

## EVALUATION OF MUTUAL COUPLING BETWEEN SLOTS FROM DIPOLE EXPRESSIONS

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**Abstract**—Two closed form approximations are given for mutual coupling between arbitrarily oriented slots with cosinusoidal distribution, using the known results for dipoles along with a new correction factor to account for the piecewise sinusoidal dipole current. Using these approximations, a scheme has been developed for calculating mutual coupling between practically used slots of arbitrary orientation and useful results are obtained from simple closed form expressions for slot separation of  $1.2 \times \text{slot length}$  or more depending upon the approximation chosen and the length of the slot. These approximations are found to be more accurate than those available in the literature, with a maximum error of less than 1.6% for slots shorter than 0.5 wavelength and separated by  $0.85 \times \text{wavelength}$  or more. Simple yet accurate expressions for mutual coupling, like the point dipole approximation developed here, result in efficient evaluation of mutual coupling for the design of large arrays of slots or for Electromagnetic Compatibility analysis.

### 1. INTRODUCTION

Slots in ground plane are commonly used as radiating elements in antenna arrays for various applications. In the design of such arrays, the mutual coupling between array elements has to be considered for reliable results [1, 2]. Different methods have been evolved for taking mutual coupling into account where the antenna consists of hundreds or thousands of inclined slots in ground plane as in Radial Line Slot Antenna (RLSA) [3]. Analysis of mutual coupling between slots is also useful in estimating the electromagnetic interference between two systems or subsystems with a common conducting plane like a rack

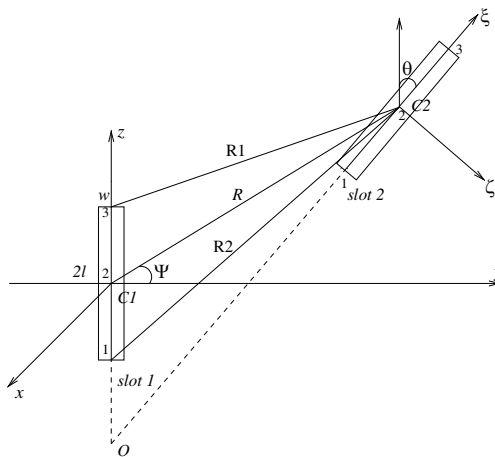
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or panel with slots for ventilation, display etc. Shielding effectiveness (SE) of enclosures is also dependent upon such arbitrarily oriented slots and seams. The effect of multiple slots and apertures on shielding effectiveness has been extensively studied in the literature. Mutual coupling between multiple apertures needs to be taken into account for accurately estimating the degradation in shielding effectiveness of enclosures [4, 5].

Mutual coupling between arbitrarily oriented coplanar dipoles with piecewise sinusoidal (PWS) current has been evaluated exactly in closed form in terms of sine and cosine integrals [6]. However, it is known that the aperture electric field or magnetic current in the slot is closer to the cosinusoidal distribution of the form  $\cos[(\pi/2l)z]$  [1]. Exact closed form expressions for mutual coupling between slots with cosinusoidal distribution are not available in the literature. Instead, a number of techniques have been used in the past for approximately evaluating the mutual coupling between slots and an excellent summary of the same can be found in [7]. Mutual coupling has been evaluated from the dipole mutual impedance in [7–9] and from asymptotic analysis in [3, 10, 11].

In [7], the mutual coupling has been evaluated as a single integral, by first simplifying the aperture distribution of one slot to a piecewise sinusoidal distribution and then using a multiplicative constant to approximately evaluate the coupling for a cosinusoidal distribution. The error due to this approximation, with respect to exact numerical results, was seen to increase with slot separation for certain slot azimuths  $\Psi$ , where  $\Psi$  is shown in Fig. 1.



**Figure 1.** Geometry of the problem.

Approximating the aperture distribution by PWS distribution immediately allows one to apply all the known and relatively simple results available for dipoles to the cosinusoidal case. Hence, although slots have been analysed using a number of techniques in the literature, e.g., [12, 13], PWS distribution along with dipole field and mutual impedance has been used in this paper to derive the mutual coupling between arbitrary slots. In order to validate the approximations and compare the results, the formulation in [7] has been taken as reference and then suitably modified in the present work. In this communication, we present another multiplicative factor that gives better agreement with numerical results at distances greater than about a wavelength for slots shorter than  $\lambda/2$  and for which the error decreases with distance or settles at a value lower than that from other methods. Also, a couple of alternative approximations in closed form are presented and are shown to give better results than those in similar works. The error due to these approximations has been studied and the results are tabulated, so that the formulation given in the next section can be used for efficient and accurate evaluation of slots in a big array or for Electromagnetic Compatibility (EMC) analysis and SE estimation.

