Progress In Electromagnetics Research M, Vol. 9, 93–105, 2009

INVERSION OF LOSSY DIELECTRIC PROFILES USING PARTICLE SWARM OPTIMIZATION

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Abstract—An electromagnetic inversion method is proposed for the reconstruction of lossy dielectric slabs. The inversion is done using particle swarm optimization hybridized with Quasi-Newton algorithm. The inversion process is applied to reconstruct dielectric slabs with discrete or continuous profiles. Accurate reconstruction of lossy dielectric slabs is obtained from inversion of reflection coefficient data of normally incident plane waves in the specified frequency range. The proposed algorithm is also tested using noisy data and showed satisfactory performance.

1. INTRODUCTION

Electromagnetic inversion is the problem of reconstructing the electromagnetic properties of a medium (e.g., electric permittivity,

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permeability, conductivity...etc) using the data scattered from this medium. This data may be considered as reflection coefficients, input impedance, scattered electric or magnetic fields. The general methodologies of the electromagnetic inversion problems can be classified into direct based inversion methods and model based inversion methods. The term direct based inversion means that the electromagnetic properties of the medium are obtained from direct calculations applied to the scattered data. These methods include analytical approximation techniques [1–3] and layer stripping techniques [4, 5]. Model based methods try to match the data calculated from the forward problem with the data obtained from the measurements or simulation process. The medium parameters that support this match are composing the solution of the inverse problem. This is usually done through the utilization of an optimization method that minimizes the error between the observed and calculated data. While direct based methods include faster techniques, model based methods have superior performance when the data is incomplete or contaminated with noise [6]. Model Based methods can be classified to local and global optimization methods. Local Optimization methods such as quasi-Newton and Gauss-Newton techniques [7–9] are relatively fast but have the possibility of being trapped in local minima due to the non-linear nature of the problem. For this reason, these techniques are recommended only when sufficient priori information about the inverted model is available. On the other hand, global optimization techniques do not require priori information about the model, for this reason convergence is reached after relatively large number of iterations. The most recent global techniques which are used in electromagnetic inversion problems are the genetic algorithms (GA) [10, 11] and the particle swarm optimization techniques (PSO) [12–15]. The swarm technique has proven to outperform GA due to many reasons. Actually, through a good selection of the swarm parameters, the rate of convergence can be controlled [16, 17]. Also, it is simpler than GA and much easier to be adjusted to obtain faster convergence [18, 19]. Recently, some researchers use hybrid techniques of different methods to utilize their advantages [15, 20, 21]. In this paper, an electromagnetic inversion method is proposed for the reconstruction of a lossy dielectric slab without any priori information. The slab may be formed of discrete layers or continuous profile. The proposed method consists of a hybrid of PSO and the quasi-Newton method. The PSO is first applied to obtain proper priori information about the model which is then used by the quasi-Newton algorithm to achieve faster convergence.

2. FORMULATION OF THE PROBLEM

Electromagnetic inversion using optimization techniques is achieved by solving out for the profile which minimizes the error between the observed data and the synthetic one which is obtained by solving the forward problem. The forward problem is introduced first.

Consider a dielectric slab which is embedded in the air and formed of an M discrete homogeneous layers as shown in Fig. 1. The mth layer of the slab is characterized by its relative permittivity ε_{rm}, electric conductivity σ_m and height h_m. All layers of the slab have magnetic permeability μ_0. Let an incident TEM polarized wave fall on the left surface of the slab with time dependence $e^{j\omega t}$. The reflection coefficient in the air at the surface of the slab is defined as $\Gamma = (E_r/E_i)|_{\text{slab\_surface}}$ and is expressed as

$$\Gamma = \frac{Z_{in}(1) - Z_c(0)}{Z_{in}(1) + Z_c(0)} \tag{1}$$

where $Z_{in}(1)$ represents the input surface impedance at the left surface of the first layer of the slab and is obtained according to the recurrence relation

$$Z_{in}(m) = Z_c(m)\frac{Z_{in}(m+1) + Z_c(m)\tanh(\gamma_m h_m)}{Z_c(m) + Z_{in}(m+1)\tanh(\gamma_m h_m)} \tag{2}$$

