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THE PROPAGATION AND CUTOFF FREQUENCIES OF THE RECTANGULAR METALLIC WAVEGUIDE PARTIALLY FILLED WITH METAMATERIAL MULTILAYER SLABS

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Abstract—In this paper, the wave propagation and the cutoff frequencies of a rectangular metallic waveguide, partially filled the metamaterial multilayer slabs have been studied. The equations of the TMM method are not complex and the numerical examples show that we can easily obtain the characteristics of the metamaterial multilayer's rectangular waveguide satisfyingly. The cutoff frequencies of the metamaterial waveguide show very different characteristics compared with the usual waveguide.

1. INTRODUCTION

In 1968, Veselago [1] proposed the concept of left-handed medium (LHM) theoretically and predicted many unusual physical properties such as the plane-wave propagation exhibited by the negative refraction media in which permittivity and permeability are both negative. These materials have been termed as metamaterials, left-handed materials, backward-wave materials and so on. People have been proposed many applications, such as a thin sub wavelength cavity resonators contained with metamaterials. Many authors have studied the guiding devices using metamaterials. The wave-guide properties of a planar two-layered wave-guide, one magnetodielectric and the other metamaterial have been theoretically considered [2]. Eleftheriades [3] has presented experimental verification of focusing using an implementation of artificial transmission-line media in planar form. Alu [4] has analysed wave propagation in a parallel-plate waveguide filled with a pair of

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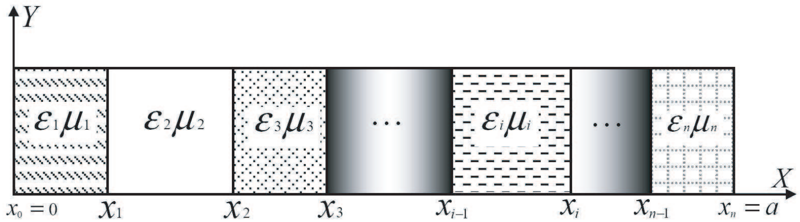


Figure 1. Geometry of a waveguide filled with N layers metamaterial multilayer slabs.

lossless slabs. Krowne [5] has studied a microstrip structure containing a metamaterial. Hrabar [6] has analyzed a rectangular metallic waveguide filled with metamaterial. H. Cory [7] has studied the wave propagation in a rectangular metallic waveguide, loaded with only one layer longitudinal metamaterial slab adjacent to air.

In this paper, we have developed the transfer matrix method (TMM) to study a rectangular metallic waveguide partially filled with the metamaterial multilayer slabs. By using the TMM, the values of the field from one boundary are transmitted to another by involving multiplication of transfer matrixes only. The order of the solved matrixes employed in the TMM is still two and an iterative process is not required. The propagation and the cutoff frequencies have been obtained for amount of the metamaterial multilayer slabs. Although only the TE mode has been studied here, the TM mode case can be treated in a similar way.

2. FORMULATION

The structure is shown in Fig. 1. The waveguide has N layers slabs in which layer is filled with metamaterial or normal materials. In this example, the first layer is filled by air, the second layer is a metamaterial-filled layer, then the third layer is air-filled, the fourth is a metamaterial layer and so on in the whole structure. The metamaterial layers' permittivity and permeability are $\varepsilon_i = -\varepsilon_r \varepsilon_o$, and $\mu_i = -\mu_0$, $i = 2, 4, 6, \ldots$. From the Solution to Maxwell's equation, we know the Borgnis'method [8], about the x direction, each components can be expressed as:

$$E_x = (k^2 - k_x^2)U \quad E_y = \frac{\partial^2 U}{\partial y \partial x} - j\omega\mu \frac{\partial V}{\partial z} \quad E_z = \frac{\partial^2 U}{\partial z \partial x} + j\omega\mu \frac{\partial V}{\partial y} \tag{1}$$

$$H_x = (k^2 - k_x^2)V \quad H_y = \frac{\partial^2 V}{\partial y \partial x} + j\omega\varepsilon \frac{\partial U}{\partial z} \quad H_z = \frac{\partial^2 V}{\partial z \partial x} - j\omega\varepsilon \frac{\partial U}{\partial z} \tag{2}$$

