

SIDELOBES REDUCTION USING SIMPLE TWO AND TRI-STAGES NON LINEAR FREQUENCY MODULATION (NLFM)

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Abstract—The Linear Frequency Modulation (LFM) waveform is the most commonly and extensively used signal in practical radar system. However a compressed LFM signal at the receiver will produce the first sidelobe at a level of -13 dB to the peak of the main lobe. A weighting function is needed to apply in order to reduce the sidelobes. However, the penalty of mismatch loss is clearly evident. It may reduce output SNR, typically by 1 to 2 dB. Every single dB of additional SNR can have great effects in reducing false alarm rates in target detection applications. In an effort to achieve low autocorrelation sidelobe level without applying weighting function, Non-Linear Frequency Modulation (NLFM) signal has been investigated. This paper describes the sidelobe reduction techniques using simple two-stage FM waveform, modified two-stage FM waveform and tri-stage FM waveform. Simulation results of the proposed NLFM signal are presented. Sidelobe reduction of more than -19 dB can be achieved by this design without any weighting technique applied.

1. INTRODUCTION

The matched filter and pulse compression concepts are the basic of radar processing algorithms [1, 2]. Since radar return is always susceptible to noise and interference from all kinds of objects illuminated by the antenna beam, the receiver must be optimized. The matched filter is a filter whose impulse response, or transfer function is determined by a certain signal, in a way that will result in the maximum attainable signal to noise ratio [3–6]. Pulse compression

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involves using a matched filter to compress the energy in a signal into a relative narrow pulse.

Pulse compression is a classical signal processing technique to increase the range resolution of transmitted pulse without having to increase the peak transmit power [1, 3, 5, 7]. Instead of a fixed frequency pulse, the transmitted pulse is modulated by a specific phase or frequency pattern during a wider pulse interval. The receiver uses a pulse-matched filter to pass reflected pulses that match the pattern of the outgoing pulse and at the same time, reject noise and other signals.

The LFM, or chirp waveform, has superior performance in pulse compression radar since they can be easily generated and processed. Many diverse techniques and devices have been developed to provide the required pulse compression processing for these signals [2]. However, the LFM has large sidelobes with respect to the mainlobe. Reducing the sidelobes can be accomplished by linear filtering the output, i.e., applying window functions or data tapering. However, since the cumulative filtering is no longer precisely matched to the signal, it necessarily reduces output SNR as well, typically by 1–2 dB.

The non-linear frequency modulation (NLFM) signal is another continuous phase modulation waveform applicable to pulse compression radar. It has been claimed to provide range sidelobe suppression since the modulation is designed to provide the desired amplitude spectrum with reduced sidelobes [3, 8–10]. The NLFM requires no time or frequency weighting for range sidelobe suppression.

2. LINEAR FREQUENCY MODULATION (LFM) WAVEFORM

The LFM waveform is the most widely discussed pulse compression waveform in the literature and most extensively used in practice [3, 4, 11]. The phase function of the LFM signal consists of a quadratic phase coefficient k which results in a linear frequency over the duration of the signal. Thus the chirp waveform can be described by the $\text{Re}\{s(t)\}$ with

$$\begin{aligned} s(t) &= \exp \left\{ j2\pi \left(f_c t - \frac{1}{2} k t^2 \right) \right\}, & -\tau/2 \leq t \leq \tau/2 \\ &= 0, & \text{elsewhere} \end{aligned} \quad (1)$$

where f_c is the carrier frequency of transmitted waveform, k is the chirp rate or the sweep rate of the waveform. The bandwidth of the signal is given by

$$B = k\tau \quad (2)$$

The instantaneous frequency $f(t)$ can be obtained by differentiating the argument of the exponential in Equation (3),

$$f(t) = \frac{1}{2\pi} \frac{d(2\pi(f_c t + kt^2/2))}{dt} = f_c + kt \quad (3)$$

Although the LFM signal has a duration of τ , it can behave like a pulse with duration equivalent to the inverse of its bandwidth, i.e., $\tau_{eq} = 1/B$. The signal processing that allows this to happen is known as pulse compression. The amount of this compression is given by $\tau/\tau_{eq} = \tau B = D$, which is the *time-bandwidth product* of the waveform. The LFM chirp exhibits the interesting property of possessing very large time-bandwidth products [4, 6]. The transform of the chirp is essentially flat over its range of frequencies. When the time-bandwidth product of the LFM signal increase, the signal's spectrum shape will become more rectangular.

The matched filter of the LFM has an impulse response, $h(t)$, that is time inverse of the signal at receiver input as given below

$$h(t) = K \cos \left\{ 2\pi \left(f_c t - \frac{1}{2} k t^2 \right) \right\}, \quad -\tau/2 \leq t \leq \tau/2 \quad (4)$$

where K is factor that result in unity gain. Since the echo from the target at time t_o is delayed replica of the transmitted signal, the return signal is given as

$$s(t_o) = \cos \left\{ 2\pi (f_c + f_d) t_o + \frac{1}{2} k t_o^2 \right\} \quad (5)$$

where $s(t_o)$ represent the return echo from target and f_d is the shifted in frequency cause by the Doppler effect.

From above equation, the general output of matched filter can be written as

$$\begin{aligned} g(t_o, \omega_d) = & K \int_{-\tau/2}^{\tau/2} \cos \left\{ (\omega_c + \omega_d) t + \frac{1}{2} k (2\pi) t^2 \right\} \cos \left\{ \omega_c (t_o - t) \right. \\ & \left. + \frac{1}{2} k (2\pi) (t_o - t)^2 \right\} dt \end{aligned} \quad (6)$$

where $\omega_c = 2\pi f_c$ and $\omega_d = 2\pi f_d$.

