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PERFORMANCE AND COMPLEXITY IMPROVEMENT OF TRAINING BASED CHANNEL ESTIMATION IN MIMO SYSTEMS

M. W. Numan

Department of Electrical, Electronic and Systems Engineering
Universiti Kebangsaan Malaysia
Bangi, Selangor 43600, Malaysia

M. T. Islam

Institute of Space Science (ANGKASA)
Universiti Kebangsaan Malaysia
Bangi, Selangor 43600, Malaysia

N. Misran †

Department of Electrical, Electronic and Systems Engineering
Universiti Kebangsaan Malaysia
Bangi, Selangor 43600, Malaysia

Abstract—Multiple-input multiple-output (MIMO) systems play a vital role in fourth generation wireless systems to provide advanced data rate. In this paper, a better performance and reduced complexity channel estimation method is proposed for MIMO systems based on matrix factorization. This technique is applied on training based least squares (LS) channel estimation for performance improvement. Experimentation results indicate that the proposed method not only alleviates the performance of MIMO channel estimation but also significantly reduces the complexity caused by matrix inversion. The performance evaluations are validated through computer simulations using MATLAB® 7.0 in terms of bit error rate (BER). Simulation results show that the BER performance and complexity of the proposed method clearly outperforms the conventional LS channel estimation method.

Corresponding author: M. W. Numan (mwnuman@gmail.com).

† Also with Institute of Space Science (ANGKASA), Universiti Kebangsaan Malaysia, Bangi, Selangor 43600, Malaysia.

1. INTRODUCTION

Wireless communication systems continue to strive for ever higher data rates. To cater to both higher transmission rates and higher spectral efficiencies in order to increase the performance of communication systems, the wireless industry is already looking ahead and embracing multiple-input multiple-output (MIMO) systems [1, 2]. Using multiple transmit as well as receive antennas, a MIMO system exploits spatial diversity, higher data rate, greater coverage and improved link robustness without increasing total transmission power or bandwidth. However, MIMO relies upon the knowledge of channel state information (CSI) at the receiver for data detection and decoding. It has been proved that when the channel is Rayleigh fading and perfectly known to the receiver, the performance of a MIMO system grows linearly with the number of transmit or receive antennas, whichever is less [3]. Therefore, an accurate and robust estimation of wireless channel is of crucial importance for coherent demodulation in MIMO system.

A considerable number of channel estimation methods have already been studied by different researchers for MIMO systems. In certain channel estimation methods, training symbols that are transmitted over the channels are investigated at the receiver to render accurate CSI [4–7]. Compared with blind and semiblind channel estimations, training based estimations generally require a small data record. Hence, they are not limited to slowly time-varying channels and entail less complexity. One of the most efficient training based methods is the least squares (LS) method, for which the channel coefficients are treated as deterministic but unknown constants [8, 9]. When the full or partial information of the channel correlation is known, a better channel estimation can be achieved by minimum mean square error (MMSE) method [5]. The fundamental difference between these two techniques is that the channel coefficients are treated as deterministic but unknown constants in the former, and as random variables of a stochastic process in the latter. The MMSE estimation has better performance than LS estimation at the cost of higher complexity as it additionally exploits prior knowledge of the channel coefficients. But practically this kind of information is sometimes not known beforehand, which makes MMSE-based technique infeasible.

