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BAND STRUCTURE AND DISPERSION PROPERTIES OF PHOTONIC QUASICRYSTALS

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Abstract—In this paper, for developing analytical and semi-analytical methods to evaluate band structure in photonic quasicrystals the perturbation theory is examined. It is shown that more isotropic and complete photonic band gap can be observed under low dielectric contrast for photonic quasicrystals in comparison with ordinary crystals and because of this feature of photonic quasicrystals, perturbation theory is suitable for evaluation of these structures. In this work, we show that using perturbation semianalytical method one can obtain complete band structure for quasicrystals that are interesting for terahertz technology especially and microwave and optical engineering too. Also, we investigate that complete band gap is appeared in quasicrystals in low refractive index contrast and with increasing number of fold in quasicrystals gap size and isotropy are increased.

1. INTRODUCTION

Quasicrystals have inherent potentials that have been subject of important theoretical developments in both solid-state physics and photonics engineering [1–5]. Quasicrystals were discovered in nature first in AlMn metallic alloys, and exhibit a lot of unique electronic properties [6]. While electronic properties of quasicrystals have been studied thoroughly, their photonic counterparts have been subject of less attention and deserve further investigations. Quasicrystals are like regular electronics crystal in that it has long-range translational order and long-range orientational order. However, the translational order is not periodic and the structure does not have rotational point

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symmetry [7]. Photonic quasicrystals are demonstrated to be more beneficial for getting interesting photonic band gap features such as, more isotropic and complete photonic band gaps under lower dielectric contrast, than the conventional photonic crystals due to their high level of rotational symmetry [5, 8, 9].

For evaluation band structure of photonic crystals, some important concepts such as Brillouin zone and Bloch theorem have been used [10]. Unfortunately, for quasicrystals, Brillouin zone in the usual sense do not exist. In these structures, it is possible to construct an analogue called Pseudo-Brillouin zone [11]. The exact theoretical prediction on the photonic band structures of photonic quasicrystals is currently a numerical challenge, however it is generally believed that analytical studies based on some approximations can helpful to understand physical properties.

In most papers, for obtaining transmission properties and band structures of 2-D photonic quasicrystals, numerical methods have been used. For example, band structures of 2-D octagonal and decagonal and dodecagonal photonic quasicrystals have been calculated by finite difference time domain (FDTD) analysis [12–16]. Also, other numerical methods such as finite element method [17], multiple scattering method [18], transfer matrix method [19] and plane wave expansion method [8, 20] usually have been popular for determining optical properties of photonic quasicrystals. So far, analytical and semi-analytical approach like perturbation method has not been studied for investigating these structures. However, Ochiai and Sakoda have proposed perturbation method to determination band structure of hexagonal lattice in 2001 [21].

In this letter, we will obtain the band structure of photonic quasicrystals by using semi-analytical method such as perturbation theory. In this direction, Pseudo-Brillouin zone for quasicrystal will be defined. It will be shown that 2-D photonic quasicrystals have more isotropic complete photonic band gap under lower dielectric contrast in comparison with ordinary photonic crystals.

The organization of this paper is as follows.

In Section 2, a mathematical model for band structure evaluation of 2-D photonic quasicrystals is developed. Simulation results and discussions are illustrated in Section 3. Finally, the paper ends with a conclusion.

2. BAND STRUCTURE EVALUATION OF 2-D PHOTONIC QUASICRYSTAL

As we mentioned in introduction, the photonic quasicrystals have many exciting properties. One of the most important characteristic of photonic quasicrystals is an isotropic complete photonic band gap under lower dielectric contrast. To demonstrate these properties, band structure of these structures must be obtained. In this direction, first, refractive index of photonic quasicrystal is expressed as a sum of periodic functions, where the amplitudes of periodic functions in comparison with the constant refractive index are much small. Then, pseudo-Brillouin zone for these structures must be defined. After that, perturbation theory can be considered for solving wave equation and the band structure is determined. Finally, a parameter that is used for measuring isotropy of photonic band gap is defined. In the following, the method of band structure evaluation of 2-D photonic quasicrystal will be investigated step by step.