The closed form solution of above Equation can be obtained through a considerable amount of trigonometric and algebraic manipulation. The result of this calculation is given as,

$$g(t_o, \omega_d) = G \cos \left\{ \left(\omega_o + \frac{\omega_d}{2} \right) t_o \right\} \frac{\sin \left\{ \frac{\omega_d + 2\pi k t_o}{2} (\tau - |t_o|) \right\}}{\frac{\omega_d + 2\pi k t_o}{2}}, \quad -\tau/2 \leq t \leq \tau/2 \quad (7)$$

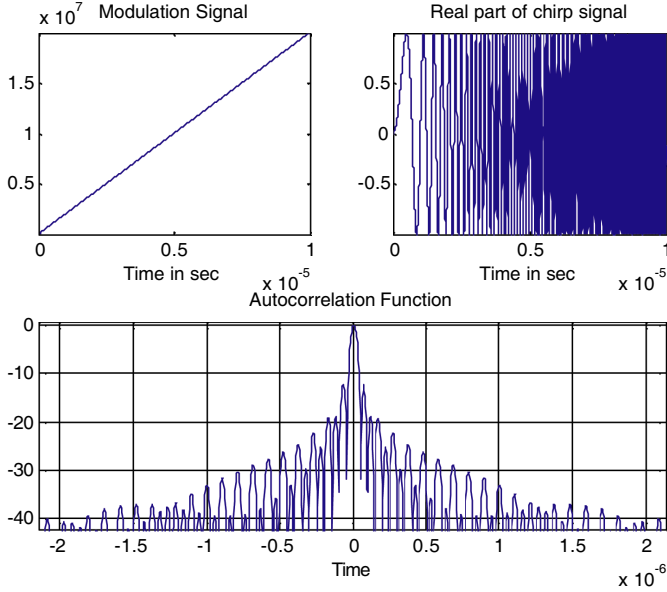


Figure 1. Linear FM waveform.

where $|t_o|$ is the absolute value of t_o . Note that the matched filter output is in the form of $\frac{\sin X}{X}$ (sinc X). The sidelobe levels of LFM correlation function is approximate -13.2 dB. Figure 1 shows the plots of LFM signal, the modulation signal and its auto-correlation function. The LFM signal is generated based on the specification of pulse width $10 \mu\text{sec}$ and bandwidth of 20 MHz.

LFM increases the bandwidth and subsequently improved the range resolution of the signal by a factor equal to the time bandwidth product. However, relatively high sidelobes remain in the autocorrelation function. Such autocorrelation function is unacceptable in some radar applications, where more than one target is present, giving rise to echo of different amplitudes. Three major techniques have been implemented to obtain lower sidelobes level, i.e., weighting in time domain, frequency domain weighting and NLFM [12, 13].

Time domain weighting is equals to the amplitude modulation of the transmitted signal. However such implementation will lead to reduction of transmitted power, and therefore a signal to noise ratio loss will occur. Frequency domain weighting spectrum shaping using well knows weighting windows such as Hann and Hamming [13]. However the implementation of weighting windows may lead to the

penalty of mainlobe broadening. NLFM waveform is designed such that its matched filter response satisfies the sidelobe requirements. Since the receiver is matched with to the signal shape, no mismatch losses as in time domain weighting and frequency domain weighting.

3. NON-LINEAR FREQUENCY MODULATION (NLFM) WAVEFORM

NLFM is a general class of continuous phase coding in which the sweep rate is not restricted to a constant as compare with LFM. For an arbitrary FM modulation signal,

$$s(t) = \exp[j\phi(t)] \quad (8)$$

where $\phi(t)$ is the frequency modulation function. Differential the phase modulation function will obtain $f(t)$, the instantaneous frequency of the NLFM signal. Figure 2 shows one of the example of NLFM waveform; the Cosine Modulation signal, its waveform and its autocorrelation function.

NLFM signal has been investigated in an effort to overcome the mismatch loss but still achieve low auto correlation sidelobe levels. However, it was found that most of the NLFM signal is not able to

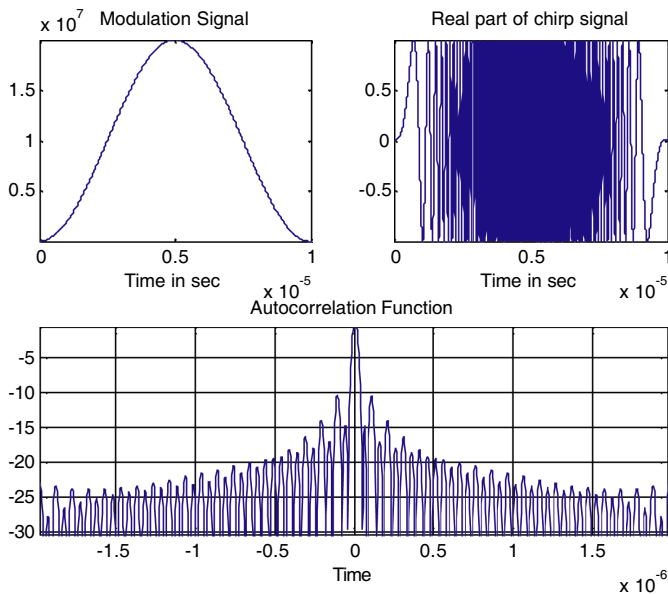


Figure 2. Cosine modulation.

provide sufficient sidelobe reduction. Only special designed NLFM modulation signal is able to obtained lower sidelobe levels. Precision NLFM signal are more difficult to design. Windowing or Weighting function has been design and used to reduce the sidelobes [14–17].