where m is an integer with $1 \leq m \leq M$, and

$$Z_c(m) = \frac{\eta_o}{\sqrt{\varepsilon'_{rm}}} \qquad \varepsilon'_{rm} = \varepsilon_{rm} + \frac{\sigma_m}{j\omega\varepsilon_o}$$

$$\gamma_m = \sqrt{j\omega\mu_o(\sigma_m + j\omega\varepsilon_o\varepsilon_{rm})} \qquad \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi\,\Omega$$

$$Z_{in}(M+1) = Z_c(M+1) \quad \text{and} \quad Z_c(M+1) = Z_c(0) = \eta_o$$

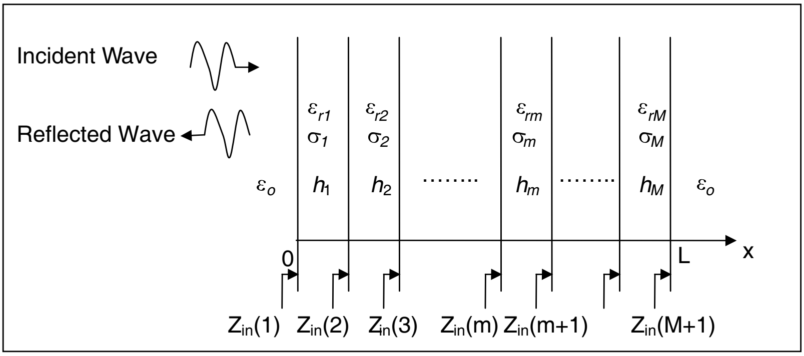

Figure 1. Illustration of the multilayer discrete dielectric slab.

It is worth noting that the above formulation can be used to calculate the reflection coefficient due to continuous dielectric profiles by dividing the profile to M homogeneous layers provided that M is sufficiently large to include all the dielectric profile variations.

Let $r_1, r_2, \ldots, r_N$ be the reflection coefficient data at different frequencies within a specified frequency range, and $\Gamma_1, \Gamma_2, \ldots, \Gamma_N$ be the synthetic reflection coefficient data calculated by solving the forward problem as explained above. The error function to be minimized is empirically proposed as

$$f(\vec{X}) = \sum_{f_i=f_{\min}}^{f_{\max}} \left[1 - \frac{f_i - f_{\min}}{f_{\max}}\right] \left|\Gamma_i(\vec{X}) - r_i\right|^2 + 3 \sum_{f_i=f_{\min}}^{f_{\min}+\frac{f_{\max}-f_{\min}}{5}} \left|\Gamma_i(\vec{X}) - r_i\right|^2 \tag{3}$$

where $f_{\min} = 30\,\mathrm{MHz}$, $f_{\max} = 300\,\mathrm{MHz}$ and $\vec{X}$ is a hyper-dimensional vector whose components represent the unknown profile parameters of the dielectric slab. It is worth noting that the error function given in Eq. (3) is better in the search process than the commonly used error formula given by

$$f(\vec{X}) = \sum_{f_i=f_{\min}}^{f_{\max}} \left|\Gamma_i(\vec{X}) - r_i\right|^2 \tag{4}$$

The modified error function of Eq. (3) has the advantage of relatively enhancing the contribution of low frequency samples. This weighting is useful to enhance the contribution of the conductivity terms in the expression of Γ in Eq. (1). This helps in increasing the accuracy of reconstructing the conductivity of the slab if compared with that of classical error function of Eq. (4).

The value of $\vec{X}$ which minimizes the error function is supposed to reconstruct a dielectric slab profile close to the original slab profile. In reconstructing the lossy dielectric slab, it is recommended to use the piecewise homogeneous model for discrete layers and any suitable expansion function for the continuous profile models. Expansion functions such as the cosine, sinc and legendre forms can be used. However, it is recommended to use the model which can perform the profile reconstruction with minimum number of terms. Through the inversion process, it is found that the cosine expansion model has the best performance for the proposed slab profiles over other expansion functions. The piecewise and the cosine model parameters are explained below.