U and V are the functions referred in Borgnis' method [8]. If $U = 0$, they can be called LSE (TEx) modes. While if $V = 0$, they can be called LSM (TMx) modes. When discussing LSE (TEx) modes, we can easily find

$$E_x = 0 \qquad E_y = -j\omega\mu \frac{\partial V}{\partial z} \qquad E_z = j\omega\mu \frac{\partial V}{\partial y} \tag{3}$$

$$H_x = (k^2 - k_x^2)V \quad H_z = \frac{\partial^2 V}{\partial z \partial x} \qquad H_y = \frac{\partial^2 V}{\partial y \partial x} \tag{4}$$

And we can obtain:

$$H_z = \frac{\partial^2 V}{\partial z \partial x} = \frac{j}{\omega\mu} \frac{dE_y}{dx} \quad H_y = \frac{\partial^2 V}{\partial y \partial x} = \frac{j}{\omega\mu} \frac{dE_z}{dx} \tag{5}$$

Using the same method, we can obtain the formula about LSM (TMx) modes as following:

$$E_z = \frac{\partial^2 U}{\partial z \partial x} = \frac{j}{\omega\varepsilon} \frac{dH_y}{dx} \quad E_y = \frac{\partial^2 U}{\partial y \partial x} = \frac{j}{\omega\varepsilon} \frac{dH_z}{dx} \tag{6}$$

For simplicity, we discuss LSE (TEx) modes and suppose $k_y = 0$, the electric field E_y components are given as follows:

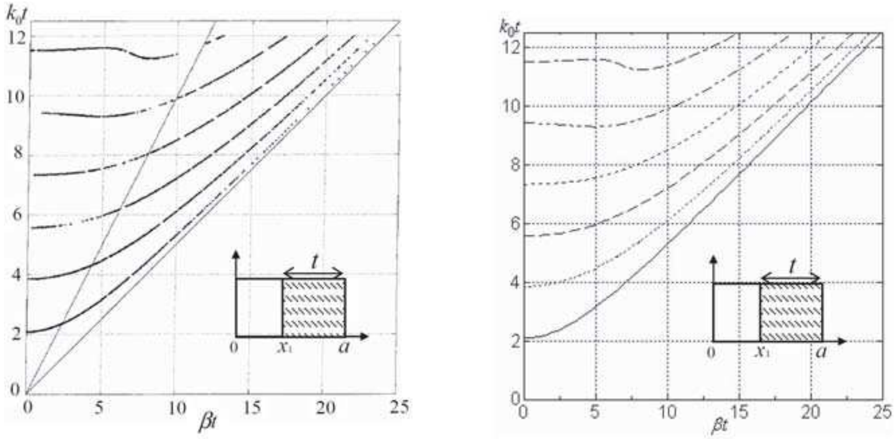
$$\begin{aligned} E_i(x) &= A_i \sin(k_i x) + B_i \cos(k_i x), \\ \frac{dE_i(x)}{dx} &= k_i (A_i \cos(k_i x) - B_i \sin(k_i x)) \end{aligned} \tag{7}$$

A_i and B_i, are the constants to be determined, and $k_i^2 = \omega^2 \mu_i \varepsilon_i - \beta^2$.

By applying the boundary condition between every two neighboring layers, the coefficients A_i, B_i for all layers can be connected. From the Eq. (5), we can know $H_z \propto dE_y/d_x$. By using the field continuity conditions at interfaces $x = x_{i-1}$ and $x = x_i$, the coefficients A_i, B_i and the field values of the neighboring layers are connected as follows:

$$\begin{aligned} \begin{bmatrix} A_i \\ B_i \end{bmatrix} &= [M_i(x_{i-1})]^{-1} [M_{i-1}(x_{i-1})] \begin{bmatrix} A_{i-1} \\ B_{i-1} \end{bmatrix} \\ \begin{bmatrix} E_i(x_i) \\ \frac{dE_i(x_i)}{dx} \end{bmatrix} &= [M_i(x_i)] [M_i(x_{i-1})]^{-1} \begin{bmatrix} E_{i-1}(x_{i-1}) \\ \frac{dE_{i-1}(x_{i-1})}{dx} \end{bmatrix} \end{aligned} \tag{8}$$

$$[M_i(x_i)] = \begin{bmatrix} \sin(k_i x_{i-1}) & \cos(k_i x_{i-1}) \\ k_i \cos(k_i x_{i-1}) & -k_i \sin(k_i x_{i-1}) \end{bmatrix}, \quad i = 1, 2, \ldots, n \tag{9}$$



(a) the result in Ref. 7

(b) the present method

Figure 2. The propagation coefficients at $t = 3/4a$ when $N = 2$.

