discontinuities are inevitable in microwave integrated circuit (MIC). Their electromagnetic property is a core issue in the design procedure. The discontinuity can be characterized by the spectral domain approach [3–5] or the spatial domain technique [6, 7]. When the geometry of the circuit is complex, no matter which domain is used, the active ports, i.e., conductors in a microstrip-type problem or apertures in slot-type problem, are divided into sub-regions for accurately determining the circuits parameters [8].

Far from discontinuities, and in TEM zones, two waves are propagating in the same direction but with inverted senses. The current distribution is presented under an exponential form and descritized in a roof-top basis functions [1, 2, 9]. Each of these waves is modelled by an attachment function and a set of basis ghost functions. These functions don't introduce any news unknowns in the MoM matrix, which permits the reduction of elements' matrix number. The MoM matrix size will be reduced and the CPU time of our new method called "Ghost-AS-MoM" will be compared with other given methods [1, 10] and ADS software. Numerical results are presented to prove its efficiency.

A similar technique including fundamental mode propagation inside the structure was developed by [11]. [11] and used a half sinusoidal basis function to model the current distribution in these zones. The principle advantage of our technique in relation with [11] results in its better convergence in integrals' resolution in the AS-MoM method developed by [1]. This later method (AS-MoM) is the original tool of our new technique.

2. ANALYTICAL SPATIAL METHOD OF MOMENTS (AS-MoM)

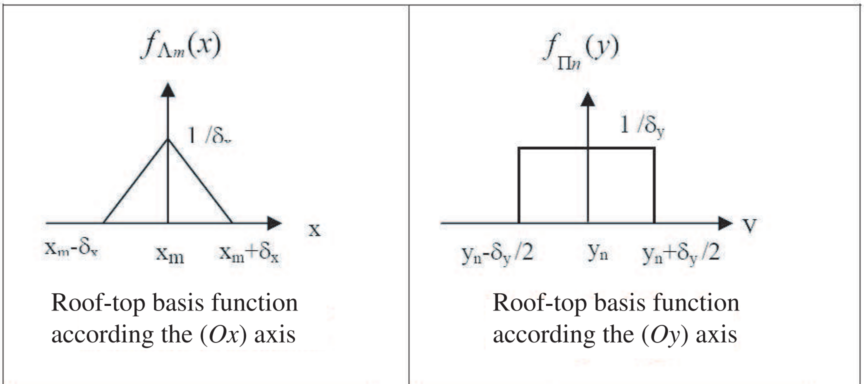
The principle of Method of Moment [1, 9, 12] is to reduce a functional equation to a simple solvable matrix equation. The development of analytical evaluation of these different techniques consists in first, to approximate Green functions in the spectral domain by the complex exponential functions [13], then to determine Green functions expressions directly in the spatial domain with the help of the Sommerfeld identity [13]. The problem seems in determining the current distribution on the interface air / dielectric of the conductor.

$$\vec{J}(x,y) = J_x(x,y)\,\vec{X} + J_y(x,y)\,\vec{Y} \tag{1}$$

Using the mixed potential integral MPIE formulation [4], the spatial domain MoM matrix entries of planar geometry can be expressed as follows [1]:

$$Z_{mn} = \left\langle J_{xm}, G_{xx}^{A} * J_{xn} \right\rangle + \frac{1}{\omega^2 \varepsilon_0 \mu_0} \left\langle J_{xm}, \frac{\partial}{\partial x} \left(G_q * \frac{\partial J_{xn}}{\partial x} \right) \right\rangle \tag{2}$$

where (G_A^{xx}, G_q) are Green's functions for the vector potential and scalar. J_{xm} and J_{xn} are the surface current density and they are decomposed in roof-top basis functions in (Ox) and (Oy) directions, respectively. Roof-top basis functions are defined as follows:



$$f_{\Lambda}(x) = \begin{cases} \frac{1}{\delta_x}\left(1 - \frac{|x-x_m|}{\delta_x}\right), & |x-x_m| \le \delta_x \\ 0, & \text{elsewhere} \end{cases}$$

$$f_{\Pi}(y) = \begin{cases} \frac{1}{\delta_y}, & |y-y_n| \le \frac{\delta_y}{2} \\ 0, & \text{elsewhere} \end{cases}$$

Figure 1. Roof-top basis function.