The complexity of channel estimation mainly increases due to matrix inversion. To reduce the complexity of MIMO detection, several matrix factorization techniques have been applied on MIMO systems recently. Orthogonal matrix triangularization is a matrix factorization technique that reduces a full rank matrix into simpler form. Some other matrix factorizations like lower upper (LU) decomposition, singular value decomposition (SVD) can also be used to avoid explicit matrix inversions. But orthogonal matrix triangularization is preferable over other methods as it guarantees numerical stability by minimizing errors caused by machine roundoffs [10]. Matrix factorization is used to conduct large matrix calculations in alternate ways, and applied for system complexity reduction. The decoding algorithm for layered space-time codes based on matrix triangularization is presented in [11]. The authors in [12] proposed a low-complexity maximum-likelihood decoding approach based on matrix factorization for signal detection in MIMO systems. In [13], a combined detection algorithm based on matrix triangularization is proposed to reduce the complexity of the MIMO detection algorithm. A reduced complexity hardware architecture for MIMO symbol detector using matrix factorization is proposed in [14] which supports two MIMO schemes of space-frequency block codes and space division multiplexing of the codes. However, in all works matrix factorizations decrease complexity, but performance improvement is not justified. In this paper, a channel estimation method is proposed for MIMO system by employing orthogonal matrix triangularization on LS estimation which minimizes the computational complexity and at the same time improves the performance. The coding scheme of MIMO considered in this paper is space-time block coding (STBC) which is an attractive approach for improving quality in wireless links [3, 15, 16].

The rest of the paper is organized as follows. In Section 2, a model of MIMO system employing space-time block coding is introduced. Section 3 presents a new channel estimation method applying orthogonal matrix triangularization on LS estimation. Section 4 comprises a number of experimentations validating the proposed method, showing its significant advantages over traditional LS estimation in terms of bit error rate (BER) performance and complexity improvement. Finally Section 5 highlights some of the distinct features of the proposed approach and draws the conclusion.

Throughout the paper most notations are standard. Matrices are represented by boldface capital letters, e.g., $\mathbf{A}$, and vectors are boldface small letters, e.g., $\mathbf{a}$. Complex conjugate, Hermitian transpose and Estimated value are denoted by $(\cdot)^*$, $(\cdot)^H$ and $\overset{\wedge}{(\cdot)}$ respectively. $\mathbf{I}_m$ stands for the $m \times m$ identity matrix, $\mathbf{0}$ denotes the all zeros matrix of appropriate dimensions. $\|\mathbf{A}\|_F$ is used for the Frobenius norm of a matrix $\mathbf{A}$. $[\mathbf{A}]_{ij}$ stands for the element in the i-th row and j-th column of $\mathbf{A}$.

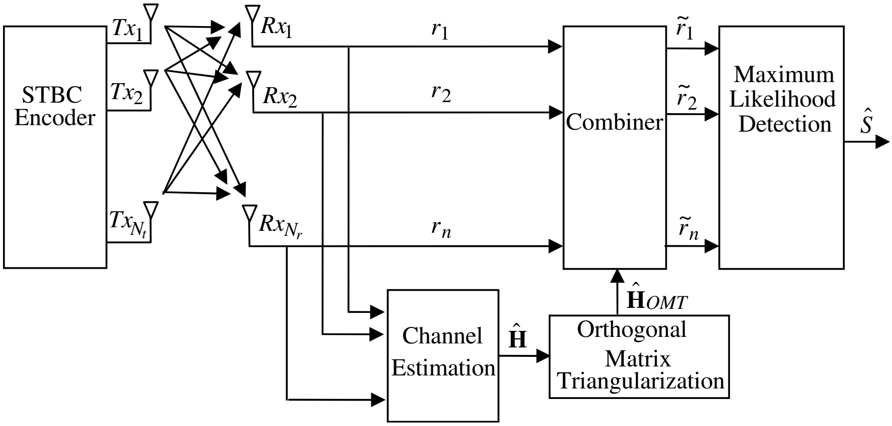

Figure 1. MIMO system using space-time block coding.