Repeated applications of Equation (8) throughout all the layers lead to the connections of the coefficients in the first layer A_1 and B_1 to the coefficients A_n and B_n in the last layer. Shown in the following,

$$\begin{bmatrix} E_n(x_n) \\ \frac{dE_n(x_n)}{dx} \end{bmatrix} = \prod_{i=1}^{n} \left\{ [M_i(x_i)] \, [M_i(x_{i-1})]^{-1} \right\} \begin{bmatrix} E_0(x_0) \\ \frac{dE_0(x_0)}{dx} \end{bmatrix} \tag{10}$$

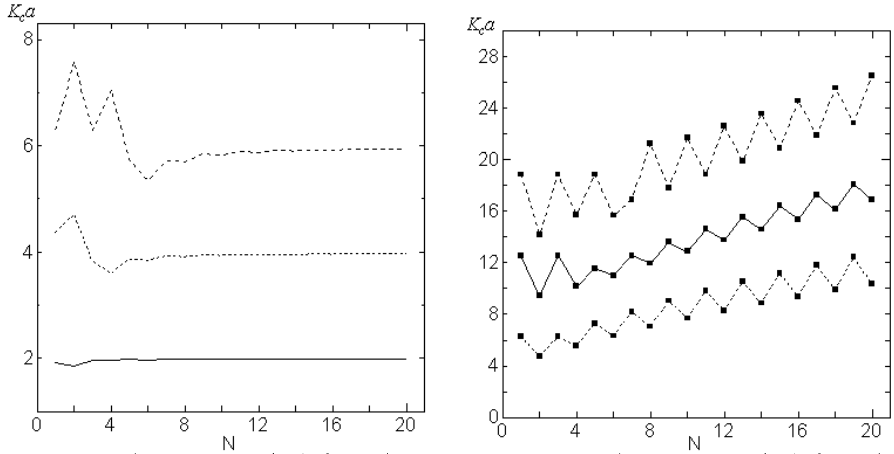
$$\begin{bmatrix} E_n(a) \\ \frac{dE_n(a)}{dx} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} E_0(0) \\ \frac{dE_0(0)}{dx} \end{bmatrix} \tag{11}$$

Using the boundary conditions at $x_0 = 0$ and $x_n = a$, we know $E_n(a) = E_0(0) = 0$, then $M_{12} = 0$. We can get the cutoff frequencies and dispersion relationships.

3. NUMERICAL RESULTS

In order to validate the proposed method, first we studied a structure shown in Fig. 2, $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$ and $\varepsilon_2 = -4\varepsilon$, $\mu_2 = -\mu_0$ at $t = 3/4a$, we showed the Graphs of k_0t versus t when the layers' number is $N = 2$ in Fig. 2, compared with the results calculated by usual analysis method in the Ref. [7], it is found that the agreement between two methods are very good, we can say that the present methods is effective for this problem.

Last we calculated the cutoff frequencies of the metamaterial multilayer waveguide. The results have been shown in Fig. 3, when every layer's width has a relation to $x_i - x_{i-1} = a/N$, $(i = 1, 2, 3, \ldots, n)$



(a) $\varepsilon_i = 4\varepsilon_0,\ \mu_i = \mu_0\ (i = 1, 3, 5\cdots)$
$\varepsilon_i = \varepsilon_0,\ \mu_i = \mu_0\ (i = 2, 4, 6\cdots)$

(b) $\varepsilon_i = -4\varepsilon_0,\ \mu_i = -\mu_0\ (i = 1, 3, 5\cdots)$
$\varepsilon_i = \varepsilon_0,\ \mu_i = \mu_0\ (i = 2, 4, 6\cdots)$

Figure 3. The cut-off frequencies in the waveguide loaded with N-layers' slabs when $x_i - x_{i-1} = a/N, (i = 1, 2, 3, \ldots, n)$.

We can see that when the layers' number N is added, the cutoff frequencies do not get convergent in the metamaterial filled waveguide, while the all normal material filled waveguide is going to a constant. We may say when the layers' number N is added, the normal material filled waveguide's permittivity and permeability can be approximated to a constant, while the metamaterial mixed with normal material filled waveguide is difficult to be done in this way.

4. CONCLUSION

We have studied the propagation and cutoff frequencies of the rectangular metallic waveguide partially filled the metamaterial multilayer by the method called TMM which is rapidly and satisfying for this problem. The waveguide loaded with the metamaterial multilayer displayed very interesting and different characteristics compared with the usual waveguide.

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