The spatial-domain Green's functions employed in (2) are obtained in closed form using the DCIM two-level approximation described in [13], and Sommerfeld identity and they are given by:

$$G = \sum_{i=1}^{M} a_i \frac{e^{-jk_0 r_i}}{r_i} \tag{3}$$

where $r_i = \sqrt{(x+x_0)^2 + (y+y_0)^2 + b_i^2}$ is the complex distance. (G stands for either G_{xx}^{A} or G_q), a_i, b_i, and M are respectively the complex coefficients, and number of complex images obtained from the application of GPOF technique [13]. Furthermore, the scalar product of the tangent electric field and the test functions T_{xm} and T_{ym} is calculated while taking account of borders conditions if those are applicable on the structure. After having determined unknowns I_{xn} and I_{yn}, we can deduce the current expression while solving a matrix equation. The scalar product being defined like follows:

$$\langle f, g \rangle = \iint f^*(x,y)\, g(x,y)\, dxdy \tag{4}$$

The convolution product of the roof-top basis functions leads to the polynomial. The computation of the entries of MoM comes back to calculate integrals of this type [1]:

$$\iint_D \frac{e^{-jk_0 r}}{r} x^P y^q dx \cdot dy \tag{5}$$

$$p = 0,\ 1,\ 2,\ 3; \quad q = 0,\ 1; \qquad D = \begin{cases} -2\delta_x \le x \le +2\delta_x \\ -\delta_y \le y \le +\delta_y \end{cases} \tag{6}$$

and $r = \sqrt{(x+x_0)^2 + (y+y_0)^2 + b^2}$.

According to the distance that separates two cells of the meshing structure; two analytical calculating techniques of these integrals are present. A first technique developed in [10], and our second technique developed in [1, 2]. The originality of the technique developed by [1] and which our contribution is based on is in the approximation on Taylor series around $r_0 = \sqrt{x_0^2 + y_0^2 + b^2}$ of $\frac{e^{-jk_0 r}}{r}$ instead of $e^{-jk_0 r}$ given by [10]. The error given is presented in Eq. (9) on [1].

The analytical integration technique developed in [10] is used for r_0 lower than $2 \cdot \delta_r$. The use of the technique integration in [10] is minimized, what improves the global calculating time [1].

3. FORMULATION OF THE PROBLEM WITH MODELLING OF DOMINATING TEM ZONES

3.1. Principle

The principle of our technique is to take advantage of zones in which TEM Mode is dominant to represent the current distribution as an exponential form. Here, we consider a structure with adapted access and we take into account the existence of TEM zones inside the structure. Far from discontinuities, only TEM mode is considered. Other excited modes in discontinuities are already attenuated. This permits to model current distribution by exponential functions. These functions will be decomposed after that in roof-top basis functions called "ghost" that are not considered in the analysis of interactions between different cells of the structure meshing. Thus, they don't introduce any new unknowns to the MoM matrix and the matrix size will be reduced. By the way, memory consumption and time are considerably minimised.

Figure 2 shows a new meshing according to the Ghost_AS-MoM technique of microwave structure assimilated to a quadruple and presenting discontinuities. In this structure, we note two types of zones: TEM zones which are located far from discontinuities and full-wave zones which are in discontinuities form. The propagating waves in TEM zones are already divided into two categories: those which are propagating through the access that are due to the source and the load adaptation. These waves have for expression $a_i \cdot e^{-j\beta x}$ for the incident wave, $a_r \cdot e^{+j\beta x}$ for the reflected wave and $a_t \cdot e^{-j\beta(x-L)}$ for the transmitted wave. The second category of waves characterizes all of those which are propagating in TEM zones located in the middle of the structure and far from discontinuities. There, we have to consider the existence of a superposition of two waves characterized by inverted senses. Each of these waves is written by an attachment function and a set of ghost basis functions and have the expressions: $a_P \cdot e^{-j\beta(x-l_1)}$ for the propagate wave and $a_R \cdot e^{+j\beta(x-l_2)}$ for the return wave.