## 2. FORMULATIONS AND EQUATIONS

The geometry of the problem is illustrated in Fig. 1. Slot 1 is assumed to be along the  $z$ -axis and slot 2 has a tilt  $\theta$  with respect to slot 1.  $\Psi$  is the azimuth angle of slot 2 with respect to slot 1. Similarly,  $\Psi'$  is the azimuth of slot 1 with respect to slot 2. The centre of slot 1,  $C_1$  is at the origin  $(0, 0, 0)$  and that of slot 2,  $C_2$  is at  $(0, y_{c2}, z_{c2})$ .  $C_2 \equiv (\rho, \pi/2, z_{c2})$  in cylindrical co-ordinates and  $C_2 \equiv (R, \pi/2 - \Psi, \pi/2)$  in spherical co-ordinate system. Similarly, the slot 2 is centred at the origin of  $x', \zeta, \xi$  coordinate system and  $C_1 \equiv (\rho', -\pi/2, z'_{c1})$  in cylindrical and  $C_1 \equiv (R, \pi/2 - \Psi', -\pi/2)$  in spherical coordinate system with reference to slot 2. The centre to centre distance is  $R$ . Both the slots are assumed to be of equal length  $2l$  and width  $w$ . The width  $w$  is assumed to be much less than length, i.e., narrow slot approximation is employed. As shown in [7], for  $w/2l < 10$ , this introduces negligible error in the coupling amplitude and phase, while for wider slots, accurate results can be obtained with two transverse integrations along each slot width.

The expression for mutual coupling may be written as [1]

$$Y_{21} = \frac{-1}{V_1 V_2} \iint_{slot\ 2} H_{21} K_2 d\zeta d\xi \quad (1)$$

where  $V_1, V_2$  are the voltages in slot 1 and 2 respectively,  $K_2 = \frac{V_2}{w} e_2 \hat{\xi}$  is the magnetic current in slot 2 and  $H_{21}$  is the magnetic field in the

aperture of slot 2 due to magnetic current distribution  $K_1 = \frac{V_1}{w} e_1 \hat{z}$  in the aperture of slot 1.  $e_1, e_2$  are the aperture current distributions with unit amplitude in slot 1 and slot 2, respectively.

$$H_{21} = \iint_{slot\ 1} K_1 G(R) dz dx \quad (2)$$

where  $G(R) = \frac{e^{-jkR}}{2\pi R}$  is the Green's function for slots in an infinite conducting ground plane [8]. Then,

$$Y_{21} = \frac{-1}{w^2} \iint_{slot\ 2} H_{21} \cdot e_2 d\zeta d\xi \quad (3)$$

where  $H_{21}$  is only due to  $e_1$  in slot 1. As the evaluation of  $H_{21}$  itself involves a double integral, (3) requires numerical evaluation of a quadruple integral.

Assuming narrow slot approximation, the integration along the width can be taken to be constant. The magnetic current is assumed directed along the length of the slot and constant across the width. The phase is assumed to be constant over each slot. Then,

$$Y_{21}^S = - \int_{\xi} H_{21} \cdot e_2 d\xi \quad (4)$$

where  $H_{21}$  involves a single integral only along the centreline of the slot 1. The evaluation of (4) involves a double integral.

For evaluating  $Y_{21}$  in the following analysis, two aperture distributions need to be considered for the slot magnetic currents  $e_1$  and  $e_2$  viz.

$$e^S = \cos\left(\frac{\pi}{2l}z\right) \quad \text{for cosinusoidal distribution}$$

$$e^D = \sin(k|l - z|) \quad \text{for dipole-like PWS approximation to } e^S$$

In the following analysis, the subscript 1 or 2 refers to slot 1 or slot 2, respectively, while the superscripts  $D$  and  $S$  are used to signify the PWS and cosinusoidal aperture distribution, respectively.

### 2.1. Mutual Coupling Evaluation with Single Dipole Approximation

Mutual coupling between narrow slots with cosinusoidal distribution,  $Y_{21}^S$ , has been evaluated in [7] as a single integral, say  $Y_{21}^r$ , using dipole-like PWS aperture distribution  $e_1^D$  for slot 1 in (4) to calculate  $H_{21}^D$  in closed form assuming narrow slots and then correcting for cosine distribution using a correction factor.

$$Y_{21}^r = -\gamma_r \int_{-l}^l H_{21}^D \cdot e_2^S d\xi \quad (5)$$

where  $\gamma_r$  is the correction factor for cosinusoidal distribution in [7].

