Progress In Electromagnetics Research M, Vol. 9, 1–8, 2009

EFFECTIVE SKIN DEPTH FOR MULTILAYER COATED CONDUCTOR

H.-W. Deng and Y.-J. Zhao

College of Information Science and Technology
Nanjing University of Aeronautics and Astronautics
Nanjing 210016, China

C.-J. Liang

School of Mechano-electronic Engineering
Xidian University
Xi'an 710071, China

W.-S. Jiang and Y.-M. Ning

The 41th Research institute of China Electronics Technology Group Corporation
Qingdao 233006, China

Abstract—In this paper, the effective skin depth which provides a useful evaluation for field penetration in multilayer coated conductor is proposed. The reflection on the interface between the adjacent conductors is considered in theoretical derivation. It is found that the effective skin depths of the gold and gold-nickel coated copper rapidly vary with the thickness of the outer layer (gold) when the gold thickness is less than twice the gold skin depth and achieve stabilization as the gold thickness increases to five times the gold skin depth.

1. INTRODUCTION

In practical engineering, multilayer metal plating technique is widely applied to high corrosion resistance or small metal loss in microwave and millimeter wave band. The plated noble metal layer thicknesses need to be properly selected to reduce the plating cost and simultaneously keep the performance of devices. So the characteristics

Corresponding author: Y.-J. Zhao (yjzhao@nuaa.edu.cn).

of multilayer coated conductor need to be predicted accurately. The skin depth describes the field penetration as an important parameter in a conductor; nevertheless, the conventional calculation of skin depth assumes a semi-infinite slab of conductive material. HIRAOKA et al. gave a modified skin depth of the finite thickness conductor without wave reflection at the backside [1]. A further consideration is that the incident field combining with the reflected field gives the actual field distribution in a finite thickness conductor, and the skin depth with reflection is given in [2].

In this paper, the general formulation of effective skin depth of multilayer coated conductor, which considers the reflection on the interfaces between the conductors, is presented. As practical engineering cases, the gold and gold-nickel coated copper are investigated, and some useful conclusions are obtained.

2. THEORY

2.1. Conventional Skin Depth

The incident electric field on a semi-infinite slab of material with conductivity (σ) that occupies $z > 0$ is shown in Fig. 1. Just at or below the surface of the conductor, the complex amplitude of the field is given as E_{x0}. As the field propagates into the slab, the complex amplitude decreases as $E_x = E_{x0}e^{-\alpha z}$. The corresponding current density is $J_x = \sigma E_{x0}e^{-\alpha z}$.

The current through a surface extending from zero to infinity in the z direction and of width w in the y direction is

$$I = \int_{z=0}^{\infty} \int_{y=0}^{w} \sigma E_{x0} e^{-\alpha_c z} dy dz = \sigma E_{x0} w \int_{0}^{\infty} e^{-\alpha z} dz \tag{1}$$

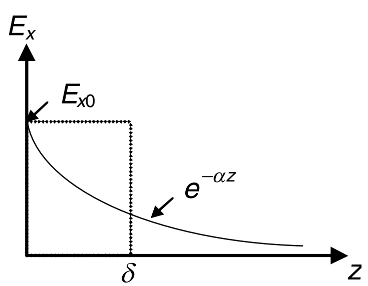

Figure 1. Transmission line model for N layers coated conductor.

The skin depth (δ) is found by evaluating the integral.

$$\delta = \int_0^\infty e^{-\alpha_c z} dz = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{2}$$

where μ and σ are the conductor material's permeability and conductivity respectively. The field is an exponentially decaying function in the conductive slab. The electric field at the surface is assumed to be constant down to a skin depth on account of calculating current. As shown in Fig. 1, the area of a rectangle of sides E_{x0} and δ is equivalent to the area under the $e^{-\alpha z}$ exponential curve.

2.2. Effective Skin Depth

The multilayer coated conductor is shown in Fig. 2. It is assumed that the base conductor thickness is much larger than its skin depth, the transmission line model can be hence used to represent multilayer coated conductor with permeability μ_i electrical conductivity σ_i and thickness t_i of each coated layer as shown in Fig. 3. The field at any point on the first coated layer is given by

$$\begin{aligned} E_x(z) &= E_1^+ e^{-\gamma_{c1}(z-t_1)} + E_1^- e^{\gamma_{c1}(z-t_1)} \\ &= E_1^+ \left(e^{-\gamma_{c1}(z-t_1)} + \Gamma_1 e^{\gamma_{c1}(z-t_1)} \right), \quad z < t_1 \end{aligned} \tag{3}$$

