

## PATTERN SYNTHESIS FOR PLANAR ARRAY BASED ON ELEMENTS ROTATION

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**Abstract**—Aiming at improving the radiating characteristics (pattern and polarization) and simplifying the design of feeding network, a new approach is applied and discussed. For uniformly excited planar antenna array, by rotating each element in its local coordinates and determining the rotation angles of the elements, the 3D radiating characteristic of the planar antenna array can be improved. The Differential Evolution (DE) algorithm is applied for optimizing rotation angles of the elements. Furthermore, the effects of the elements rotation are discussed in detail.

### 1. INTRODUCTION

Pattern synthesis is known as the process of choosing the parameters of an antenna array to produce desired radiating characteristics. The synthesis of antenna arrays with a main beam and low side lobes has received much attention over the years, and there are a wide variety of techniques that have been developed for the synthesis of linear and planar arrays [1–8].

However, many of these researches are concerned with the 2D pattern synthesis since the analysis and synthesis of 3D far-field radiation pattern are more complex and difficult. Meanwhile, these researches almost all assumed relatively simple element pattern modeling. To our knowledge, the element polarized pattern of the array is always ignored for simplification.

For the array with element rotated, its polarization properties depend not only on the antenna elements, but also on the rotation of each element. Aiming at achieving favorable controls over the radiation

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characteristic of the array, the 3D polarization pattern analysis and synthesis for planar antenna array are discussed and studied.

In general researches, the goal in antenna array synthesis is determining the complex excitations, amplitudes and phases of the elements for producing desired radiation pattern, such as low sidelobe level (SLL). Actually, for planar array, this also can be done by determining the rotation angles of the elements in their local coordinates for uniform excitation [9]. For the antenna elements rotated, their radiation patterns are rotated accordingly; meanwhile, the elements will be polarized in a variety of directions, which poses unique difficulties in the pattern analysis and synthesis process.

Differential Evolution (DE) was proposed by Price and Storn in 1995 [10, 11]. It is an effective, robust, and simple global optimization algorithm which only has a few control parameters. According to frequently reported comprehensive studies, DE outperforms many other optimization methods in terms of convergence speed and robustness over common benchmark functions and real-world problems.

In this paper, for minimizing the sidelobe level (SLL) of co-polarization, restraining the cross-polarization pattern, and also simplifying the design of feeding network of the array, a new approach of elements rotation is applied and discussed. The DE algorithm is applied to determine the rotation angles of the elements in their local coordinates for uniform excitation. Moreover, a sampling method is adopted to enhance the computational efficiency of the 3D pattern synthesis process.

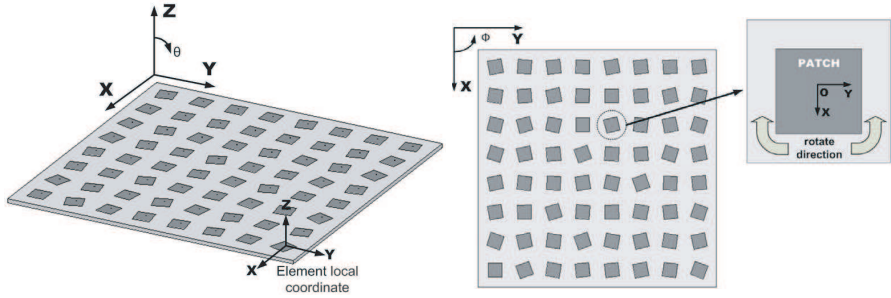
## 2. THEORY AND FORMULATION

Consider the general planar array geometry consisting  $M \times N$  identical elements equally spaced at distance  $d_x$  in  $X$ -direction and  $d_y$  in  $Y$ -direction, each element is indexed by the parameters of  $m$  ( $m \leq M$ ) and  $n$  ( $n \leq N$ ).  $f^{mn}(\theta, \phi)$  is the individual element pattern; the far-field radiated pattern produced by  $M \times N$  elements can be expressed as follows:

$$F(\theta, \phi) = \sum_{m=1}^M \sum_{n=1}^N I_{mn} f^{mn}(\theta, \phi) \exp\{jk[(m-1)d_x \cos \phi + (n-1)d_y \sin \phi] \sin \theta + j\phi_{mn}\} \quad (1)$$

where  $I_{mn}$  is the element excitation current amplitude;  $\phi_{mn}$  is the excitation current phase;  $k$  is the free-space wave number.