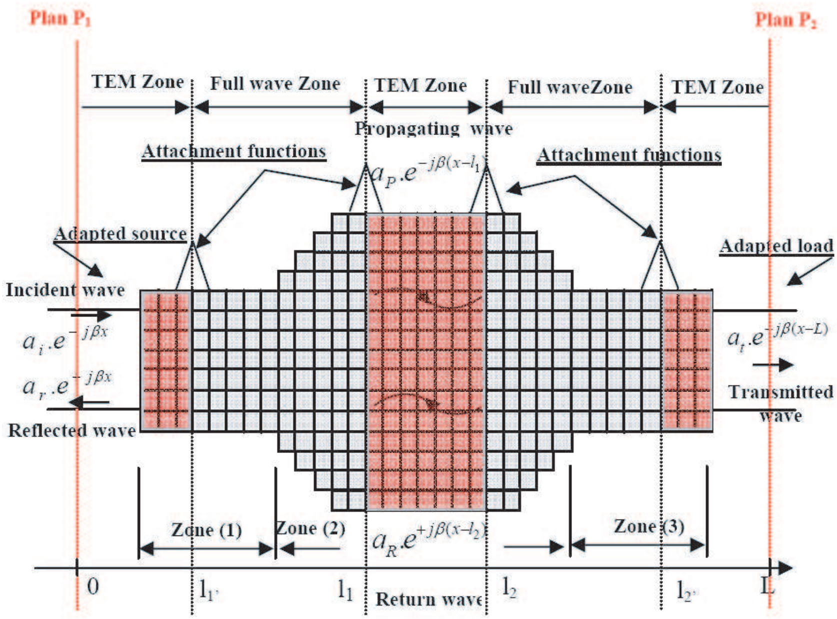


Figure 2. Meshing of microstrip structure according to the Ghost_AS-MoM technique.

In TEM zone, the current distribution has this expression:

$$J_x(x,y) = A_0 e^{-j\beta x} + A_1 e^{+j\beta x} \tag{7}$$

A_0 and A_1 are the amplitudes of these waves.

3.2. New MoM Matrix According to Ghost_AS-MoM Technique

The considered Ghost_AS-MoM technique formulation is the same as the Analytical Spatial Method of Moments (AS-MoM) formulation but with taking into account of the TEM zones inside the structure. Four columns of the MoM matrix will be modified. The new current distribution presented by the Ghost_AS-MoM technique according to the (Ox) axis decomposed with roof-top basis functions is given by the expression:

$$\begin{aligned} J_x(x,y) \;=\; & J_{xs}(x,y) + \sum_{k=1}^{G_1} J_{xk_1} + \sum_{j=1}^{N_1-G_1} a_{xj} J_{xj} \\ & + \sum_{J=1}^{K} \left(\sum_{i=1}^{X_1} a_{xi} J_{xi} + \sum_{m=1}^{G_2} J_{xm} + \sum_{n=1}^{G_2} J_{xn} + \sum_{i=1}^{X_2} a_{xi} J_{xi} \right) \\ & + \sum_{j=1}^{N_3-G_3} a_{xj} J_{xj} + \sum_{k=1}^{G_3} J_{xk_2} \end{aligned} \tag{8}$$

- J_{xk_1}, J_{xk_2} are respectively the current distribution of the TEM zones in both sides source and load. G_1, G_3 respectively the ghost basis functions number in these zones.
- X_1 and X_2 are respectively the basis roof-top functions number from one side to the other of TEM zone. G_2 is the ghost functions number in this zone.
- J_{xm} and J_{xn} are the current distributions corresponding to the propagate wave and to the return wave in the TEM zone. k is the according order line in the following (Oy) axis.