In this expression, E_1^+ and E_1^- are the complex amplitudes of the incident and the reflected waves at $z = t_1$ respectively; γ_{c1} is the propagation constant; Γ_1 is the reflection coefficient at the conductor-conductor boundary (at $z = t_1$). E_0^+ expressed in terms of the field E_{x0} at the surface ($z = 0$) is substituted to Equation (3), and then

$$\begin{aligned} E_x(z) &= E_1^+ e^{-\gamma_{c1}(z-t_1)} + E_1^- e^{\gamma_{c1}(z-t_1)} \\ &= \frac{\left(e^{-\gamma_{c1}(z-t_1)} + \Gamma_1 e^{\gamma_{c1}(z-t_1)} \right)}{\left(e^{\gamma_{c1} t_1} + \Gamma_1 e^{-\gamma_{c1} t_1} \right)} E_{x0}, \quad z < t_1 \end{aligned} \tag{4}$$

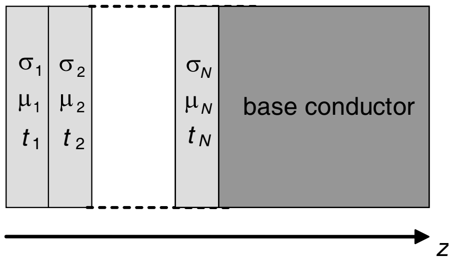

Figure 2. Multilayer coated conductor.

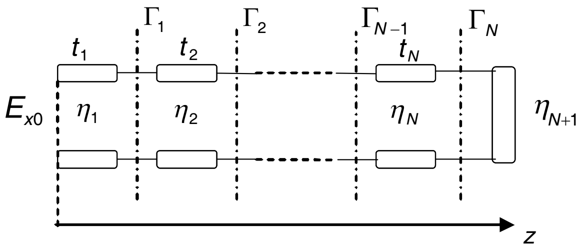

Figure 3. Transmission line model for N layers coated conductor.

The field at any point of the second coated layer is given similarly by

$$E_x(z) = E_2^+ \left(e^{-\gamma_{c2}(z-t_1-t_2)} + \Gamma_2 e^{\gamma_{c2}(z-t_1-t_2)} \right), \quad t_1 < z < t_2 \tag{5}$$

For the tangential field is equal at the surface ($z = t_1$), E_2^+ is expressed by

$$E_2^+ = \frac{1+\Gamma_1}{(e^{\gamma_{c2}t_2} + \Gamma_{L2} e^{-\gamma_{c2}t_2})(e^{\gamma_{c1}t_1} + \Gamma_1 e^{-\gamma_{c1}t_1})} E_{x0} \tag{6}$$

So the Equation (5) is written as

$$E_x(z) = \frac{(1+\Gamma_1)\left(e^{-\gamma_{c2}(z-t_1-t_2)} + \Gamma_2 e^{\gamma_{c2}(z-t_1-t_2)}\right)}{(e^{\gamma_{c1}t_1} + \Gamma_1 e^{-\gamma_{c1}t_1})(e^{\gamma_{c2}t_2} + \Gamma_2 e^{-\gamma_{c2}t_2})} E_{x0}, \quad t_1 < z < t_1 + t_2 \tag{7}$$

Similar to treatment in the second coated layer, the last coated layer is expressed by

$$E_x(z) = \frac{(1+\Gamma_1)(1+\Gamma_2)\dots(1+\Gamma_{N-1}) \left(e^{-\gamma_N(z-t_1-t_2-\dots-t_{N-1})} + \Gamma_N e^{\gamma_N(z-t_1-t_2-\dots-t_{N-1})}\right)}{(e^{\gamma_1 t_1} + \Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2} + \Gamma_2 e^{-\gamma_2 t_2})\dots (e^{\gamma_N t_N} + \Gamma_N e^{-\gamma_N t_N})} E_{x0} \quad \sum_{i=1}^{N-1} t_i < z < \sum_{i=1}^{N} t_i \tag{8}$$

As a result of the base conductor thickness much larger than its skin depth, the reflected field on the interface between the base conductor and air has no additional consideration. So the field of the base conductor is written as

$$E_x(z) = A e^{-\gamma_N(z-t_1-t_2-\dots-t_N)} \tag{9}$$

where A is amplitude of the incident wave and attained by the tangential field. We have

$$A = \frac{(1+\Gamma_1)(1+\Gamma_2)\ldots(1+\Gamma_N)}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2}+\Gamma_2 e^{-\gamma_2 t_2})\ldots(e^{\gamma_N t_N}+\Gamma_N e^{-\gamma_N t_N})} E_{x0} \tag{10}$$