Consider the planar array with elements rotated individually as shown in Figure 1, each element is rotated around its feeding point



**Figure 1.** A graphical representation of the planar antenna array with elements rotated.

either in the clockwise direction or anticlockwise direction in its local coordinates. Accordingly, the radiation pattern is rotated as well. The radiated field of the planar array with elements rotated individually can be expressed as:

$$F_{\theta}(\theta, \phi) = \sum_{m=1}^M \sum_{n=1}^N I_{mn} E_{\theta}(\theta, \phi_{mn}) \exp\{jk[(m-1)d_x \cos \phi + (n-1)d_y \sin \phi] \sin \theta + j\phi_{mn}\} \quad (2)$$

$$F_{\phi}(\theta, \phi) = \sum_{m=1}^M \sum_{n=1}^N I_{mn} E_{\phi}(\theta, \phi_{mn}) \exp\{jk[(m-1)d_x \cos \phi + (n-1)d_y \sin \phi] \sin \theta + j\phi_{mn}\} \quad (3)$$

$$\phi_{mn} = \begin{cases} \phi - y_{mn} - 360 & 360 \leq \phi - y_{mn} \\ \phi - y_{mn} & 0 \leq \phi - y_{mn} \leq 360 \\ \phi - y_{mn} + 360 & \phi - y_{mn} < 0 \end{cases} \quad (4)$$

where  $y_{mn}$  are the variables representing the element rotation angles, and  $y_{mn} > 0$  means that the element is rotated in the clockwise direction, while  $y_{mn} < 0$  means in the anticlockwise direction.  $E_{\theta}(\theta, \phi_{mn})$  and  $E_{\phi}(\theta, \phi_{mn})$  are the  $\hat{\theta}$ - and  $\hat{\phi}$ -components of the radiated field for the element indexed by position parameters  $m$  and  $n$ , which is transformed from the radiated field  $E_{\theta}(\theta, \phi)$  and  $E_{\phi}(\theta, \phi)$  of isolated element without rotation.

Based on the third definition of cross-polarization by Ludwig [12], for a transmitted field polarized in the  $\hat{i}_x$  direction at  $\theta = 0$ , the radiated field of co-polarization  $F_{co}(\theta, \phi)$  and cross-polarization  $F_{cross}(\theta, \phi)$  can be expressed as follows. Expression (7) demonstrates the relation-

ship between the  $U$ - $V$  coordinates and spherical coordinates.

$$F_{\text{co}}(\theta, \phi) = F_{\theta}(\theta, \phi) \cos \phi - F_{\phi}(\theta, \phi) \sin \phi \quad (5)$$

$$F_{\text{cross}}(\theta, \phi) = F_{\theta}(\theta, \phi) \sin \phi + F_{\phi}(\theta, \phi) \cos \phi \quad (6)$$

$$U = \sin \theta \cos \phi \quad V = \sin \theta \sin \phi \quad (7)$$

### 3. NUMERICAL RESULTS

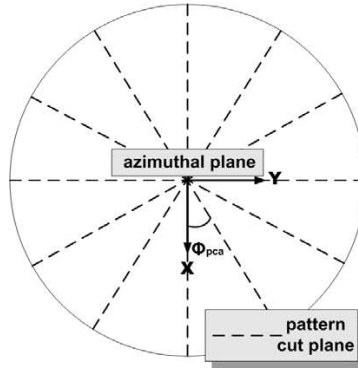
In this paper, a planar antenna array consisting  $8 \times 8$  linearly polarized microstrip patch antennas on a plane ( $XY$ -plane), equally spaced from its neighbors by  $0.5\lambda$  in  $X$ -direction and  $Y$ -direction is investigated. The isolated radiated field of the microstrip patch antenna is attained by applying Ansoft HFSS.