For each of the four modified columns corresponding to the TEM zones limits, the current distribution is written as an attachment function and a set of ghost basis functions where their weights are deducted from the corresponding attachment function. The current distribution of every column is defined as follows:

$$J_{xNX+1} \;=\; \sum_{i=0}^{G-1} L_i J_{xNX+1+i} \tag{9}$$

where:

- $L_0 = 1; l_1 = \alpha \cdot a_{NX+1}, l_2 = \alpha^2 \cdot a_{NX+2}; \ldots; L_{G-1} = \alpha^{G-1} \cdot a_{NX+1}$.
- G is the ghost basis functions number of the TEM zone, X is the corresponding column order of the TEM zone limit and $\alpha = e^{-j\beta\delta_x}$.

We consider:

- N_i, the number of basis roof-top functions on zone (i).
- $N_T = \sum_i N_i$, the total basis functions number in different zones (cells number).
- G_i, the number of basis ghost roof-top functions on zone (i).
- $G_T = \sum_i G_i$, the total ghost basis functions number.

$M'_{m,n}$ is the new matrix, K'_m the new excitation vector and $[a_r, \ldots, A_{NT1+NT3+2}]$ are unknowns the problem. If $M'_{m,NX+1}$, $m = [1,\ NX+1]$ is the new MoM matrix elements of the column (NX+1), then, these elements are expressed from reaction terms M_{mn} as follows:

$$M'_{m,NX+1} = M_{m,NX+1} + \alpha M_{m,L_1} + \alpha^2 M_{m,L_2} + \ldots + \alpha^{G-1} M_{m,L_{G-1}} \quad (10)$$

When we apply AS-MoM to compute the excitation vector using adapted terminations, K' is written according to terms of M'_{mn} reaction as follows:

$$K'_m = -\sum_{p=1}^{p=Ng} \left(e^{+j\beta\delta_x}\right)^{p-1} \cdot\ M'_{m,p} \quad (11)$$

The new expression of this vector is:

$$\begin{aligned} K'_m \ =\ & -M_{m,1} - \alpha^{-1} M_{m,c_{-1}} - \alpha^{-2} M_{m,c_{-2}} - \ldots \\ & -\alpha^{-N1'} M_{m,c_{-(N1')}} - \ldots - \alpha^{-N1} M_{m,c_{-(N1)}} - \alpha^{-N2} M_{m,c_{-(N2)}} \\ & -\alpha^{-N2} M_{m,c_{-(N2)}} - \ldots - \ldots - \alpha^{-N2'} M_{m,c_{-(N2')}} \alpha^{-Ng} M_{m,c_{-Ng}} \end{aligned} \quad (12)$$

Ng is number of ghost basis functions used to adapt the excitation. N_1, N_2, $N_{1'}$ and $N_{2'}$ correspond to modified columns of the MoM matrix.

Weights of ghost basis functions relative to the excitation are inspired from coefficient a_0 are: $c_{-1} = \alpha^{-1} \cdot a_0; c_{-2} = \alpha^{-2} \cdot a_0; \ldots; c_{-Ng} = \alpha^{-Ng} \cdot a_0$, where, $a_0 = 1$ is the incidental wave amplitude in the input of the transmission line.

Finally, the new matrix system using ghost basis functions inside the structure becomes:

$$\left(M'_m\right) \cdot (I) = \left(K'_m\right) \quad (13)$$

(K') is the new excitation vector; (M') the new moment's matrix and (I') always remain the problem unknowns. This new matrix system using ghost roof-top basis functions is:

$$\left[M'_{mn}\right] \cdot \begin{bmatrix} a_r \\ a_{N1'} \\ \vdots \\ a_{N1+N1'} \\ a_{N2} \\ \vdots \\ a_{N2+N2'} \\ a_{N1'+N1+N2+N2'+4} \end{bmatrix} = \left[K'_m\right] \tag{14}$$