In [7],  $\gamma_r$  is taken as a ratio of the first moments of the aperture distributions

$$\gamma_r = \frac{2l}{\pi} \frac{k}{1 - \cos(kl)} \quad (6)$$

and

$$H_{21}^D = H_z^D \cos \theta + H_\rho^D \sin \theta \quad (7)$$

$$H_\rho^D = \frac{-1}{j\eta} \frac{1}{2\pi} \left[ \frac{z_{c2} - l}{\rho} \frac{e^{-jkR_1}}{R_1} + \frac{z_{c2} + l}{\rho} \frac{e^{-jkR_2}}{R_2} - 2 \cos(kl) \frac{z_{c2}}{\rho} \frac{e^{-jkR}}{R} \right] \quad (8)$$

$$H_z^D = \frac{1}{j\eta} \frac{1}{2\pi} \left[ \frac{e^{-jkR_1}}{R_1} + \frac{e^{-jkR_2}}{R_2} - 2 \cos(kl) \frac{e^{-jkR}}{R} \right] \quad (9)$$

where  $k = 2\pi/\lambda$  and  $\eta$ ,  $\lambda$  are the free space impedance and wavelength respectively.  $R_1 = \sqrt{\rho^2 + (z_{c2} - l)^2}$ ,  $R_2 = \sqrt{\rho^2 + (z_{c2} + l)^2}$  and  $R = \sqrt{\rho^2 + (z_{c2})^2}$ .

The error in mutual coupling expressions for the two distributions, i.e.,  $Y_{21}^S$  evaluated numerically from (4) using  $e^S$  above and  $Y_{21}^r$  from (5), was found to increase with spacing between the slots for certain azimuths  $\Psi$  [7]. For slot lengths greater than  $0.5\lambda$ , the PWS and cosinusoidal distributions being significantly different, the error increases rapidly with slot separation  $R$ .

Hence, a better correction factor is desired, particularly for slots separated by more than a wavelength or so. In [14], the near field of extended dipoles was evaluated using a correction factor taken as a ratio of the far fields for extended and point dipoles. Here, we propose another correction factor  $\gamma_f$ , that is a ratio of the far field factors for the two distributions, cosinusoidal and PWS.

$$\gamma_f(\Psi) = \frac{E_f^S(\Psi)}{E_f^D(\Psi)} \quad (10)$$

where

$$E_f^S(\Psi) = \frac{2(\pi/2l) \cos(k_z l)}{(\pi/2l)^2 - k_z^2} \quad (11)$$

$$E_f^D(\Psi) = \frac{-2k [\cos(kl) - \cos(k_z l)]}{k^2 - k_z^2} \quad (12)$$

and  $k_z = k \sin(\Psi)$ . Unlike  $\gamma_r$ , the  $\gamma_f$  factor accounts for the variation in relative coupling between the two distributions with azimuth  $\Psi$ , leading to better accuracy. Then

$$Y_{21}^f = -\gamma_f \int_{-l}^l H_{21}^D \cdot e_2^S d\xi_2 \quad (13)$$

The mutual coupling single integrals  $Y_{21}^r$  from (5) and  $Y_{21}^f$  from (13) have been evaluated numerically. It is noted that the latter expression gives an error decreasing with the spacing  $R$ , in contrast to the former, where, above a certain angle  $\Psi$ , the error was seen to increase with  $R$  in comparison with exact results evaluated using numerical integration of (4). Numerical results are presented in Section 3.

## 2.2. Mutual Coupling with Double Dipole Approximation

In Section 2.1, the mutual coupling was evaluated using PWS aperture distribution for slot 1 and cosinusoidal for slot 2. This required a numerical single integral. It is possible to carry out this analysis by assuming a dipole-like PWS distribution in each slot and then correcting the resulting mutual coupling expression for cosinusoidal distribution in each. The mutual coupling between arbitrarily oriented coplanar slots with PWS current distribution can be derived in closed form from [6]. Although the detailed analysis can be found in [6], the end result is given below in brief for the sake of completeness. If  $Y_{21}^{DD}$  is the closed form expression derived from [6] for mutual coupling between slots in Fig. 1 with dipolar distribution  $e^D$ ,