Since the 3D pattern synthesis lacks computational efficiency, sampling method is applied. By sampling the data of 3D radiation pattern on specific cut planes, the 3D pattern synthesis problem is transformed into multi-2D pattern synthesis problem.

As shown in Figure 2,  $\phi_{\text{pca}}$  ( $45^\circ$  in this design) is the azimuthal angle difference between adjacent pattern cut planes. The destination of co-polarization  $F_{\text{dco}}(\theta, \phi_{\text{cut}-v})$  in the pattern cut planes of  $\phi = \phi_{\text{cut}-v}$  can be written as follows, where NULL represents the first null.

$$F_{\text{dco}}(\theta, \phi_{\text{cut}-v}) = \begin{cases} 1 & \theta = 0 \\ 0 & \theta \notin \theta_{\text{NULL}-v} \\ F_{\text{co-nor}}(\theta, \phi_{\text{cut}-v}) & \theta \in \theta_{\text{NULL}-v} \text{ and } \theta \neq 0^\circ \end{cases} \quad (8)$$

$$\phi_{\text{cut}-v} = (v-1)\phi_{\text{pca}} \quad \left( v = 1, \dots, \frac{\pi}{\phi_{\text{pca}}} \right) \quad (9)$$



**Figure 2.** A graphical representation of the radiation pattern cut plane.

HPBW is the half power beamwidth. The destination of cross-polarization  $F_{\text{dcross}}(\theta, \phi_{\text{cut}-v})$  in the pattern cut planes of  $\phi = \phi_{\text{cut}-v}$  can be written as

$$F_{\text{dcross}}(\theta, \phi_{\text{cut}-v}) = \begin{cases} F_{\text{cross-nor}}(\theta, \phi_{\text{cut}-v}) & \theta \notin \theta_{\text{HPBW}-v} \\ 0 & \theta \in \theta_{\text{HPBW}-v} \end{cases} \quad (10)$$

Aiming at minimizing the SLL of co-polarization and restraining the cross-polarization in the HPBW region, the fitness function is defined as:

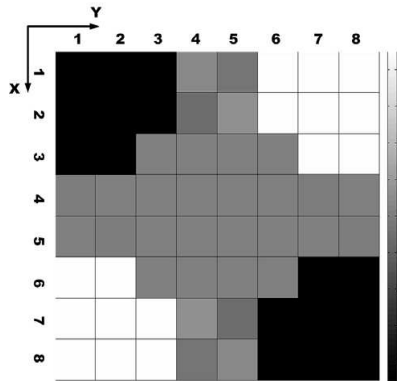
$$\begin{aligned} \text{Fitness} = & \omega_1 \max |(F_{\text{co-nor}}(\theta, \phi) - F_{\text{dco}}(\theta, \phi))| \\ & + \omega_2 \max |(F_{\text{cross-nor}}(\theta, \phi) - F_{\text{dcross}}(\theta, \phi))| \\ & \left( \phi = \phi_{\text{cut}-v} \left( v = 1, \dots, \frac{\pi}{\phi_{\text{pca}}} \right) \right) \end{aligned} \quad (11)$$

$$F_{\text{co-nor}}(\theta, \phi) = F_{\text{co}}(\theta, \phi) / \max(F_{\text{co}}(\theta, \phi)) \quad (12)$$

$$F_{\text{cross-nor}}(\theta, \phi) = F_{\text{cross}}(\theta, \phi) / \max(F_{\text{co}}(\theta, \phi)) \quad (13)$$

$\omega_1$  and  $\omega_2$  stand for the weight coefficients which are 1 and 0.4 respectively.  $F_{\text{co-nor}}(\theta, \phi)$  and  $F_{\text{cross-nor}}(\theta, \phi)$  are the normalized radiation pattern of co- and cross-polarization to equal unity.