By the AS-MoM, the number of unknowns in departure was: $(N_T = N_1 + N_2 + N_3)$. In our contribution which is the Ghost-AS-MoM, this number has decreased of $(G_T = G_1 + G_2 + G_3)$ unknown. The MoM matrix order becomes $[(N_T - G_T) + 4]^2$. That means $[(N_1 - G_1) + (N_2 - G_2) + (N_3 - G_3) + 4]^2$ instead of $[N_1 + N_2 + N_3]^2$. Here, we add 4 attachment functions corresponding to TEM zones limits.

We note that in general, for any meshing by N_T roof-top basis functions and assimilated to a quadruple with two adapted accesses, if the matrix departure order is $(N_T)^2$, then it becomes $[(N_T - G_T) + (2 \cdot N_Z - 2)]^2$ according to our new technique. The reduction is therefore of $[G_T - (2 \cdot N_Z - 2)]^2$ MoM matrix elements. We take in to account that the structure includes N_Z TEM zones on which we can define a total of G_T ghost basis functions (including the two inputs of structure). The $(2 \cdot N_Z - 2)$ functions correspond to the two attachment functions that model the propagating wave and the return wave in every TEM zone inside the structure, and add one function for each of the two extreme zones.

3.3. Ghost_AS-MoM Convergence

When we study the convergence of our technique, we note:

$$\tau = \frac{N_T - G_T}{N_T} \tag{15}$$

τ is the report of the number of basis functions in full-wave zones and their interactions constitute the MoM matrix elements by the total number of these functions without taking into account of the TEM zones. The report τ can be written also as follows:

$$\tau = \frac{N_T - G_T}{N_T} = 1 - \frac{G_T}{N_T} = 1 - \rho \tag{16}$$

where $\rho = \frac{G_T}{N_T}$ is the report of ghost basis roof-top functions by the total number of basis functions without considering TEM zones. In term of measurements, this report corresponds to the TEM zone length by the total length of the structure. We note that $\tau = 1$ if $\rho = 0$, when it is about using the AS-MoM alone. That means, when we don't consider any TEM zones in side the structure ($G_T = 0$).

4. NUMERICAL RESULTS

To validate our results, we apply our new approach to analyze a rectangular patch antenna.

In Figure 4, we represent the reflection coefficient "Γe" to analyze different report "ρ". We note that the different curves are confounded from ρ near 40%. The reflection coefficient is about −12 dB. Our results appear conform to those given by Advanced Design System (ADS) (Γe about −12.3 dB). For another report where we are increasing the length of the TEM zone in relation to the length of the rectangle, we note that different curves will be distinct. Results are not anymore the same, and the solution diverges. To prove the efficiency of our technique, we analyze the relative error compared to ADS software. Compared to ADS, the relative error of our technique for different report ρ is 3% for $\rho = 20\%$, it increases to about 3.5% for $\rho = 40\%$ which is the convergent point and it remains acceptable for $\rho = 50\%$.

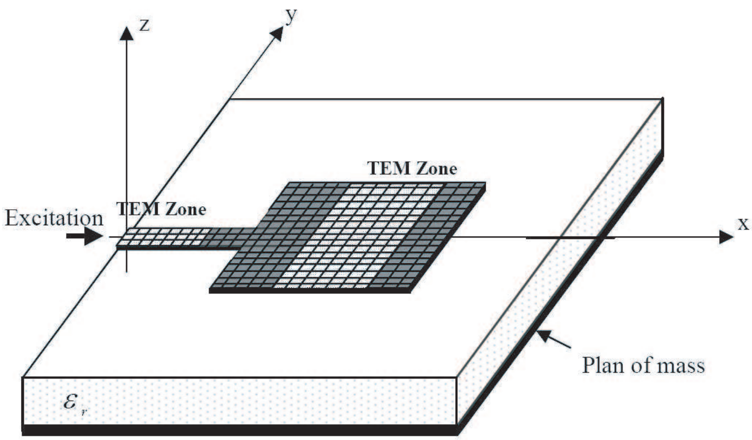


Figure 3. New meshing of a microstrip patch antenna according the "Ghost_AS-MoM" technique. Excitation strip length: 7.62 mm, rectangular patch length and width: 7.62 mm, $\varepsilon_r = 2.33$, $H = 1.58$ mm.