3.1. Simple Two Stages NLFM

A simple two stage NLFM signal has been investigated. This NLFM modulation function has two LFM stages as shown in Figure 3. It can be divided into two parts which has two distinct LFM sweep rate. The frequency of each part is linearly swept through the given time frame. The instantaneous frequency of this NLFM can be written as,

$$f(t) = \begin{cases} \frac{B_1}{T_1}t, & 0 \leq t \leq T_1 \\ B_1 + \frac{B_2}{T_2}t, & T_1 \leq t \leq T_2 \end{cases} \quad (9)$$

Thus, the phase of this two stages NLFM signal can be derived by integrating Equation (9).

$$\phi(t) = \int f(t)dt = \begin{cases} \frac{B_1}{T_1} \times \frac{t^2}{2}, & 0 \leq t \leq T_1 \\ B_1 \times t + \frac{B_2}{T_2} \times \frac{t^2}{2}, & T_1 \leq t \leq T_2 \end{cases} \quad (10)$$

A few simulations have been carried out for $T = 10\mu\text{s}$ and $B = 20\text{ MHz}$ with different values of T_1 , T_2 , B_1 and B_2 . Several

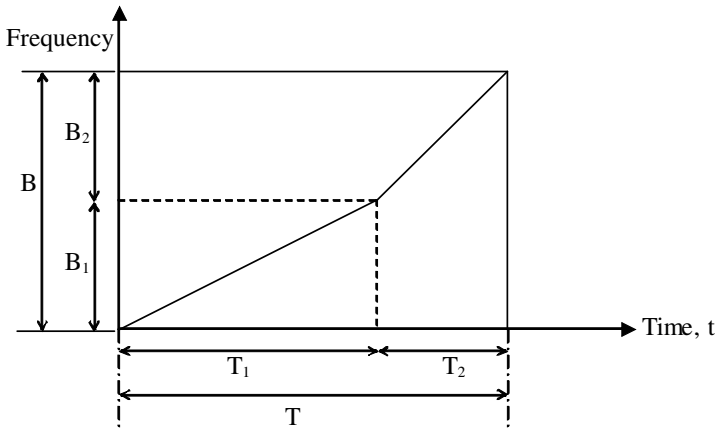


Figure 3. Modulation signal for simple two stages NLFM waveform.

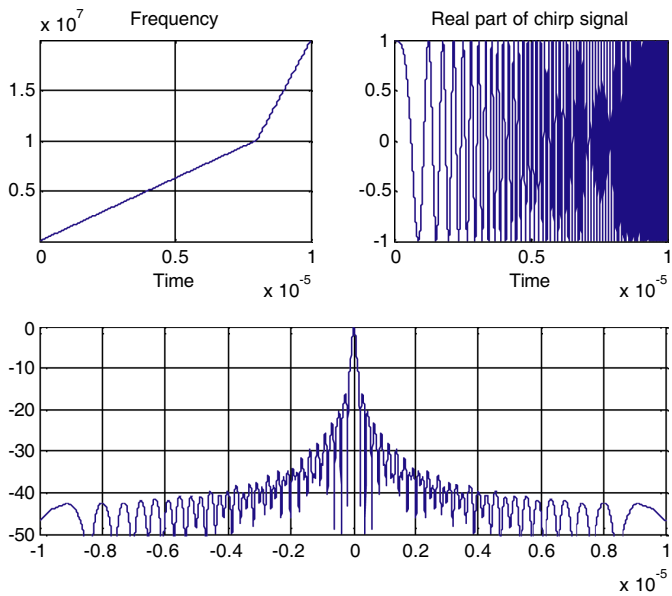


Figure 4. Two stages NLFM waveform response for configuration 1.

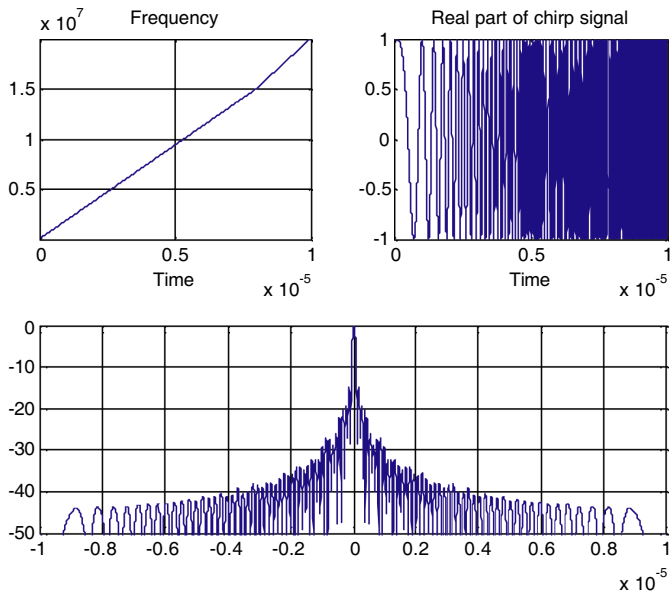


Figure 5. Two stages NLFM Waveform response for configuration 2.

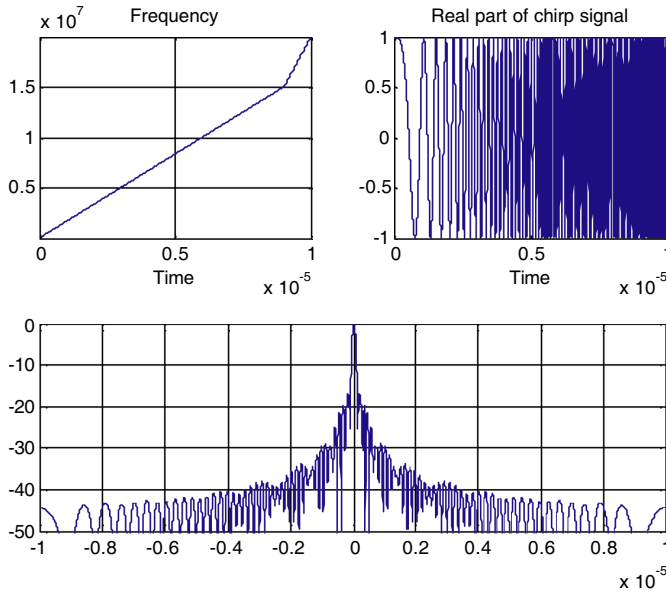


Figure 6. Two stages NLFM waveform response for configuration 3.