2.1. Refractive Index Modeling

Right now, we can develop refractive index of 2-D photonic quasicrystals as a superposition of refractive indexes for periodic crystals [7]. A function is defined as quasi-periodic if it expressed as a sum of periodic functions with periods, which at least some of these periods are incommensurate (i.e., their ratio is irrational). Using presented definition, we can express the refractive index of 2-D photonic quasicrystals in the same way provided that the ratio of periods of periodic terms must be irrational. Then, the refractive index of photonic quasicrystals can be given as

$$n(\rho) = n_0 + \sum_{i=1}^{N} n_i \cos(G_i.\rho), \tag{1}$$

where n_0, n_i, ρ and G_i are average refractive index, amplitude of periodic terms, the projection of r in the x-y plane and the ith reciprocal lattice vector of quasicrystal respectively. The number N determines the order of the rotational symmetry of the quasicrystal pattern. The reciprocal lattice vectors are given by

$$G_i = \frac{2\pi}{a}\left(\cos\left(\frac{2\pi i}{N}\right), \sin\left(\frac{2\pi i}{N}\right)\right), \tag{2}$$

2.2. Mathematical Model

The Maxwell equations for linear and lossless materials are considered. It is remarkable fact that many interesting and useful properties arise from the elementary case of linear and lossless materials. In addition the theory of these materials is much simpler to understand, making it an excellent foundation on which to build the theory of more complex media. Because the Maxwell equations are linear, however, one can separate the time dependence from the spatial dependence by expanding the fields into a set of harmonic modes. It should be considered that there is no great limitation, because one knows by Fourier analysis that any solution using an appropriate combination of these harmonic modes can be obtained. As it mentioned in many photonics text books [10], the equations governing the modes profile for a given frequency are given by

$$\begin{gathered} \nabla \times \left(\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}) \mathbf{H}(\mathbf{r}), \\ \nabla \cdot \mathbf{H}(\mathbf{r}) = 0, \end{gathered} \tag{3}$$

where $\mathbf{H}(\mathbf{r})$, $\varepsilon(\mathbf{r})$, ω, c are the magnetic field, dielectric constant and Eigen-frequency of the mode and the vacuum speed of light respectively. Also, electric field can be obtained using

$$\mathbf{E}(\mathbf{r}) = \frac{1}{\omega \varepsilon_0 \varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \tag{4}$$

We identify the left side of the Eq. (3) as operator Θ acting on $\mathbf{H}(\mathbf{r})$ similar to eigenvalue equation. It should mention that the operator Θ is a Hermitian operator and thus the Maxwell equation is arranged as an eigenvalue problem for the magnetic field $\mathbf{H}(\mathbf{r})$ and then the $\mathbf{E}(\mathbf{r})$ via Eq. (4) can be calculated.

For 2-D case, the eigenvalue equations are much simplified if the K vector is parallel to the 2-D plane. In this case, the modes must be oscillatory in the z-direction; with no restrictions on the wave vector $\mathbf{k}_z$, because the system is homogeneous in that direction. In the case of 2-D photonic crystal, the system has discrete translational symmetry in the x-y plane. So by applying Bloch's theorem, one can focus on the value of $\mathbf{k}_{||}$ that are in the Brillouin zone. Unfortunately, for quasiperiodic systems, Brillouin zone in the usual sense do not exist. In these systems, the reciprocal lattice vectors of a quasiperiodic structure densely fill all reciprocal space. However, it is often useful to choose a subset of basic reciprocal lattice vectors that corresponds to the relatively intense spots in the diffraction pattern. In addition, although quasicrystals do not possess a Brillouin zone, it is possible to construct an analogue called the pseudo-Brillouin zone which is defined by lines

bisecting the basic reciprocal lattice vectors [11]. By defining pseudo-Brillouin zone for quasicrystal, one pays attention to the values of $\mathbf{k}_{||}$ in to this zone. The label n (band number) is used to label the modes in order of increasing frequency. Indexing the modes of quasicrystal by $\mathbf{k}_z$ and $\mathbf{k}_{||}$ and n, they take the familiar form of Bloch states.

$$\mathbf{H}_{\left(n,k_z,k_{||}\right)}\left(\mathbf{r}\right) = e^{ik_{||}\rho}e^{ik_z z}u_{\left(n,k_z,k_{||}\right)}\left(\rho\right) \tag{5}$$