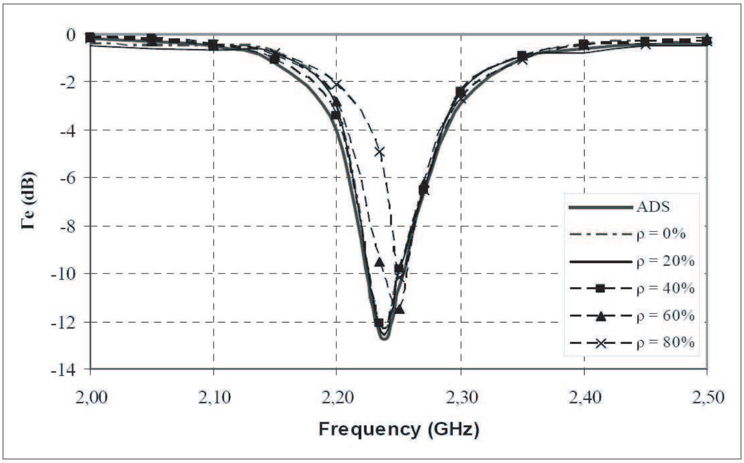


Figure 4. Reflection Coefficients "Γe" given by Ghost_AS-MoM technique to analyze different report "ρ" of a microstrip patch antenna at resonance frequency.

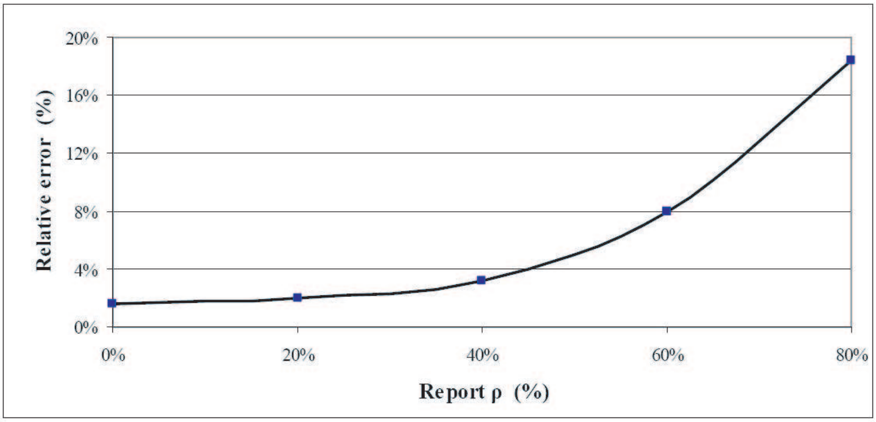


Figure 5. Relative error of different techniques compared to momentum of ADS (at the resonance).

All presented results, compared with other method (AS-MoM) and ADS simulator are sufficient to prove the validity and usefulness of our method that allows us to search comparison of the time consumption.

In Figure 5, we present the CPU times for different report ρ when we simulate the patch antenna.