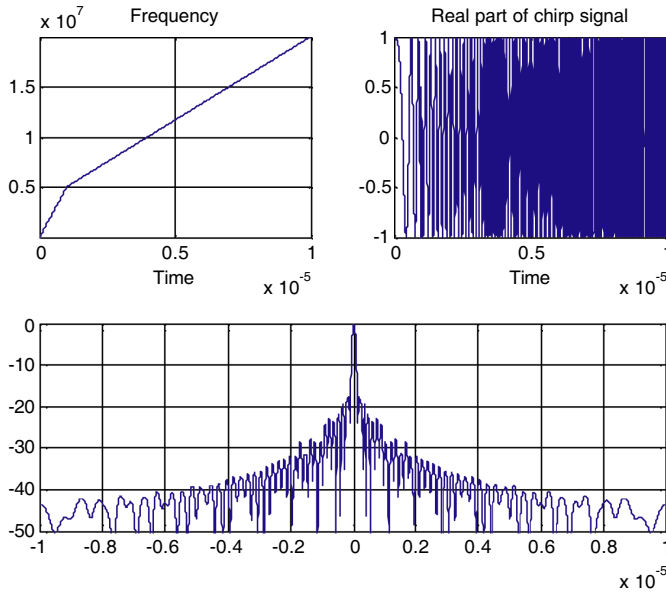


Figure 7. Two stages NLFM waveform response for configuration 4.

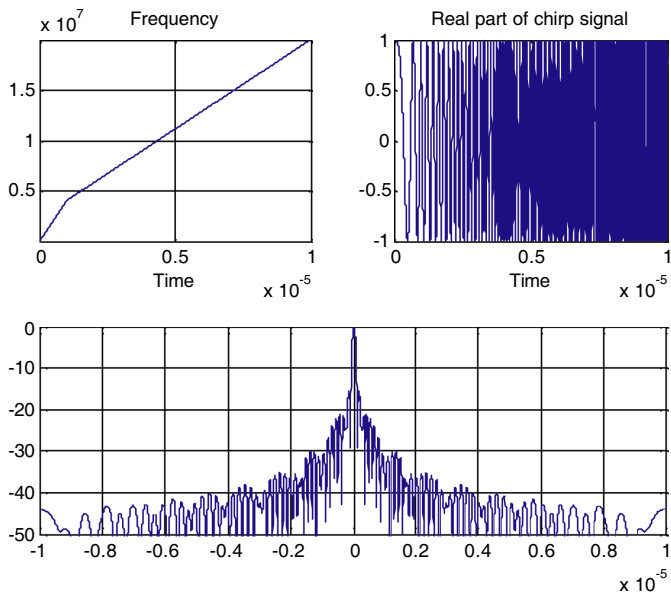


Figure 8. Two stages NLFM waveform response for configuration 5.

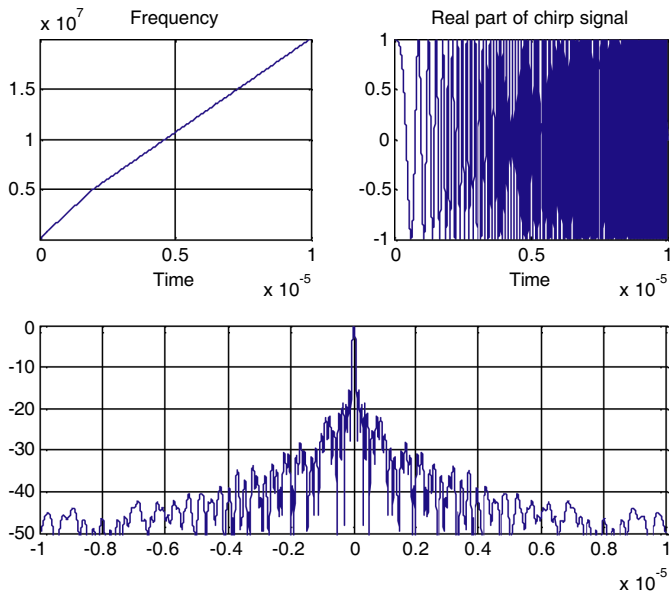


Figure 9. Two stages NLFM waveform response for configuration 6.

potentially suitable functions have been investigated with varying the parameters stated above. Figure 4 to Figure 9 show some of the promising functions and their corresponding autocorrelation functions output. The highest sidelobe level for each of the configuration is listed in Table 1.

Table 1. Highest sidelobe level for two-stages LFM signal.