In this equation, ρ is the projection of $\mathbf{r}$ in the x-y plane and $u(\rho)$ is a profile of modes. Also, in this case $\mathbf{k}_z$ is unrestricted. As we know, there are two kinds of polarizations, transverse-electric (TE) modes and transverse-magnetic field (TM). The band structure for TE and TM modes can be completely different. In this paper, we mainly restrict ourselves to in-plane ($\mathbf{k}_z = 0$) propagation. As mentioned before, the refractive index of quasicrystal can be developed as a superposition of periodic crystals refractive index.

$$n\left(\rho\right) = n_0 + \mu\sum_{i=1}^{N} n_i \cos\left(G_i.\rho\right), \tag{6}$$

where μ is the small perturbation parameter. If the amplitude of periodic terms n_i in comparison with constant part n_0 are much small, the perturbation theory can be used for determining the eigenfrequency of quasicrystal.

The idea is to begin with the modes of idealized homogeneous medium (as an unperturbed system), and using analytical tools to approximately evaluate the effect of small changes in the dielectric function (as a profile of quasiperiodic function) on the modes and their frequencies. For many realistic problems, the error in this approximation is negligible. The derivation of perturbation theory for Hermittian Eigen-problem is straightforward and is covered in many texts on quantum mechanics [22, 23]. Suppose a Hermitian operator Θ is altered by a small amount $\Delta\Theta$. The resulting eigenvalues and eigenvectors of the perturbed operator can be written as series expansions, in terms that depend on increasing powers of the perturbation strength $\Delta\Theta$. The resulting equation can be solved order-by-order using only the eigenmodes of the unperturbed operator. Now, we briefly review this method according to our system. The Eq. (3) for TM polarization and 2-D case is considered.

$$\begin{aligned} -\left(\frac{\partial}{\partial x}\frac{1}{\varepsilon\left(\rho\right)}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\varepsilon\left(\rho\right)}\frac{\partial}{\partial y}\right)\mathbf{H}_z\left(\rho\right) &= \frac{\omega^2}{c^2}\mathbf{H}_z\left(\rho\right), \\ \Theta^{(2)}\mathbf{H}_z\left(\rho\right) &\equiv \frac{\omega^2}{c^2}\mathbf{H}_z\left(\rho\right), \end{aligned} \tag{7}$$

where superscript (2) denotes the 2-D operator. Then, by substituting Eq. (5) in this equation and assuming the in-plane propagation ($\mathbf{k}_z = 0$), the 2-D operator can be written as,

$$\begin{aligned}
&\Theta^{(2)} u_{(n,k_{||})}(\rho) \equiv \frac{\omega^2}{c^2} u_{(n,k_{||})}(\rho), \\
&\Theta^{(2)} = -\left(\frac{1}{\varepsilon(\rho)}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\frac{\partial}{\partial x}\frac{1}{\varepsilon(\rho)}\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\frac{1}{\varepsilon(\rho)}\frac{\partial}{\partial y}-k_{||}^2\right),
\end{aligned} \tag{8}$$

where in this notation the subscript n and $\mathbf{k}_{||}$ denotes the band number and corresponding $\mathbf{k}_{||}$ in the pseudo-Brillion zone. After that, Eq. (6) is substituted into the 2-D eigenvalue problem given by Eq. (8). The unperturbed Hermittian operator $\hat{\Theta}^{(2)}$ and the perturbation operator $\Delta\hat{\Theta}^{(2)}$ are obtained as follows,

$$\begin{aligned}
&\left(\hat{\Theta}^{(2)}+\Delta\hat{\Theta}^{(2)}\right) u'_{(n,k_{||})}(\rho) \equiv \Omega'_{(n,k_{||})} u'_{(n,k_{||})}(\rho), \\
&\hat{\Theta}^{(2)} \equiv -\left(\frac{1}{n_0^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-k_{||}^2\right), \\
&\Delta\hat{\Theta}^{(2)} \equiv 2n_0\sum_{i=1}^{N} n_i\cos(G_i.\rho)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) \\
&+\frac{\partial}{\partial x}\left(2n_0\sum_{i=1}^{N} n_i\cos(G_i.\rho)\right)\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\left(2n_0\sum_{i=1}^{N} n_i\cos(G_i.\rho)\right)\frac{\partial}{\partial y},
\end{aligned} \tag{9}$$