2. MIMO SYSTEM MODEL

Data symbols $s_1, s_2, s_3, \ldots, s_L$ are encoded by $\mathbf{S}$ that is the transmission matrix of an STBC, where $[\mathbf{S}]_{ij}$, $i = 1, 2, \ldots, N_t$ and $j = 1, 2, \ldots, T$ represent element of the linear combination of symbols and their complex conjugates, which are transmitted simultaneously from the i-th transmit antenna in the j-th symbol periods [17–19]. $\mathbf{S}$ is independent identically distributed (i.i.d) Gaussian random signals with zero mean and variance matrix given by

$$E\left\{s_n s_m^H\right\} = \begin{cases} \sigma^2, & n = m \\ 0, & n \neq m \end{cases} \tag{1}$$

where $E\{\cdot\}$ implies the expectation and σ^2 is the power accompanying one symbol. Since transmission matrix $\mathbf{S}$ is orthogonal, $\mathbf{S}\mathbf{S}^H = c\sum_{l=1}^{L} |s_l|^2 \mathbf{I}_{N_t}$ where c is a constant dependent on $\mathbf{S}$. For example, $c = 1$ if G_2, H_3 and H_4 and $c = 2$ if G_3 and G_4 in [18]. Here L symbols transmitted over T symbol periods. So the code rate is $R = \frac{L}{T}$.

The input-output relation of the system can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{W} \tag{2}$$

where $\mathbf{S}$ is the $N_t \times T$ transmit matrix, $\mathbf{Y}$ is the $N_r \times T$ received matrix and $\mathbf{W}$ is an $N_r \times T$ i.i.d. Gaussian random noise matrix with zero mean. The MIMO channel response is described by $N_r \times N_t$ matrix $\mathbf{H}$. A general entry of the channel matrix $\mathbf{H}$ is denoted by $\{h_{ij}\}$. This represents the complex gain of the channel between the j-th transmitter and the i-th receiver and can be written as

$$\mathbf{H} = \begin{pmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \cdots & h_{N_rN_t} \end{pmatrix} \tag{3}$$

where

$$\begin{aligned} h_{ij} &= \alpha + j\beta \\ &= \sqrt{\alpha^2 + \beta^2} \cdot e^{j \arctan \beta/\alpha} \\ &= |h_{ij}| \cdot e^{j\phi_{ij}} \end{aligned}$$

In a rich scattering environment with no line-of-sight (LOS), α and β are independent and normal distributed random variables, then channel gains $|h_{ij}|$ are usually Rayleigh distributed.

The signals are transmitted over channel. The combined signal $\tilde{r}_n$ at the receiver is

$$\tilde{r}_n = \left[c \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} |h_{ij}|^2 \right] s_n + \tilde{w}_n = c \left\| \mathbf{H} \right\|_F^2 s_n + \tilde{w}_n, \quad n = 1, 2, \ldots, L \tag{4}$$

where $\tilde{w}_n$ is the noise term after combining. At the receiver the maximum likelihood (ML) decoder is used to detect the transmitted symbol. The ML decoder can be simplified using the orthogonality of $\mathbf{S}$. The received symbol, hence, can be determined by

$$\hat{s}_n = \arg\min \left| \tilde{r}_n - c \left\| \mathbf{H} \right\|_F^2 s \right|^2 \tag{5}$$

This is a low complexity orthogonal system model that can be considered highly attractive for practical applications.

3. PROPOSED CHANNEL ESTIMATION METHOD

The knowledge of CSI is required at the receiver to recover the transmitted signals properly in MIMO systems. In training based channel estimation, the training symbols that are known to the receiver are multiplexed along with the data stream and examined in the receiver to estimate the channel. In practice, training based LS estimation is more frequently used due to its acceptable performance. But this estimation involves matrix inversions, which result in high computational complexity and hence undesirable for hardware implementation. The orthogonal matrix triangularization is a very convenient technique to avoid matrix inversion and is preferable because of its clever implementation in highly parallel array architecture [20].

In a $N_t \times N_r$ MIMO system, totally $(N_t \times N_r)$ channels are needed to be estimated between transmitters and receivers. The received training symbols can be expressed as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{6}$$

where $\mathbf{x}$ is the transmitted training signal, $\mathbf{y}$ is the received signal and $\mathbf{n}$ is the noise response. The channel response $\mathbf{H}$ is assumed to be random and quasi-static within two transmission blocks.