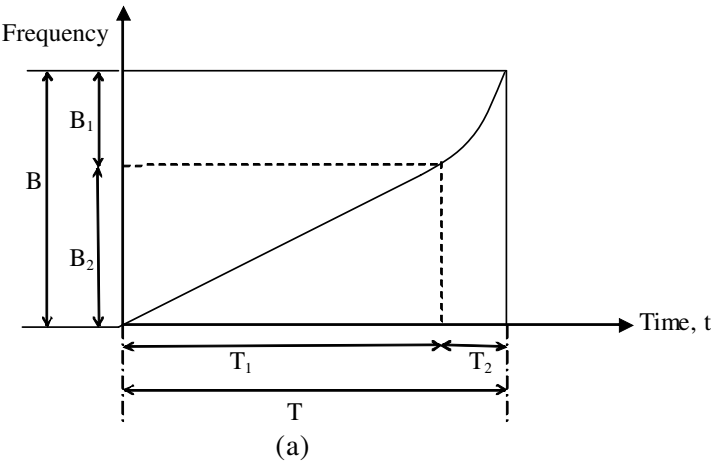
| Configuration | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------------|--------|--------|-------|-------|-------|-------|
| Sidelobe Level (dB) | -16.15 | -14.88 | -16.8 | -17.7 | -15.4 | -15.4 |

3.2. Modified Two Stage NLFM Signal

Instead of LFM sweep for both segments, one of them can be modulated using polynomial function of order 3 to 5. Two possible NLFM modulation signals are shown in Figure 10. It is a combination of LFM chirp signal followed by a polynomial function or LFM chirp signal after a polynomial function. Both configurations are able to produce promising low sidelobes as shown in Figure 11 to Figure 16.

Table 2. Highest sidelobe level for modified two stages LFM signal.

| Configuration | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------------|--------|-------|-------|-------|--------|-------|
| Sidelobe Level (dB) | -15.65 | -15.4 | -19.2 | -16.8 | -14.78 | -16.5 |



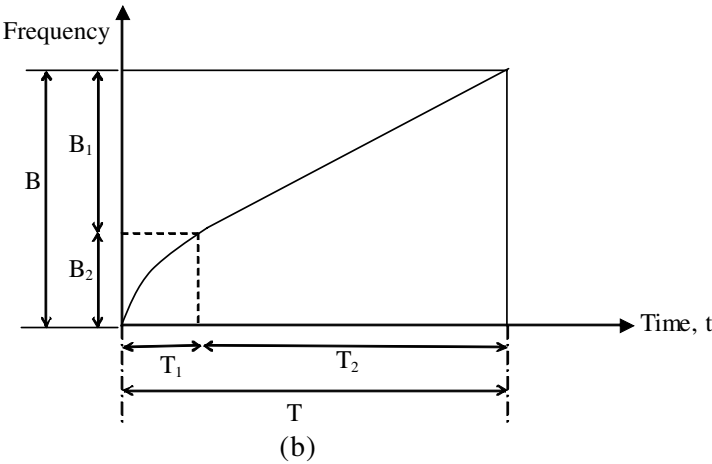


Figure 10. Modulation signal for modified two stage LFM signal.

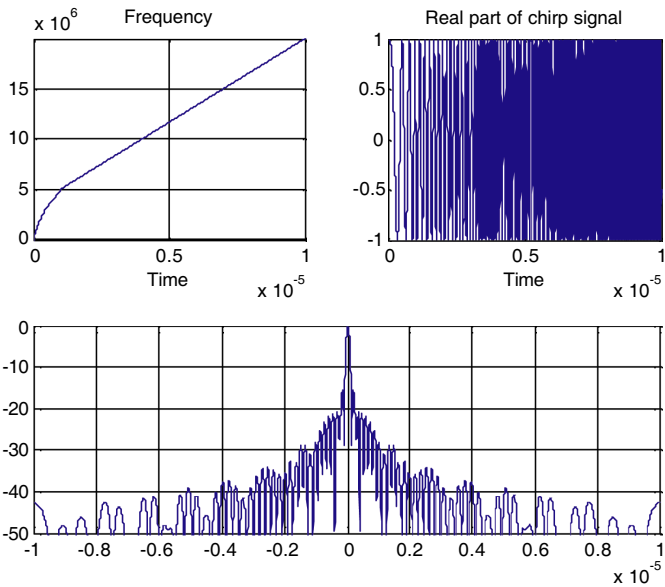


Figure 11. Modified two stages NLFM waveform response for configure 1.

All of them are simulated with $T = 10 \mu\text{s}$ and $B = 20 \text{ MHz}$ but with different LFM sweep rates and polynomial functions. Table 2 lists the corresponding sidelobe level of some potentially NLFM functions which may apply to modern radar system for the aid of better performance.

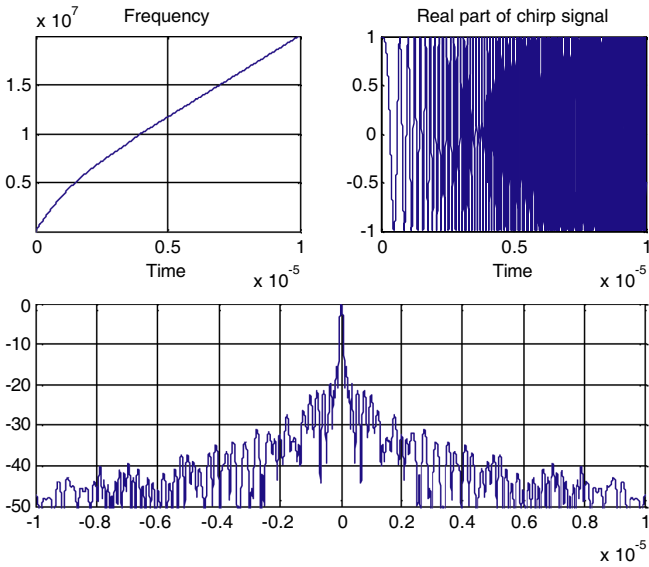


Figure 12. Modified two stages NLFM waveform response for configuration 2.