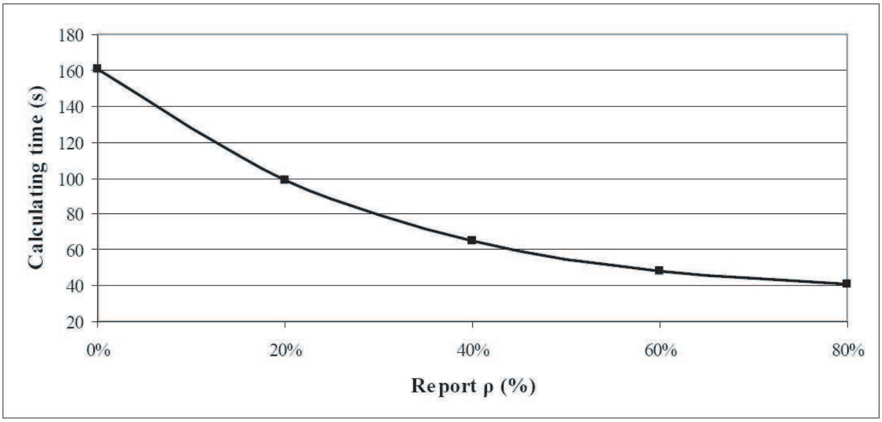


Figure 6. CPU time of MoM matrix elements obtained for different report ρ in the analysis of the microstrip rectangular patch antenna.

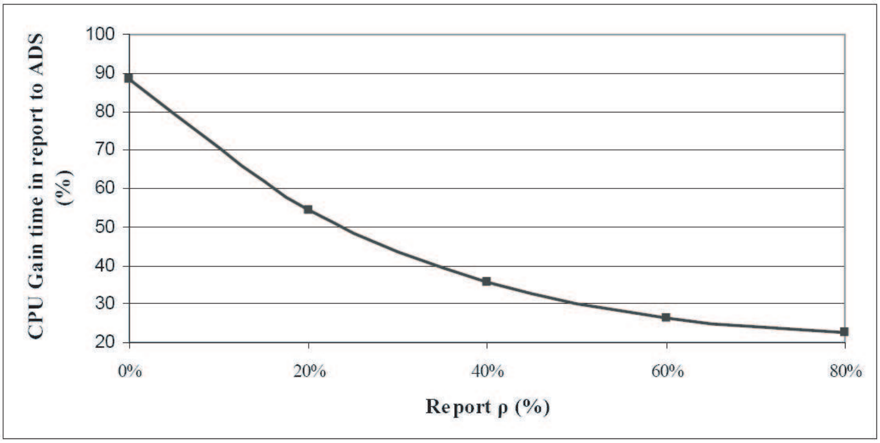


Figure 7. CPU gain time of MoM matrix elements in report to ADS obtained for different report ρ in the analysis of the microstrip rectangular patch antenna.

We note that the calculating time of the AS-MoM method corresponding to $\rho = 0$ is 161 seconds. It decreases to 65 seconds for $\rho = 40\%$ and it is less than 50 seconds for $\rho = 60\%$.

In Figure 7, we present the CPU gain time in relation to ADS.

We note that, at the resonance frequency and at the convergent point ($\rho = 40\%$), our technique puts 65 seconds, so 117 seconds less than ADS software (182 seconds) which corresponds to the gain of about 64% of the CPU time. At this point, the CPU time by our technique represents 36% in relation with the one obtained by ADS.

In term of rapidity, our technique is about three times faster than ADS and two times and half faster than AS-MoM. All of this analysis concludes the validity and usefulness of our method compared with other given method in term of time and memory consumption.

5. CONCLUSION

In this paper, we have presented the Ghost_AS-MoM technique. It is a new electromagnetic modelling analysis based on the Analytical Spatial Method of Moments (AS-MoM) using roof-top basis functions in conjunction with technique permitting to reduce the number of unknowns in the MoM matrix and to minimize time and memory consumption

The main idea of this technique is to take benefits from the existence of TEM zones inside the structure. This technique takes into account only TEM mode propagation in zones those are far from discontinuities what permits to represent the current distribution as an exponential form. The current distribution in these zones is modelled by ghost roof-top basis functions. The size of the MoM matrix is reduced and consequently the CPU time has decreased considerably. We notice that the principle advantage for the AS-MoM method, what our new technique is based on, was the efficient integrals' evaluation. So, the new Ghost_AS-MoM technique has shown its well adaptability with the AS-MoM and even takes the advantage in zones that are far from discontinuities.

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