The LS approach solves the estimation (6) by minimizing the cost function as,

$$J(\mathbf{H}) = (\mathbf{y} - \mathbf{Hx})^H (\mathbf{y} - \mathbf{Hx}) \tag{7}$$

The gradient of (7) is given below,

$$\frac{\partial J(\mathbf{H})}{\partial \mathbf{H}} = -2\mathbf{x}^H \mathbf{y} + 2\mathbf{x}^H \mathbf{x} \tag{8}$$

Minimizing the gradient to zero yields the LS estimation $\hat{\mathbf{H}}$ of the channel response obtained by

$$\hat{\mathbf{H}} = \left(\mathbf{x}^H \mathbf{x}\right)^{-1} \mathbf{x}^H \mathbf{y} \tag{9}$$

The inversion of $\mathbf{x}^H\mathbf{x}$ in (9) has a high complexity and will significantly increase when the size of $\mathbf{x}$ increases, which is dependent on the number of transmit antennas. To avoid complexity because of matrix inversion, orthogonal matrix triangularization is applied on $\mathbf{x}$. The matrix triangularization can be calculated via Householder transformation, or Givens rotation. The Givens rotation is a recursive method that requires a larger number of floating point operations as compared to the Householder transformation method [10]. In this work, Householder transformation is chosen to minimize the required operations. In the Householder approach, a series of reflection matrix is applied to the matrix, $\mathbf{x}$, column by column to annihilate the lower triangular elements. The reflection transformations are orthonormal matrices that can be written as

$$\mathbf{A} = \left(\mathrm{I} + \beta \mathbf{v}\mathbf{v}^H\right) \tag{10}$$

where $\mathbf{v}$ is the Householder vector and $\beta = -2 \left\|\mathbf{v}\right\|_2^2$. For the matrix $\mathbf{x}$, to annihilate the lower elements of the k-th column the $\mathbf{A}_k$ is constructed as follows:

i. Let $\mathbf{v}$ equal the k-th column of $\mathbf{x}$
ii. Update $\mathbf{v}$ by $\mathbf{v} = \mathbf{x} + \|\mathbf{x}\|_2 \varphi$, where $\varphi = [1, 0, \ldots, 0]^T$
iii. Determine β by $\beta = -2\,\|\mathbf{v}\|_2^2$
iv. $\mathbf{A}_k$ is calculated according to (10).

The $\mathbf{A}_k$ formed from the above steps are pre-multiplied by $\mathbf{x}$ sequentially as follows

$$\mathbf{A}_n, \ldots, \mathbf{A}_1\mathbf{x} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{11}$$

where, $\mathbf{R}$ is a $n \times n$ upper triangular matrix, $\mathbf{0}$ is a null matrix, and the sequence of reflection matrices form the complex transpose of the orthogonal matrix $\mathbf{Q}^H$, i.e., $\mathbf{Q}^H = \mathbf{A}_n, \ldots, \mathbf{A}_1$ and $\mathbf{I} = \mathbf{Q}^H\mathbf{Q}$. Thus (11) can be written as

$$\mathbf{x} = \mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{12}$$

The error function for estimation (9) can be expressed as

$$\varepsilon = \mathbf{y} - \hat{\mathbf{H}}\mathbf{x}, \text{ if } \varepsilon = 0, \text{ then } \mathbf{y} = \hat{\mathbf{H}}\mathbf{x} \tag{13}$$

By combining (12) and (13) the received signal stands

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{x} = \hat{\mathbf{H}}\mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{14}$$

The Hermitian of $\mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}$ is multiplied to both sides of (14) to derive the proposed channel estimation

$$\hat{\mathbf{H}}_{OMT} = \mathbf{y}\mathbf{Q}^H\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}^H \tag{15}$$

As $\mathbf{R}$ is an upper triangular matrix, $\hat{\mathbf{H}}_{OMT}$ can be solved through back-substitution. The proposed estimation is a numerically stable low complexity solution to channel estimation of MIMO systems.