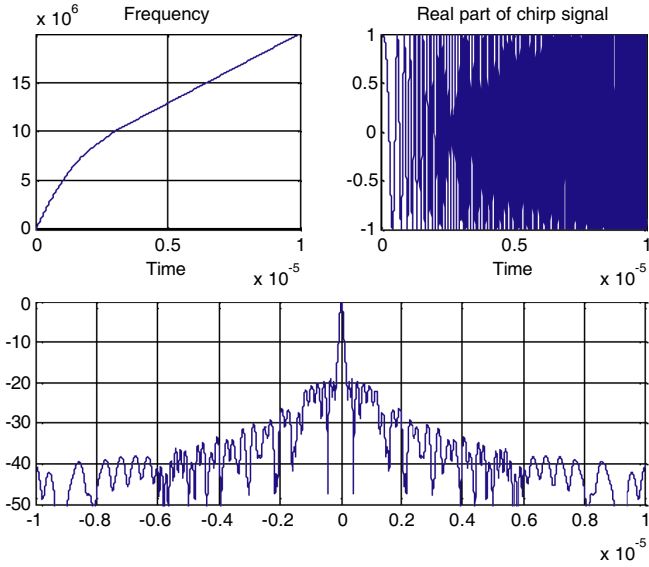


Figure 13. Modified two stages NLFM waveform response for configuration 3.

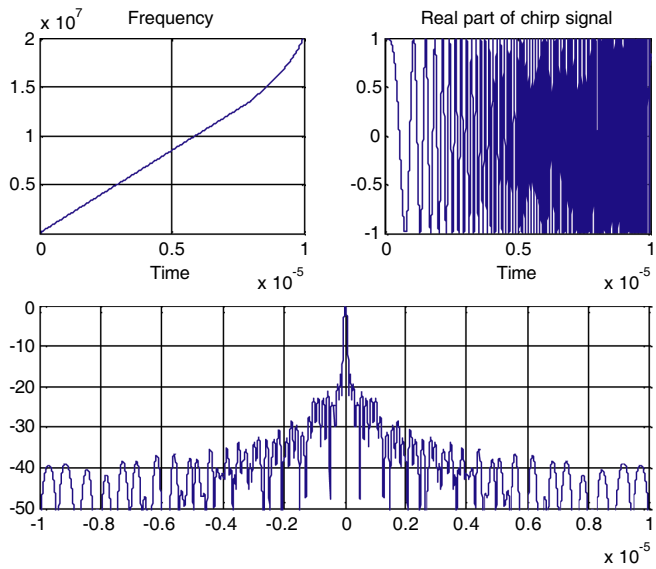


Figure 14. Modified two stages NLFM waveform response for configuration 4.

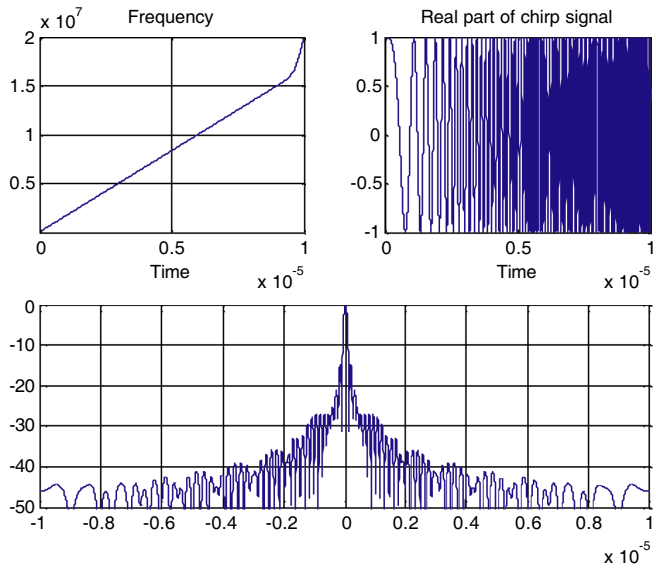


Figure 15. Modified two stages NLFM waveform response for configuration 5.

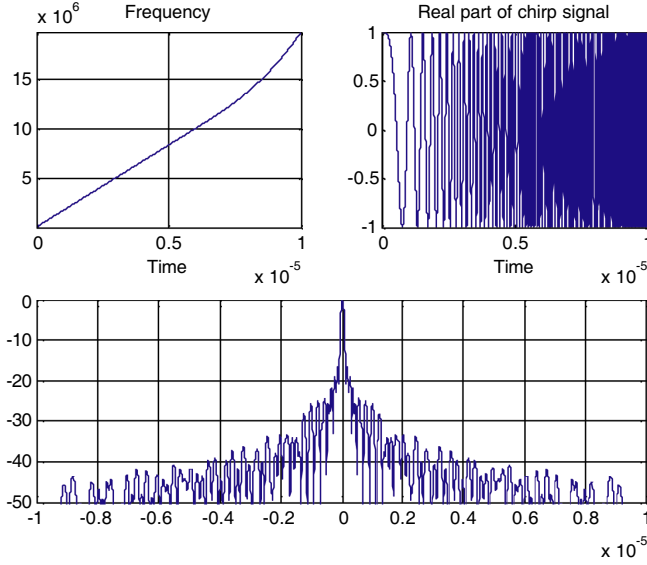


Figure 16. Modified two stages NLFM waveform response for configure 6.

3.3. Tri-stages NLFM

Beside the possible configurations shown in Section 3.1 and Section 3.2, another possible FM modulation i.e., Tri-stages NLFM modulation signal can be applied in order to suppress the sidelobe level. Its general waveform is shown in Figure 17, which can be subdivided into three stages. Each of the segments is linearly sweep with a particular chirp rate. It composes of two LFM segments that increase the modulation rate at the leading and trailing edges of the waveform.