2.1. Piecewise Homogeneous Model

In this model, it is assumed that the slab can be divided into M piecewise homogeneous regions, and the $3M$-dimensional vector $\vec{X}$ contains the relative permittivity, electric conductivity and length of each layer. Thus, $\vec{X} = (\varepsilon_{r1}, \sigma_1, h_1, \varepsilon_{r2}, \sigma_2, h_2, \ldots, \varepsilon_{rM}, \sigma_M, h_M)$ where ε_{r1}, ε_{r2}, $\ldots$, ε_{rM} are the relative permittivities of the different layers, σ_1, σ_2, $\ldots$, σ_M are their electric conductivities and h_1, h_2, $\ldots$, h_M are the layers lengths.

2.2. Cosine Expansion Model

In this model, we use some terms of the Fourier cosine series of the permittivity and conductivity profiles [22], which is given by

$$\varepsilon_r(z) = a_o + \sum_{m=1}^{M} a_m \cos\left(\frac{m\pi z}{L}\right), \qquad 0 \le z \le L \tag{5a}$$

$$\sigma(z) = b_o + \sum_{m=1}^{M} b_m \cos\left(\frac{m\pi z}{L}\right), \qquad 0 \le z \le L \tag{5b}$$

In this case, the model parameters can be defined by the vector $\vec{X}$ which is expressed as $\vec{X} = (a_o, a_1, a_2, \ldots, a_M, b_o, b_1, b_2, \ldots, b_M)$. In either case, the target of the optimization inversion algorithm is to find the most suitable vector $\vec{X}$ which corresponds to the global minimum of the error function given in Eq. (3).

3. PARTICLE SWARM OPTIMIZATION

The particle swarm optimizer (PSO) is a population based stochastic optimization algorithm modeled after the simulation of social behavior of bird flocks [23]. In the PSO system, a swarm of particles fly through a hyper-dimensional search space. Each particle represents a candidate solution to the optimization problem. The position of a particle is influenced by the best position visited by itself which represents its own experience and the position of the best particle in its neighborhood representing the experience of neighboring particles [16]. The performance of each particle is measured using a fitness function that varies depending on the optimization problem. The position of each particle is updated by a stochastic velocity which depends on the distance of the particle from its own best and from that of the swarm best. The velocity and position update equations are given at any time step t by [16]:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_{1,j}(t)(pbest_{ij}(t) - x_{ij}(t)) + c_2 r_{2,j}(t)(gbest_j(t) - x_{ij}(t)) \quad (6)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (7)$$

with $i = 1, \ldots, s$ and $j = 1, \ldots, n$ where

- s is the number of particles in the swarm
- n is the number of dimensions of the problem, i.e., the number of parameters of the function to be optimized
- c_1 and c_2 are acceleration coefficients
- w is inertia weight factor
- $r_1(t), r_2(t) \sim U(0,1)$
- $x_i(t)$ is the position vector of particle i, at time step t
- $v_i(t)$ is the velocity vector of particle i, at time step t
- $pbest_i(t)$ is the best solution found by particle i, until time step t,
- $gbest(t)$ is the best solution found by all the swarm at time step t,

The global and personal bests are updated using the following equation in case of a minimization problem

$$pbest_i(t+1) = \begin{cases} x_i(t+1), \text{ if } f(x_i(t+1)) < pbest_i(t) \\ pbest_i(t), \text{ if } f(x_i(t+1)) \geq pbest_i(t) \end{cases} \quad (8)$$

$$gbest(t+1) = pbest_i(t+1) \quad \text{where } f(pbest_i(t+1)) = \min_j f\left(pbest_j(t+1)\right) \quad (9)$$

The first term of the velocity update in Eq. (6) represents the inertia of the particle. This term serves as a memory of previous velocities. The inertia weight controls the impact of the previous velocity: a large inertia weight favors exploration, while a small inertia weight favors exploitation [24]. The second term known as the cognitive component represents the tendency of the individual particle to return to the places of their best performance, and the third term known as the social term represents the tendency of individuals to follow the success of others [16].