where $u'_{(n,k_{||})}(\rho)$ and $\Omega'_{(n,k_{||})} = \frac{\omega^2}{c^2}$ are the eigenvectors and eigenfrequencies of the perturbed operator. They can be written as series expansion,

$$\begin{aligned}
u'_{(n,k_{||})}(\rho) &= u^{(0)}_{(n,k_{||})}(\rho) + \mu u^{(1)}_{(n,k_{||})}(\rho), \\
\Omega'_{(n,k_{||})} &= \Omega^{(0)}_{(n,k_{||})} + \mu\Omega^{(1)}_{(n,k_{||})},
\end{aligned} \tag{10}$$

where the superscripts denote the order of eigenfrequencies and eigenvectors corrections. Now, perturbed eigenfrequencies and eigenvectors given by Eq. (10) is substituted into the Eq. (8). Then, terms are collected and factorized order-by-order in μ. For the zeroth order in μ (i.e., O (1)), the unperturbed system is considered, which can be solved by different methods such as finite difference method and eigenvectors and eigenvalues $u^{(0)}_{(n,k_{||})}(\rho)$, $\Omega^{(0)}_{(n,k_{||})}$ can be obtained. For the first order, O (μ), is obtained as

$$\hat{\Theta}^{(2)} u^{(1)}_{(n,k_{||})}(\rho) + \Delta\hat{\Theta}^{(2)} u^{(0)}_{(n,k_{||})}(\rho) = \Omega^{(0)}_{(n,k_{||})} u^{(1)}_{(n,k_{||})}(\rho) + \Omega^{(1)}_{(n,k_{||})} u^{(0)}_{(n,k_{||})}(\rho), \tag{11}$$

when, Eq. (11) is multiplied by $u^{*(0)}_{(n,k_{||})}(\rho)$ and is integrated over the surface of crystal, $\Omega^{(1)}_{(n,k_{||})}$ is obtained.

$$\Omega^{(1)}_{(n,k_{||})} = \frac{\iint u^{*(0)}_{(n,k_{||})}(\rho)\,\Delta\hat{\Theta}^{(2)} u^{(0)}_{(n,k_{||})}(\rho)\,d\rho}{\iint \left|u^{(0)}_{(n,k_{||})}(\rho)\right|^2 d\rho} \tag{12}$$

It can also be found the correction to the eigenvector by multiplying Eq. (11) to another mode at a different band, m.

It must be noticed that if there are degeneracy in the eigenfrequency of unperturbed system, that it means any two zeroth-order eigenvectors had the same frequency; the perturbation theory is no longer valid. In this case, the degenerate perturbation theory can be used to identify the eigenfrequency of perturbed system. This theory is covered in some text book [22, 23] and it is briefly reviewed here. Let us assume that the zeroth-order solution of Maxwell equation has g-fold degeneracy, which they can be written as $u^{(0)}_{(n,k_{||},m)}(\rho)$. The subscripts n and $\mathbf{k}_{||}$ represent the band number and the in-plane k-vector, respectively. The subscript m represents which one of the g degenerate states is being considered. The m value is between 1 to g. Each of the states $u^{(0)}_{(n,k_{||},m)}(\rho)$ is an eigenvector of $\hat{\Theta}^{(2)}$ with precisely the same eigenfrequency $\Omega^{(1)}_{(n,k_{||})}$. As we know, any linear combination of the g degenerate eigenvectors is also an eigenvector.

$$u^{(0)\alpha}_{(n,k_{||})}(\rho) = \sum_{m=1}^{g} a^{\alpha}_{m} u^{(0)\alpha}_{(n,k_{||},m)}(\rho)\,, \tag{13}$$

where the coefficients are labeled as a^{α}_{m}. The coefficients a^{α}_{m} must be chosen such that the terms in the first order correction to the eigenvector vanish whenever the dominator is zero. The first order correction to the eigenvector is given as

$$u^{(1)}_{(n,k_{||})}(\rho) = \sum_{m\neq n} \frac{M_{mn}}{\left(\Omega^{(0)}_{(n,k_{||})} - \Omega^{(0)}_{(m,k_{vert|})}\right)\iint u^{(0)*}_{(m,k_{||})} u^{(0)}_{(m,k_{||})} d\rho}\, u^{(0)}_{(n,k_{||})}(\rho)\,, \tag{14}$$

where M_{mn} is defined as

$$M_{mn} = \iint u^{*(0)\alpha}_{(m,k_{||})}(\rho)\,\Delta\hat{\Theta}^{(2)} u^{(0)}_{(n,k_{||})}(\rho)\,d\rho \tag{15}$$