The DE algorithm was implemented using MATLAB for the 3D pattern synthesis. Due to the geometrical symmetry of the array and radiating characteristic of the element, variables  $y_{mn}$  to be optimized is confined in the  $\frac{1}{4}$  region ( $m, n \leq 4$ ) of the array. Meanwhile, the variables  $y_{mn}$  are fixed as integers, which means the minimum step of change for  $y_{mn}$  is  $1^\circ$ . Moreover, since large rotation angles may reduce the gain of co-polarization, while small ones may have difficulties

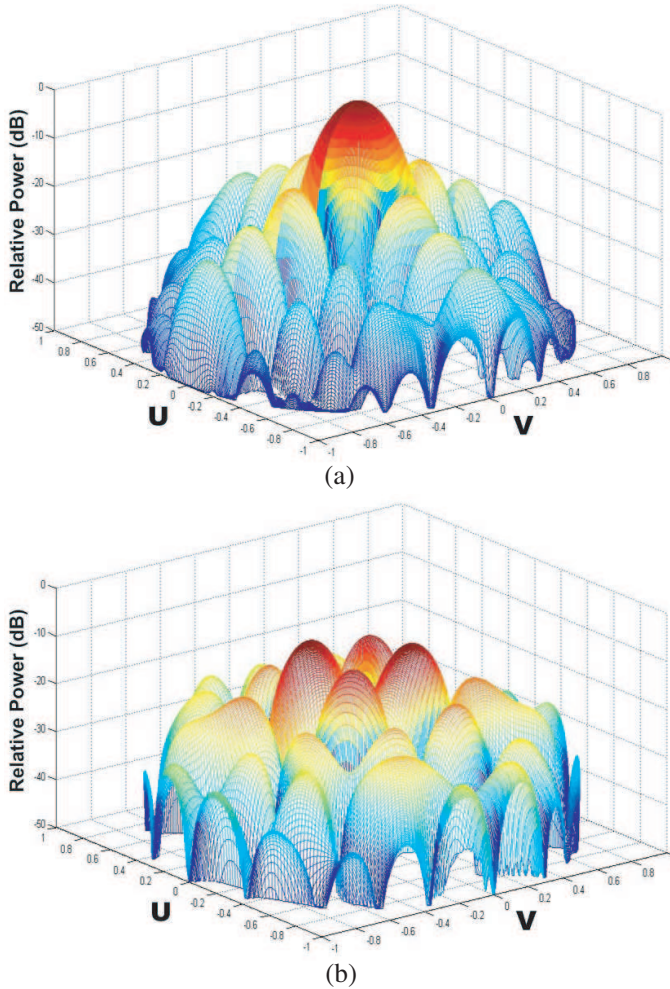


**Figure 3.** Optimized rotation angles distribution of the elements.

in improving the radiation characteristic, as a balance choice, the elements rotation angles are confined from  $-45^\circ$  to  $45^\circ$  in this design.

Figure 3 demonstrates the optimized rotation angles of the elements in the planar array. It is apparent that the elements located in the regions of four corners of the array have large rotation angles, which play an important role in sidelobe level reduction.