The stochastic behavior of the second and third terms makes each individual fluctuates between the region of its personal best and that of the global best, thus makes some search for better solutions around the global best and also searching in its area of personal best because the global minimum might not turn out to be in the valley of the global best, so by making each individual search in its personal best area, there is a better chance that one of them discovers a deeper valley.

The convergence of swarm is either detected from the velocity of particles being so small in magnitude or from the distances between particles being small enough. The algorithm can also be terminated if convergence occurred for a percentage of the swarm. There should be a lot of care taken when selecting the parameters w, c_1 and c_2 as the performance of PSO is sensitive to changes in values of these parameters.

4. THE INVERSION ALGORITHM

The inversion algorithm is started at $n = 1$ and then n is increased gradually until the fitness is achieved with the permissible error limits. Each time n is increased, one of the individuals of the new generation of the higher dimension space is obtained from the best individual found. Also, one extra individual representing the average locations of other individuals is added to the new generation. These two modifications enhance the convergence time with a negligible additional cost. The latter is called Center PSO [25]. To decrease the computation time at any prescribed n, the output of the PSO is switched to the Quasi-Newton local optimizer when expecting the trapping in the global minimum valley. The process of switching between global and local optimizers for each n is continued until final solution is obtained.

5. RECONSTRUCTION OF LOSSY DIELECTRIC SLABS

The proposed inversion algorithm is applied to three different models for a lossy dielectric slab embedded in the free space. The first dielectric slab model represents discrete dielectric variations with $\varepsilon_{rn} = (4, 7, 5)$, $\sigma_n = (7, 2, 5)\,\text{mS/m}$ and $h_n = (20, 60, 40)\,\text{cm}$. The second dielectric slab model is continuous with a tanh distribution in the form

$$\varepsilon_r = 6 + 4\tanh\left(\frac{z-60}{10}\right) \tag{10a}$$

$$\sigma = 5 - 2\tanh\left(\frac{z-60}{10}\right) \tag{10b}$$

and the third model is also continuous with a Gaussian distribution in the form

$$\varepsilon_r = 4 + 5e^{-\frac{1}{2}\left(\frac{z-60}{12}\right)^2} \tag{11a}$$

$$\sigma = 3 - 2e^{-\frac{1}{2}\left(\frac{z-60}{12}\right)^2} \tag{11b}$$

where z is in cm and σ is in mS/m. The swarm parameters used in the reconstruction algorithm are given as $c_1 = 0.172$, $c_2 = 0.172$ and $w = 0.985$ for continuous slabs and $c_1 = 0.011$, $c_2 = 0.011$ and $w = 0.999$ for discrete slabs with a population size of $5n$ where n is the number of model parameters to be reconstructed.

The reconstructed profiles are shown in Figs. 2, 3 and 4 respectively. From these figures, it is shown that the accuracy of the inversion is quite satisfactory in the three models. However, it is clear that the accuracy of inversion of discrete model is better than that of the other continuous models. It is also observed that the inverted conductivity profiles of the continuous models contain slight fluctuations around the original profiles. As shown in Fig. 4, the

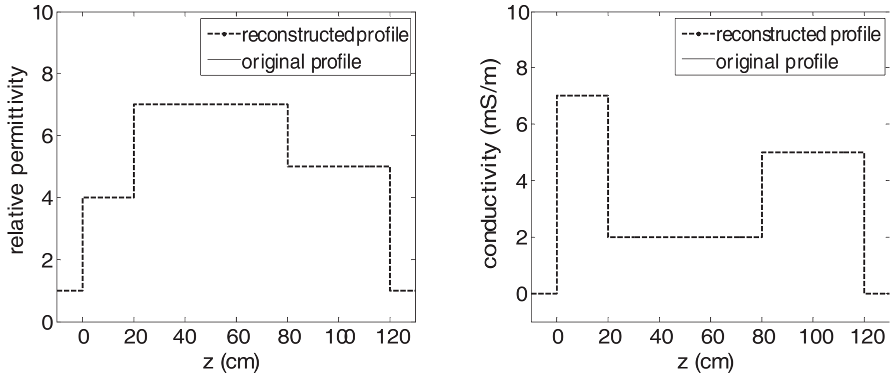

Figure 2. Reconstruction of permittivity and conductivity of discrete profile using piecewise homogenous model.