The instantaneous frequency of this NLFM can be written as,

$$f(t) = \begin{cases} \frac{B_1}{T_1}t, & 0 \leq t \leq T_1 \\ B_1 + \frac{B_2}{T_2}t, & T_1 \leq t \leq T_2 \\ B_1 + B_2 + \frac{B_3}{T_3}t, & T_2 \leq t \leq T_3 \end{cases} \quad (11)$$

Thus, the phase of this two stages NLFM signal can be derived by

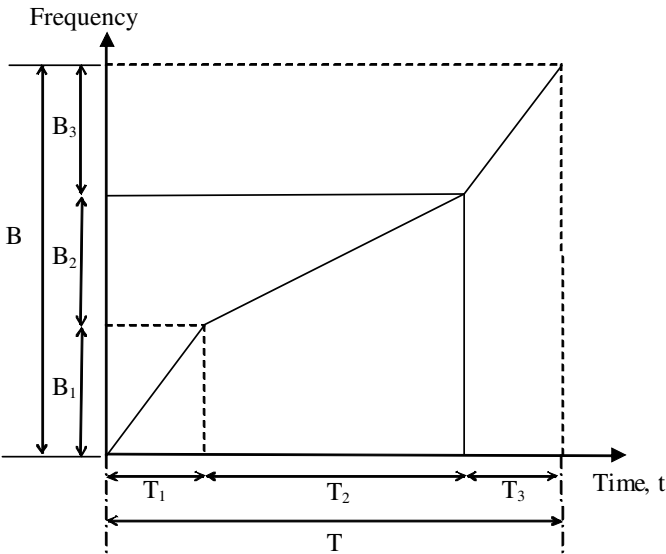


Figure 17. Modulation signal for Tri-stages NLFM waveform.

Table 3. Highest sidelobe level for Tri-stages LFM signal.

| Configuration | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------------|--------|--------|--------|-------|--------|--------|
| Sidelobe Level (dB) | -14.26 | -17.37 | -17.35 | -19.2 | -18.45 | -17.13 |

integrating Equation (10).

$$\phi(t) = \int f(t)dt = \begin{cases} \frac{B_1}{T_1} \times \frac{t^2}{2}, & 0 \leq t \leq T_1 \\ B_1 \times t + \frac{B_2}{T_2} \times \frac{t^2}{2}, & T_1 \leq t \leq T_2 \\ B_1 \times t + B_2 \times t + \frac{B_3}{T_3} \times \frac{t^2}{2}, & T_2 \leq t \leq T_3 \end{cases} \quad (12)$$

Simulations have been done with various configurations. It can be shown that the tri-stage NLFM waveform is able to produce lower sidelobe as compared to conventional LFM signal. Some of the simulation results are shown in Figures 18–23. It can be observed that some of the configurations are able to suppress the sidelobe for better radar target detection. Table 3 listed the highest sidelobe level of autocorrelation function for some potentially tri-stage NLFM function.

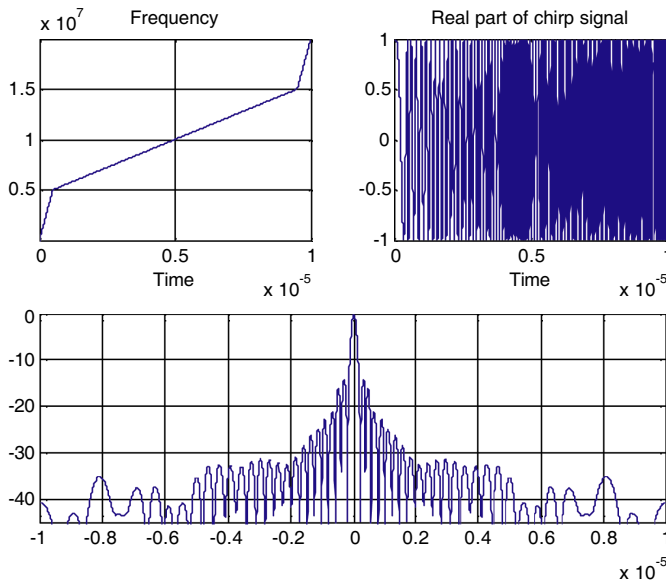


Figure 18. Tri-stages NLFM waveform response for configuration 1.

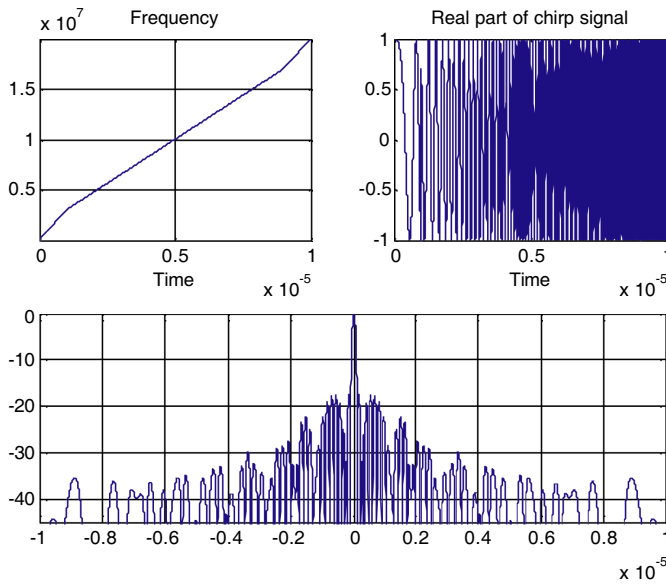


Figure 19. Tri-stages NLFM waveform response for configuration 2.