The above equations can be used to determine the field distribution, and each integral value of N layers coated conductor is written as

$$d_1 = \int_0^{t_1} \left| \frac{(e^{-\gamma_1(z-t_1)}+\Gamma_1 e^{\gamma_1(z-t_1)})}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})} \right| dz$$

$$d_2 = \int_{t_1}^{t_1+t_2} \left| \frac{(1+\Gamma_1)(e^{-\gamma_2(z-t_1-t_2)}+\Gamma_2 e^{\gamma_2(z-t_1-t_2)})}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2}+\Gamma_2 e^{-\gamma_2 t_2})} \right| dz$$

. . . .

$$d_i = \int_{t_1+t_2+\ldots+t_{i-1}}^{t_1+t_2+\ldots+t_{i-1}+t_i} \left| \frac{(1+\Gamma_1)(1+\Gamma_2)\ldots(1+\Gamma_{i-1})\left(e^{-\gamma_i(z-t_1-t_2-\ldots-t_i)} + \Gamma_i e^{\gamma_i(z-t_1-t_2-\ldots-t_i)}\right)}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2}+\Gamma_2 e^{-\gamma_2 t_2})\ldots(e^{\gamma_i t_i}+\Gamma_i e^{-\gamma_i t_i})} \right| dz$$

. . . .

$$d_N = \int_{t_1+t_2+\ldots+t_{N-1}}^{t_1+t_2+\ldots+t_{N-1}+t_N} \left| \frac{(1+\Gamma_1)(1+\Gamma_2)\ldots(1+\Gamma_{N-1})\left(e^{-\gamma_N(z-t_1-t_2-\ldots-t_N)} + \Gamma_N e^{\gamma_N(z-t_1-t_2-\ldots-t_N)}\right)}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2}+\Gamma_2 e^{-\gamma_2 t_2})\ldots(e^{\gamma_N t_N}+\Gamma_N e^{-\gamma_N t_N})} \right| dz$$

$$d_{N+1} = \int_{t_1+t_2+\ldots+t_N}^{\infty} \left| \frac{(1+\Gamma_1)(1+\Gamma_2)\ldots(1+\Gamma_N)}{(e^{\gamma_1 t_1}+\Gamma_1 e^{-\gamma_1 t_1})(e^{\gamma_2 t_2}+\Gamma_2 e^{-\gamma_2 t_2})\ldots(e^{\gamma_N t_N}+\Gamma_N e^{-\gamma_N t_N})} e^{-\gamma_N(z-t_1-t_2-\ldots-t_N)} \right| dz$$

The general integral formula of each conductor is summarized as

$$d_i = \int_{\sum_{j=0}^{i-1} t_j}^{\sum_{j=0}^{i} t_j} \left| \frac{\prod_{j=0}^{i-1}(1+\Gamma_j)}{\prod_{j=0}^{i}\left(e^{\gamma_j t_j}+\Gamma_j e^{-\gamma_j t_j}\right)} \left(e^{-\gamma_i\left(z-\sum_{j=0}^{i-1} t_j\right)} + \Gamma_i e^{\gamma_i\left(z-\sum_{j=0}^{i} t_j\right)} \right) \right| dz \tag{11}$$

The equation with known propagation constant γ_i and reflection coefficient Γ_i is solved using numerical integration. So the effective skin depth can be calculated by

$$\delta_{\mathit{eff}} = \sum_{i=1}^{N+1} d_i, \quad i = 1, 2, \ldots, N, N+1 \tag{12}$$

The propagation constant γ_i and characteristic impedance η_i are obtained by the correspond frequency ω, permeability μ_i and electrical conductivity σ_i [3, 4].

$$\gamma_i = (1+j)\cdot\sqrt{\frac{\omega\mu_i\sigma_i}{2}} \tag{13}$$

$$\eta_i = \frac{\gamma_i}{\sigma_i} = (1+j)\cdot\sqrt{\frac{\omega\mu_i}{2\sigma_i}} \tag{14}$$

In practical application, the base conductor thickness is large enough that Γ_{N+1} on the interface between base conductor and air is set to zero, and η_{N+1} can be considered as the load impedance Z_{N+1}. According to the transmission line theory, the equivalent surface impedance Z_i can be obtained by cascaded formula

$$Z_i = \eta_i \frac{Z_{i+1} + \eta_i \mathrm{th}(\gamma_i \cdot t_i)}{\eta_i + Z_{i+1}\mathrm{th}(\gamma_i \cdot t_i)} \tag{15}$$

So the Γ_i is attained by

$$\Gamma_i = \frac{Z_{i-1} - \eta_i}{Z_{i-1} - \eta_i} \tag{16}$$

3. NUMERICAL RESULTS

In the microwave and millimeter wave band, the gold or gold-nickel with nickel as an adhesive is coated on the conductor to improve the corrosion resistance in practical engineering. Fig. 4 gives the variation of effective skin depths with the thickness of outer layer (gold) of the gold and gold-nickel coated copper.