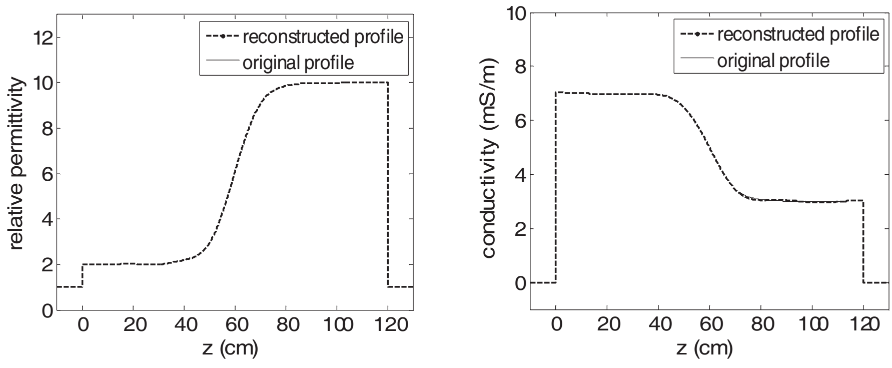

Figure 3. Reconstruction of permittivity and conductivity of tanh profile using cosine expansion model.

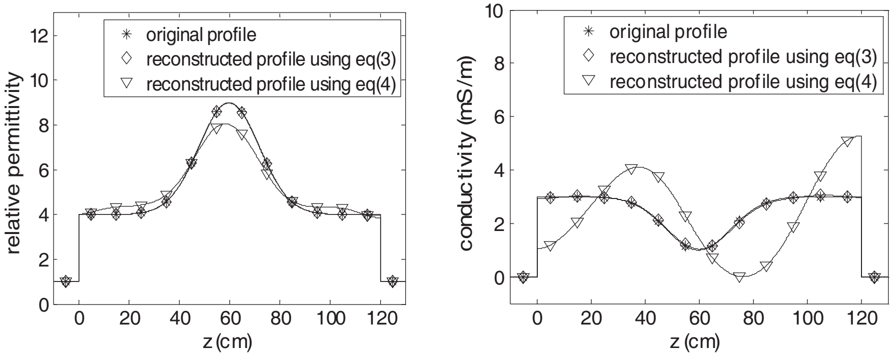

Figure 4. Reconstruction of permittivity and conductivity of gaussian profile using cosine expansion model and different error functions.

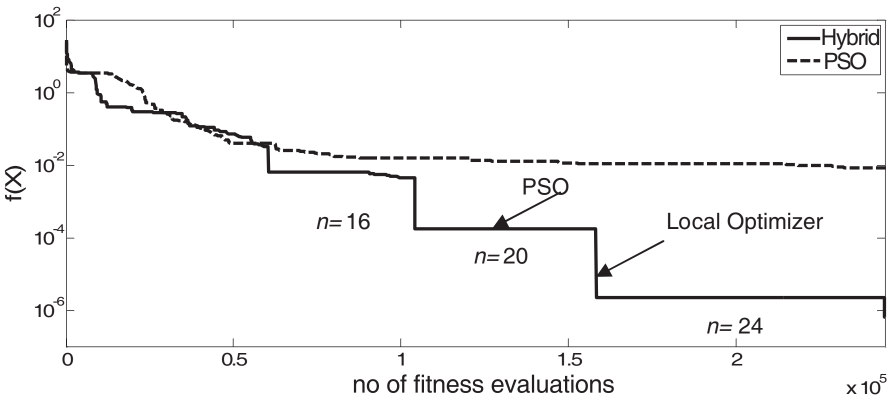

Figure 5. Value of fitness function vs. number of fitness evaluations in inversion of Gaussian slab using both hybrid algorithm and standard PSO.

reconstruction of the Gaussian profile using the proposed error function is much better than that of the conventional one.

The convergence status of the reconstruction process using the hybrid PSO algorithm is displayed together with that of the standard PSO one in Fig. 5 when reconstructing the gaussian slab. It is evident from this figure that the hybridization has much better effect on the speed of convergence.