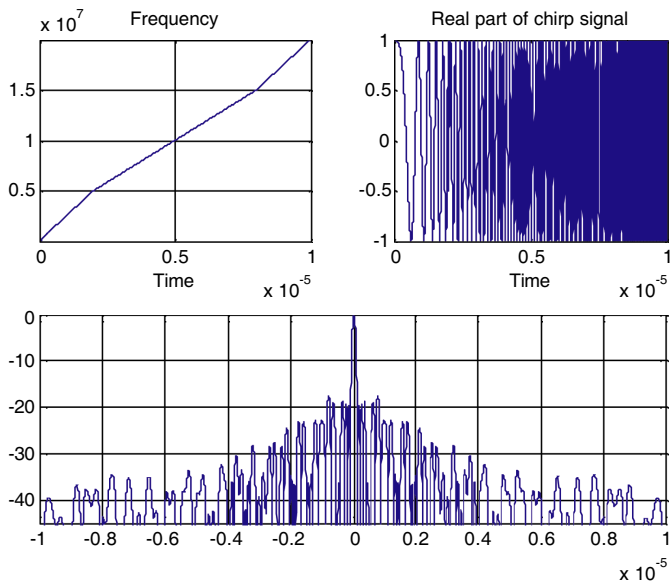


Figure 20. Tri-stages NLFM waveform response for configuration 3.

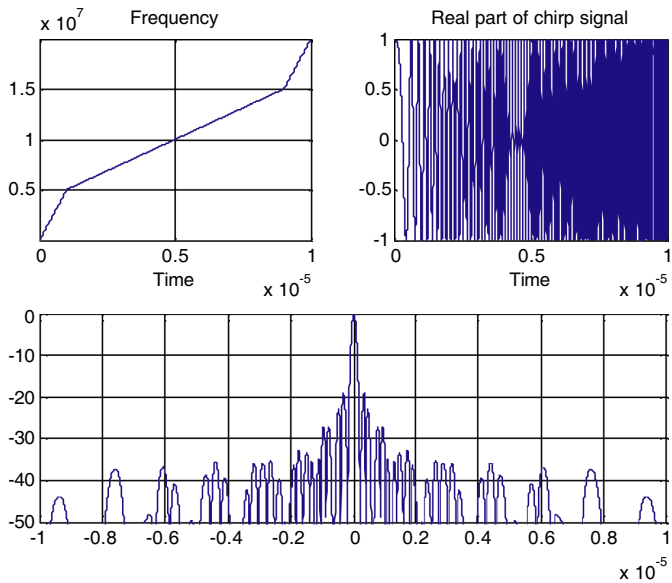


Figure 21. Tri-stages NLFM waveform response for configuration 4.

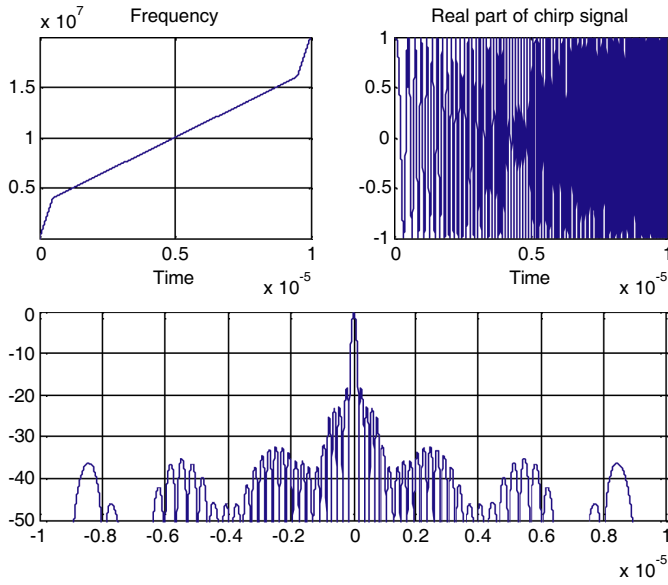


Figure 22. Tri-stages NLFM waveform response for configuration 5.

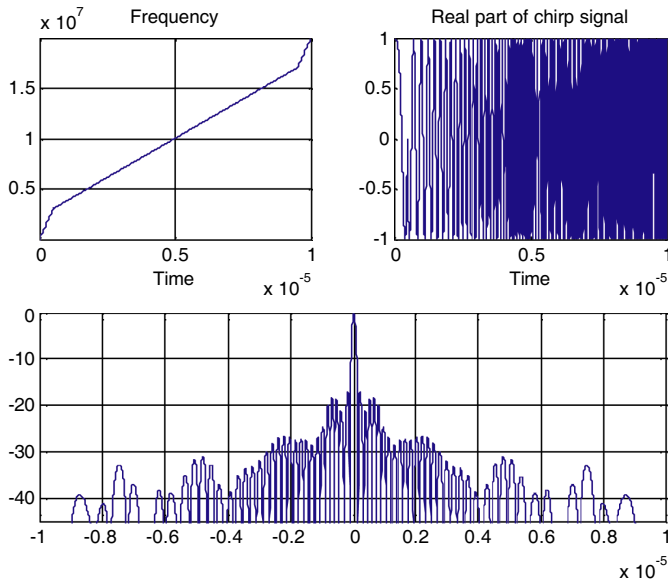


Figure 23. Tri-Stages NLFM waveform response for configuration 6.

4. CONCLUSION

This paper presents the detailed description of LFM signal in term of match filter response and pulse compression. The main characteristics of LFM waveform include linear frequency ramp, flat topped spectrum and its autocorrelation function can be approximated by a sinc function with -13.3 dB sidelobes.

The Two-stages LFM functions are attractive since they are capable of reducing sidelobes level with simple implementation scheme. The simulation results show an improvement of 1.58 dB to 4.4 dB depends on the configuration applied. It has been shown that the Modified Two-stages NLFM signal capable of achieving better sidelobe reduction as compared to simple Two-stages NLFM signal. A highest sidelobe suppression of -19.2 dB can be achieved. Beside, the Tri-stages NLFM signal is investigated with various configuration and the simulation results show the autocorrelation function exhibits attenuated sidelobes to less than -19 dB in one of the NLFM waveform.

In summary, the NLFM has been demonstrated to be an effective technique for sidelobes suppression. More importantly, its implementation scheme is simple to achieve without any SNR-robbing sidelobe filtering or window functions.

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