4. RESULTS AND DISCUSSION

The performance limit of MIMO system for different antenna configuration is quantified through BER which is particularly an attractive measurement for wireless communications. Extensive computer simulations have been conducted to demonstrate the performance and complexity of the proposed channel estimation. The comparisons are investigated by computer simulations using MATLAB® 7.0. A system equipped with two transmit antennas and arbitrary number of receive antennas is considered for this purpose. In the simulation scenarios the QPSK modulation is used and Rayleigh fading radio channel is assumed.

4.1. System Performance

In the first simulation, the STBC is applied for two transmit antennas and different number of receive antennas to demonstrate the performance of the considered system at perfect channel knowledge. Fig. 2 shows the BER performance comparison between 2×2, 2×4 and 2×6 STBC systems. As can be observed from the figure, the 2×6 system performs better than others. For example, the BER of 6×10^{-3} is achieved at SNR $= 1$ for 2×6 system, whereas the same BER is achieved at SNR $= 3$ for 2×4 and at SNR $= 7$ for 2×2 system. It reflects that the BER performance increases as the number of receive antennas increases for the same number of transmit antennas. But, at the same time, complexity increases significantly.

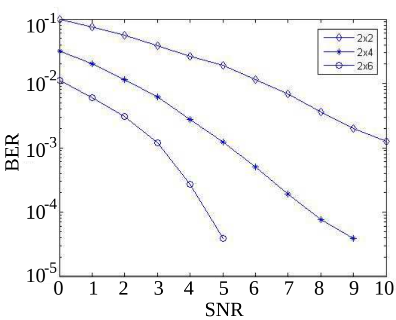

Figure 2. BER comparison between 2×2, 2×4, and 2×6 STBC.

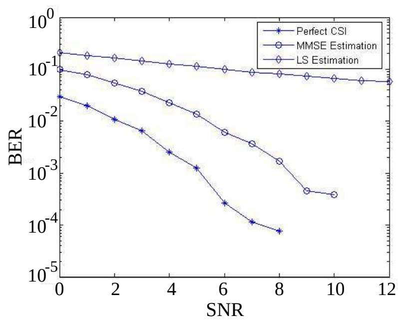

Figure 3. Performance comparison between different channel estimations.

Figure 3 illustrates a performance comparison between different training based channel estimations for 2×4 MIMO system. An ideal channel estimation is also calculated for comparison. All the channel estimations use the same training sequence. From the figure, it can be observed that the BER performance of MMSE estimation is slightly worse than perfect CSI, but much better than LS estimation.

Compared with LS-based techniques, MMSE-based techniques yield better performance because they additionally exploit and require prior knowledge of the channel correlation and SNR. However, the channel correlation is sometimes not a priori known, which makes MMSE-based techniques infeasible. To consider wider MIMO applications, this work focuses on channel estimation technique that does not require prior knowledge of the channel correlation i.e., LS channel estimation.

In this paper, the performance of LS channel estimation is improved by applying orthogonal matrix triangularization as described in Section 3. Fig. 4 shows the performance comparison between LS and proposed channel estimation for 2×2 and 2×4 antenna configurations. Though at lower SNR the BER curves of the proposed estimation closely follows the traditional LS estimation, but at higher SNR it outperforms the LS estimation. It is seen that at higher SNR the performance of the proposed estimation method is 2 dB superior to the traditional LS method.

Figure 5 demonstrates the performance of the proposed channel estimation method for different modulation techniques. As can be observed from the figure, BPSK exhibits better BER performance than others, but this is unsuitable for high data-rate applications when bandwidth is limited. Rather QPSK is a better choice as it can be used either to double the data rate compared to a BPSK system while maintaining the bandwidth of the signal or to maintain the data-rate of BPSK but halve the bandwidth needed.