As a result of the reflection on the interface between the adjacent conductors, the current concentrates more on the outer coated layer. It is shown in Fig. 4 that the effective skin depths of the gold and gold-nickel coated copper rapidly vary with the thickness of the outer layer (gold) when the thickness is less than twice the gold skin depth. As the gold thickness increases to five times the gold skin depth, the effective skin depth achieves stabilization. In addition, the effective skin depths for different coated gold thicknesses of gold-nickel coated copper are

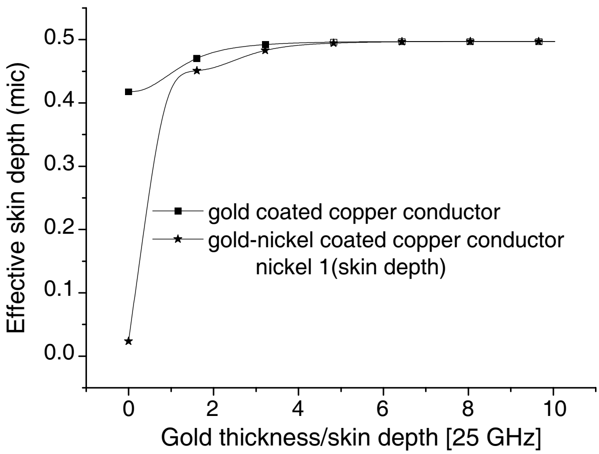

Figure 4. Effective skin depth for gold and gold-nickel coated copper conductor.

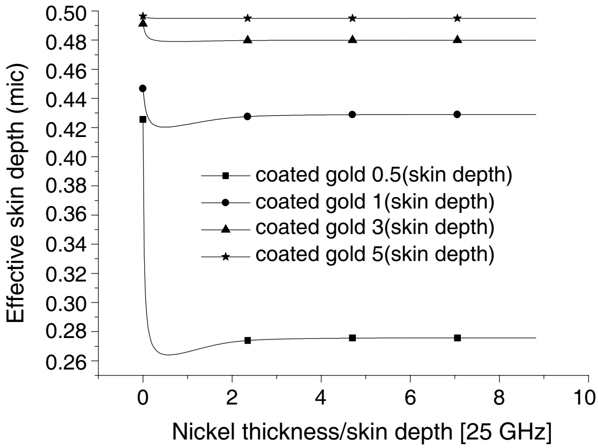

Figure 5. Effective skin depth for different coated gold thicknesses.

shown in Fig. 5. The effective skin depth varies not only with the gold thickness, but also with nickel thickness. When the coated gold thickness is more than five times the gold skin depth, the effective skin depth does not vary with coated nickel thickness.
($\sigma_{\text{gold}} = 4.1e^7$ s/m, $\mu_{\text{rgold}} = 1$; $\sigma_{\text{nickel}} = 1.45e^7$ s/m, $\mu_{\text{rnickel}} = 600$; $\sigma_{\text{copper}} = 1.5e^7$ s/m, $\mu_{\text{rcopper}} = 1$)

4. CONCLUSION

Considering the reflection on the interface between the adjacent conductors, the general formulation of effective skin depth for multilayer coated conductor is derived in detail in this paper. As a result of the reflection on the interface between the adjacent conductors, the current concentrate more on the outer coated layer. It is found that the effective skin depths of the gold and gold-nickel coated copper rapidly vary with the thickness of the outer layer (gold) when the gold thickness is less than twice the gold skin depth and achieve stabilization as the gold thickness increases to five times the gold skin depth.

ACKNOWLEDGMENT

This work was supported by basic research items of National Key Lab of Electronic Measurement Technology.

REFERENCES

1. Hiraoka, T., T. Tokumitsu, and M. Aikawa, "Very small wide-band MMIC magic T's using microstrip lines on a thin dielectric film," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 37, No. 10, 1569–1575, 1989.
2. Wentworth, S. M., M. E. Baginski, D. L. Faircloth, S. M. Rao, and L. S. Riggs, "Calculating effective skin depth for thin conductive sheets," *Antennas and Propagation Society International Symposium 2006, IEEE*, 4845–4848, 2006.
3. Thiel, W. and L. P. B. Katehi, "A surface impedance approach for modeling multilayer conductors in FDTD," *Microwave Symposium Digest, 2002 IEEE MTT-S, IEEE*, Vol. 2, 759–762, 2002.
4. Klein, C. A., "Microwave shielding effectiveness of EC-coated dielectric slabs," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 38, No. 3, 321–324, 1990.