The new eigenvector $u^{(0)\alpha}_{(m,k_{||})}(\rho)$ must be constructed such that M_{mn} vanishes whenever $\Omega^{(1)}_{(n,k_{||})} = \Omega^{(1)}_{(m,k_{||})}$. Under this condition, no

singularity occurs in the first-order correction to the eigenvectors. So, the coefficients a_m^{α} and $\Omega^{(1)}_{(n,k_{||})}$ are determined by solving the following eigenvalue equation,

$$\sum_{m=1}^{g}\left(M_{pm}-\Omega^{(1)}_{(n,k_{||})}\delta_{mp}\right)a_m^{\alpha}=0, \tag{16}$$

where index p has values from 1 to g. The same approach can be developed for TE polarization, too.

2.3. Band Gap Evaluation

The band structure of photonic quasicrystals can be obtained for TM and TE polarization using semi-analytical method such as perturbation theory investigated pervious subsection, particularly. The relative photonic band gap between bands is defined as

$$\text{Gap}=\frac{\Delta\omega}{\omega_0}=\frac{\min\limits_{k_{||}\in P-BZ}:\left\{\omega_h\left(k_{||}\right)\right\}-\max\limits_{k_{||}\in P-BZ}:\left\{\omega_l\left(k_{||}\right)\right\}}{\min\limits_{k_{||}\in P-BZ}:\left\{\omega_h\left(k_{||}\right)\right\}+\max\limits_{k_{||}\in P-BZ}:\left\{\omega_l\left(k_{||}\right)\right\}} \tag{17}$$

where the minima and maxima are taken over all $\mathbf{k}_{||}$ on the pseudo-Brillouin zone (P-BZ), and ω_l and ω_h are bands just below and just above the complete gap. Another parameter that one can be interested in about band gap, is measuring isotropy of the photonic band gap. A measure of isotropy for a photonic band gap can be defined as

$$I=\frac{\min\limits_{k_{||}\in P-BZ}:\left\{\omega_h\left(k_{||}\right)\right\}-\max\limits_{k_{||}\in P-BZ}:\left\{\omega_l\left(k_{||}\right)\right\}}{\max\limits_{k_{||}\in P-BZ}:\left\{\omega_h\left(k_{||}\right)\right\}-\min\limits_{k_{||}\in P-BZ}:\left\{\omega_l\left(k_{||}\right)\right\}} \tag{18}$$

It is clear from its definition that $I\in[-1,1]$. When I is at its greatest, the gap is perfectly isotropic and it is at its lowest in a homogeneous material.

3. RESULTS AND DISCUSSIONS

In this paper, taking account of analytical results reported in pervious section, the simulated band structure of hexagonal photonic crystal based on perturbation theory is calculated and it has good agreement with the simulated result that have been obtained by well-known software such as Rsoft based on FDTD method. So, one can trust this method and the results are calculated for photonic quasicrystals are valid. According to this point, the simulated results of the

band structure for special case 2-D 12-fold photonic quasicrystal are demonstrated. The photonic quasicrystal is composed of 150-nm-diameter rods arranged on a pitch of 260 nm and the refractive index of rods material are chosen $n = 2.45$ (materials such as porous silicon or TiO_2 that are familiar as optical materials). The index of refraction for 2-D photonic quasicrystal of considered structure described above can be given by the following parameters and is shown in Fig. 1(a). They are assumed that n_0, n_i, a, and N are chosen 1.48, 0.08, 260 nm and 12 respectively.

As we mentioned before, the parameter n_i should be much smaller than n_0 that it is satisfied by choosing these parameters. So, we can use the perturbation theory results calculated in the previous section. Reciprocal lattice for this structure can be obtained and it is illustrated in Fig. 1(b). The first pseudo-Brillouin zone is indicated in Fig. 1(b) as white dodecagon.