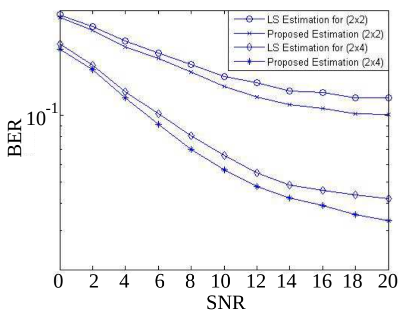

Figure 4. Performance comparison between LS and the proposed estimation for different MIMO systems.

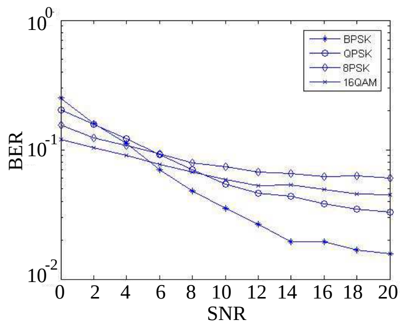

Figure 5. Performance of the proposed method in different modulations.

Table 1. Number of real operations in every complex operation.

Complex Operations	Number of Real Operations		
	Multiplication	Division	Addition/ Subtraction
Multiplication	4	2	0
Division	6	3	2
Addition/ Subtraction	0	0	2

Table 2. Operation counts for every real operation.

Real Operations	Operation Counts
Multiplication/Addition/ Subtraction	1
Division	6
Square Root	10

4.2. Complexity Comparison

In the present simulation scenario, the computational complexity of the LS estimation and the proposed estimation methods are measured and compared in terms of number of mathematical operations. For consistent comparison, the complex operations are converted to real operation equivalents. Table 1 summarizes the real equivalent operations for various complex operations. Each type of real operations has different levels of complexity when implemented in the hardware. Table 2 shows the number of floating point operations for each real operation. It should be noted that counting of the number operations is only an estimate of the computational complexity of the algorithms. A more exact measurement can be found by implementing the algorithm in hardware and counting the number of instructions and required processing time. However, in simulations, floating point operation counts can give a good indication of the relative complexity of different algorithms.

Figure 6 depicts the complexity comparison between LS and the proposed channel estimation in terms of real operations. The impacts of varying antenna configurations on the estimation methods are studied. As expected, when the number of antennas increases, the size of unknown parameters also increases and as a result complexity increases in both estimation techniques. The general trend of the proposed method is that it increases almost linearly with the number

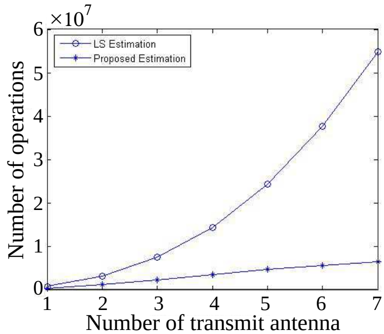

Figure 6. Complexity comparison between LS and the proposed channel estimation.

of transmit antennas. The LS method increases exponentially at a significantly higher rate than the proposed methods. Thus the matrix triangularization channel estimation method is lower in complexity and proves itself a better choice for low complexity channel estimation. This feature plus better performance improvement make the proposed method an attractive solution for MIMO channel estimation.

5. CONCLUSION

In this paper, an orthogonal matrix triangularization based channel estimation method has been proposed for MIMO systems, and a detailed analysis and computer simulations are performed. The proposed method is simulated and compared with LS estimation, showing a significant improvement in terms of BER performance of channel estimation. Moreover the computational complexity of the proposed channel estimation is much lower than conventional LS estimation. The complexity of the proposed method increases almost linearly with respect to the number of transmit antenna, whereas for LS method it increases exponentially. The proposed estimation appreciably outperforms the LS estimation at a lower complexity with a better performance and represents a good solution for MIMO channel estimation technique.

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