Figure 4 shows the optimized 3D co- and cross-polarization radiation pattern normalized to equal unity. For clearly observing



**Figure 4.** Optimized 3D radiation pattern of the proposed planar array: (a) Co-polarization; (b) cross-polarization.

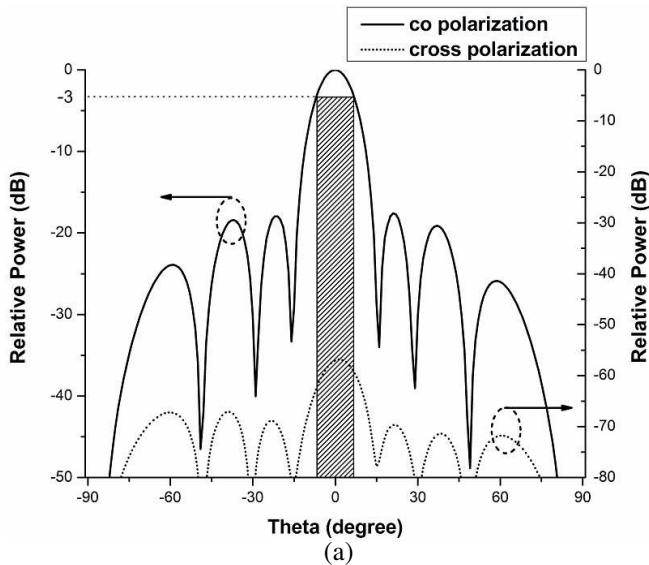
the radiating characteristic, the 2D radiation patterns cut from the 3D patterns are shown in Figure 5 (the pattern cut planes of  $\phi = 0^\circ$ ,  $45^\circ$  and  $90^\circ$ ). It is clearly seen that the sidelobe level for the multi-2D planes is below  $-17.7$  dB. Compared with the conventional uniformly excited planar antenna array, the maximum relative sidelobe can be reduced by about 5 dB. Meanwhile, it is apparent that the cross-polarization in the HPBW region is restrained below  $-17.2$  dB.

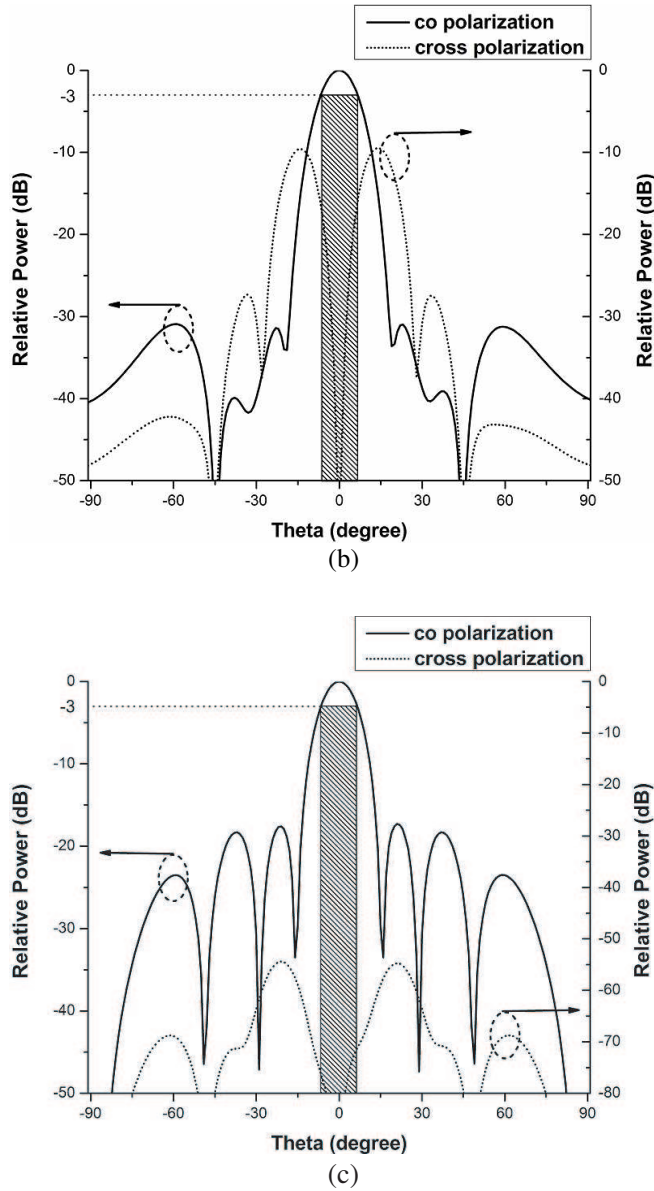
To further determine the effects of elements rotation, based on the third definition of cross-polarization by Ludwig, for a transmitted field polarized in the  $\hat{i}_x$  direction at  $\theta = 0^\circ$ , we can conclude that in the plane of  $\phi = 0^\circ$ , the  $\hat{\theta}$ -component  $E_\theta$  of the array, obtained by a coherent summation of the  $E_\theta^{mn}$ , is equal to the radiated field of co-polarization  $E_{\text{copolar}}$  and