For evaluation band structure based on perturbation theory, the band structure of unperturbed structure (empty lattice) must be obtained by using zeroth order equation of perturbation method and it is shown in Fig. 2(a) for corresponding k-vector in pseudo-Brillouin zone. There are some points with high symmetry in the unperurbed band structure which the eigenfrequency of unperturbed operator is degenerated (i.e., they have different eigenvector with

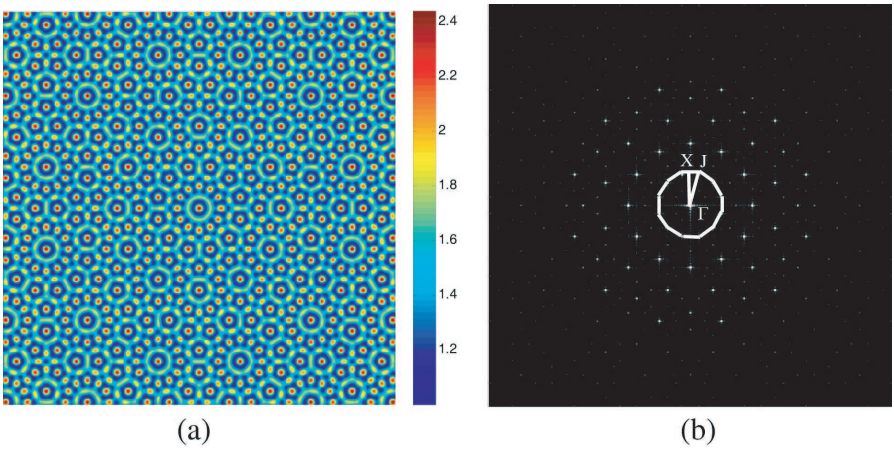

Figure 1. (a) The refractive index of 2-D photonic quasicrystal is composed of 150-nm-diameter rods arranged on a pitch of 260 nm and the refractive index of rods material are chosen $n = 2.45$. (b) Corresponding reciprocal lattice, the first pseudo-Brillouin zone is indicated as white dodecagon.

equal eigenfrequency). The correction of eigenfrequency of perturbed operator in these cases can be obtained using degenerate perturbation theory. In other points of unperturbed band structure that there is not degeneracy, correction of eigenfrequency can be calculated using Eq. (12) described in the previous section. Now, the band structure of 2-D 12-fold photonic quasicrystal calculated by using perturbation method for both TM and TE polarization is illustrated in Fig. 2. As mentioned before, the relative photonic band gap and isotropy of band gap can be calculated using Eqs. (17) and (18), respectively. The results for this structure are summarized in Table 1.

The band structure of 2-D 8-fold photonic quasicrystal can be obtained using this method in a same way. However, the pseudo-Brillouin zone for this case will be an octagonal. 4-fold and 6-fold (hexagonal) photonic crystals are modeled by this method, so the band structure of these structures is investigated using semi-analytical method.

Table 1. Band structure evaluation of considered 2-D 12-fold photonic quasicrystal.

$\%\text{Gap} = \frac{\Delta\omega}{\omega_0}$	I (isotropy)
17. 9132	0.470003

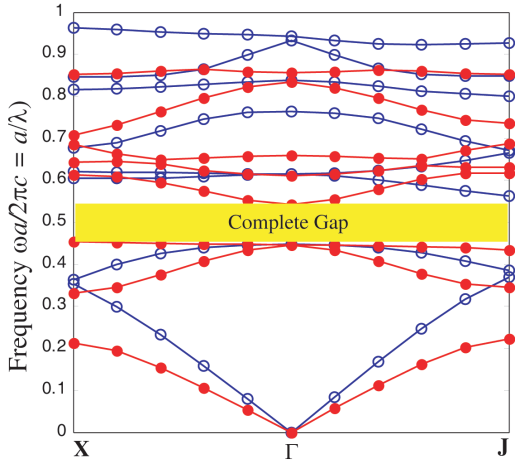

Figure 2. Band structure of considered 2-D 12-fold photonic quasicrystal for TM (blue line marked empty circle) and TE (red line marked solid circle) polarization.