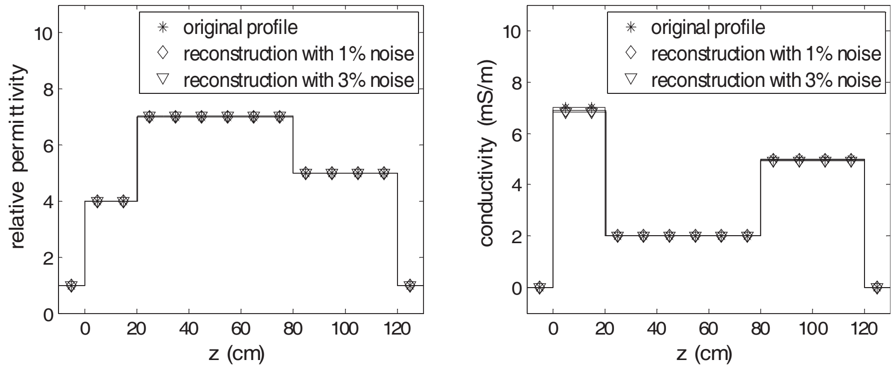

Figure 6. Inversion of discrete slab using noisy data for different noise levels.

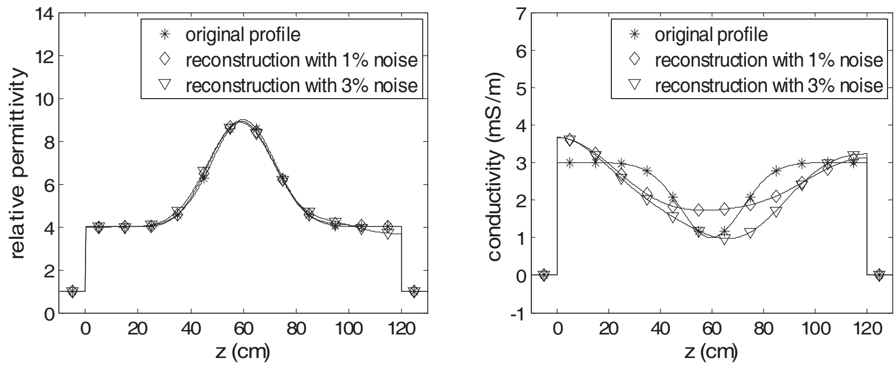

Figure 7. Inversion of the gaussian slab using noisy data for different noise levels.

6. EFFECT OF NOISE CONTAMINATION IN DATA

One of the major advantages of model based inversion methods is that its performance is superior to other inversion algorithms when the reconstruction data is contaminated with noise. This is because minimizing the error of Eq. (3) is acting as if we are making a curve fitting to find the model parameters that makes a reflection coefficient curve moving in between the noisy reflection coefficient data, and thus smoothing it and reducing the effect of noise. It is important to note that inversion of noisy data is enhanced by increasing the resolution of the data to obtain a more accurate average smoothing curve. Figs. 6 and 7 represent sample of results of applying the inversion algorithm to noisy data with 1% and 3% relative noise power level in the discrete and gaussian slabs.

From these two figures, one concludes that the discrete model is much more robust against noise than the continuous model. It is also observed that the conductivity profile is more sensitive to noise than the permittivity profile. This is expected since the loss tangent of these models is much smaller than unity in the specified frequency band. It is worth noting that the cases of large loss tangents values may lead to weak inversion of the deep regions in the slab due to large losses.

7. CONCLUSION

An inversion algorithm for the reconstruction of dielectric slab profiles has been proposed using the PSO. Inversion examples are presented for different lossy dielectric slab models. These models include both discrete and continuous models. The reflection simulated data are used in the VHF band for the specified slab dimensions. The proposed algorithm has proved to be successful, accurate and robust. The use of Quasi-Newon method together with the PSO is effective in saving a lot of computation time during the inversion process. The accuracy of inversion of the discrete models is relatively larger than that of the continuous models. It is also observed that the inversion of the permittivity profiles is more robust against noise from that of the conductivity profiles. The performance of the proposed algorithm is promising for the inversion of 2D and 3D models.

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