$$E_{\text{copolar}}(\theta, 0^\circ) = E_\theta(\theta, 0^\circ) \quad E_\theta(\theta, 0^\circ) = \sum_{m=1}^M \sum_{n=1}^N E_\theta^{mn}(\theta, 0^\circ) \quad (14)$$

For the element without rotation, Figure 6 shows the normalized magnitude and phase difference of  $E_\theta$  at  $\theta = 0^\circ$ . Due to the good polarization purity of microstrip antenna,  $E_\theta$  is the projection of  $E_{\text{copolar}}$  in the  $\phi$  plane.

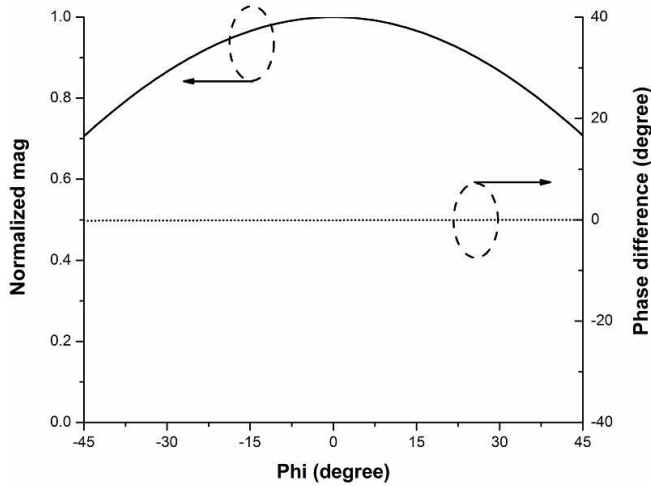
The total radiated field  $E_\theta$  of the array is a summation of the projection of  $E_{\text{copolar}}$  of each element rotated. Expression (7) gives the relationship between gain decrease of the array and elements rotation





**Figure 5.** The 2D radiation patterns at (a)  $\phi = 0^\circ$ ; (b)  $\phi = 45^\circ$ ; (c)  $\phi = 90^\circ$ .





**Figure 6.** The radiated field  $E_\theta$  of the isolated element without rotation at  $\theta = 0^\circ$ .

angles.

$$\Delta G = \left| 20 \log_{10} \frac{\left( \sum_{m=1}^M \sum_{n=1}^N \cos(y_{mn}) \right)}{M \times N} \right| \quad (15)$$

In this design, for the impact of elements rotation, the decrease in the gain is about 1.3 dB.

#### 4. CONCLUSION

For improving the radiating characteristic and simplifying the design of feeding network, a new approach has been exploited and discussed. For uniformly excited antenna array, by determining the rotation angles of the elements rotated in their local coordinates, the maximum relative sidelobe can be reduced by about 5 dB, and more importantly, compared with the planar antenna array excited by complex excitations, amplitudes and phases, the design of the feeding network is achieved with great ease. For this design, a further discussion to the effects of elements rotation shows relatively slight decrease of 1.3 dB in the gain. The simulation results demonstrate the validity of the new approach proposed, which makes some inspiration to planar arrays pattern synthesis.

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