As it was mentioned in introduction, one of the most important features of photonic quasicrystals is a more isotropic complete band gap in a low dielectric contrast due to their high level of rotational symmetry. It is promising for application in a range of optical devices. In this direction, the band structure of 12-fold and 8-fold photonic quasicrystals and also 4-fold and 6-fold photonic crystals are obtained using perturbation theory as mentioned previously for various dielectric contrasts. After that, band gaps for each case over the contrast range 1.44–5.76 where the perturbation theory is valid are plotted in Fig. 3.

As shown in Fig. 3, stop gaps increase with dielectric contrast ratio for every n-fold structures. It is clear in this figure that the higher symmetry structures have greater gaps at low contrast. The photonic quasicrystals have gaps that are more isotropic than those of the crystals for all contrasts due to the fact that their pseudo-Brillouin zones are more circular than the Brillouin zones of crystals. For example, the Brillouin zone for 6-fold symmetry is a hexagon and the pseudo- Brillouin zone for 12-fold is dodecagon.

Isotropy is plotted against dielectric contrast for all rotational symmetries in Fig. 4. The photonic quasicrystals all have higher isotropic than the crystals and I increases as the pseudo-Brillouin zone becomes more circular. The isotropy of 12-fold photonic quasicrystal is the highest.

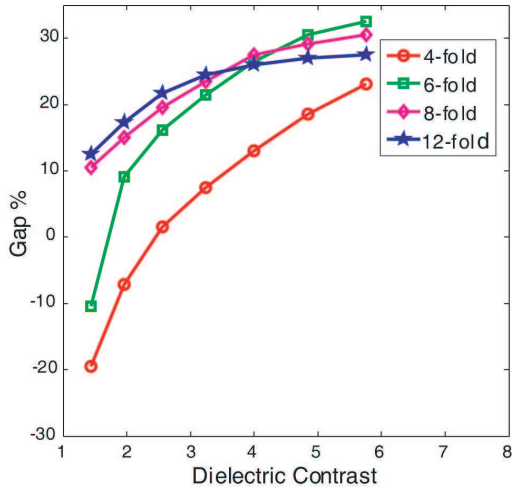

Figure 3. Band gaps of structures for rotational symmetries $n = 4$, 6, 8 and 12 as a function of the dielectric contrast of the rods and background materials.

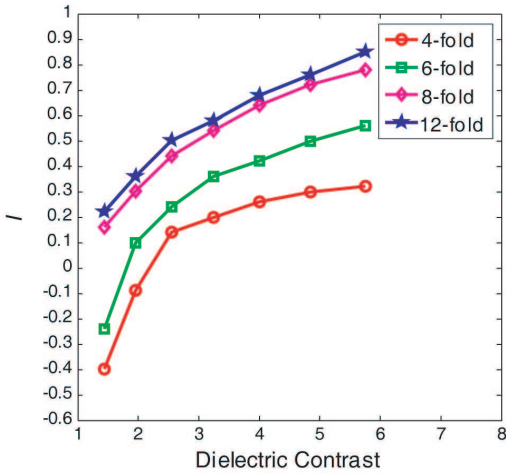

Figure 4. The degree of isotropy of structures for rotational symmetries $n = 4$, 6, 8 and 12 as a function of the dielectric contrast of the rods and background materials.

In this section, simulation results for illustration of the band structure for quasicrystals were demonstrated. In these figures quality of quasicrystals investigated and shown that quasicrystals are easy to work from band gap opening at low contrast points of view.

4. CONCLUSION

As conclusion, it should be pointed out that pseudo-Brillouin zone was defined for evaluation of the band structure for quasicrystals and consequently the Bloch theory was applied to quasicrystals. Then, we have calculated the band structure of 2-D 12-fold and 8-fold photonic quasicrystal by using perturbation method. It was shown that using first order perturbation theory band structure (eigenfrequencies) can be obtained. As we know, in quasicrystals complete band gap can be observed in low dielectric contrasts, so developing perturbation theory based band solver is interesting. Also, we used degenerate perturbation theory for points with high symmetry to extract precision band structure. The proposed method is semi-analytic and presents a conceptual view for applied designer and numerical errors which are conventional in popular methods are strongly reduced. Finally, it is shown that high symmetry photonic quasicrystals have greater and more isotropic photonic band gap in low dielectric contrast in comparison with photonic crystals.

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