$$Y_{21}^{DD} = \frac{-1}{4\pi\eta} \sum_{m=1}^3 \sum_{n=1}^3 C_m D_n \sum_{p=-1}^1 \sum_{q=-1}^1 pq \exp[jk(pz_m + q\xi_n)] \cdot E(kR_{mn} + kpz_m + kq\xi_n) \quad (14)$$

where  $p$  and  $q$  assume only the values  $\pm 1$ .  $z_m, \xi_n$  are measured from the origin  $O$  at intersection of the centreline of slots as shown in Fig. 1, where  $m(n) = 1, 3$  corresponds to endpoints of slot 1(2) while  $m(n) = 2$  corresponds to the centre.  $R_{mn}$  is the distance from point  $z_m$  on slot 1 to point  $\xi_n$  on slot 2 such that  $R_{mn} = (z_m^2 + \xi_n^2 - 2z_m\xi_n \cos \theta)$ , e.g.,  $R_{22} \equiv R$  in Fig. 1.  $E(x) = Ci(|x|) - jSi(x)$ , where  $Ci(x)$  and  $Si(x)$  are the cosine and sine integrals, respectively, and  $C_{1(3)}, D_{1(3)} = 1$ ,  $C_2, D_2 = -2 \cos(kl)$  [6].

If  $\gamma_{f1}, \gamma_{f2}$  are the far field correction factors as defined above for slot 1 and slot 2, respectively, then

$$Y_{21}^{rr} = -\gamma_r^2 Y_{21}^{DD} \text{ using the correction factor } \gamma_r \text{ in [7], and}$$

$$Y_{21}^{ff} = -\gamma_{f1}(\Psi)\gamma_{f2}(\Psi')Y_{21}^{DD}$$

The results for these are also evaluated and given in Section 3. It can be seen that the error in general is double compared to that for single integral. Hence, this formulation gives excessive error for  $Y_{21}^{rr}$  at wider angles and larger separation as compared to  $Y_{21}^{ff}$ , which gives acceptable error in this region and particularly accurate results for

slots shorter than  $\lambda/2$ . Thus, the use of double dipole approximation is practically feasible with the correction factor  $\gamma_f$  proposed here.

### 2.3. Mutual Coupling with Point Dipole Approximation

The formulation in Section 2.1 requires the numerical evaluation of single integral whereas the formulation in Section 2.2, although in closed form, requires as many as 36 sine and cosine integrals [6]. In an effort to simplify the formulation and at the same time retain an acceptable accuracy, the formulation has been carried out using point dipole approximation for the two slots, i.e., instead of two dipoles in the formulation of Section 2.2, mutual coupling is evaluated assuming point dipoles at the centre of the slots, and correction applied for extended dipoles in addition to correction for cosinusoidal distribution.

The magnetic field  $H_{21}^p$  at the centre of slot 2,  $C_2$ , due to point dipole at the centre  $C_1$  of slot 1, is given by

$$H_{21}^p = \left[ H_{z21}^p \cos(\theta) + H_{y21}^p \sin(\theta) \right] \quad (15)$$

$$H_{z21}^p = \frac{1}{j\eta k} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \frac{e^{-jkR}}{2\pi R} \quad (16)$$

$$H_{y21}^p = \frac{1}{j\eta k} \left( \frac{\partial^2}{\partial z \partial y} \right) \frac{e^{-jkR}}{2\pi R} \quad (17)$$

The correction applied for extended dipole 1,  $\gamma_1^{ext}$ , is then taken as the ratio of magnetic fields due to extended and point dipoles

$$\gamma_1^{ext} = \frac{H_{21}^D}{H_{21}^p} \quad (18)$$

Similarly,

$$\gamma_2^{ext} = \frac{H_{12}^D}{H_{12}^p} \quad (19)$$

$H_{21}^D$  is given by (7). It is to be noted that  $\gamma_1^{ext} \neq \gamma_2^{ext}$  unless the slots are symmetrically placed about  $C_1C_2$ , i.e., unless  $\Psi = \Psi'$ .

Then, the mutual coupling can be evaluated as

$$Y_{21}^p = - [\gamma_{f1}(\Psi) \gamma_{f2}(\Psi') H_{21}^p] \gamma_1^{ext}(\Psi) \gamma_2^{ext}(\Psi') \quad (20)$$

The results for mutual coupling evaluated from the above formulation are also presented in Section 3. The point dipole approximation can also be used for non-planar slots such as slots in different sides of a cabinet or between transmitting and receiving antennas. Point dipole approximation has been used previously in the literature, typically with far field approximation, thus being less accurate and applicable at longer distances compared to the present formulation.

### 3. NUMERICAL RESULTS AND DISCUSSION

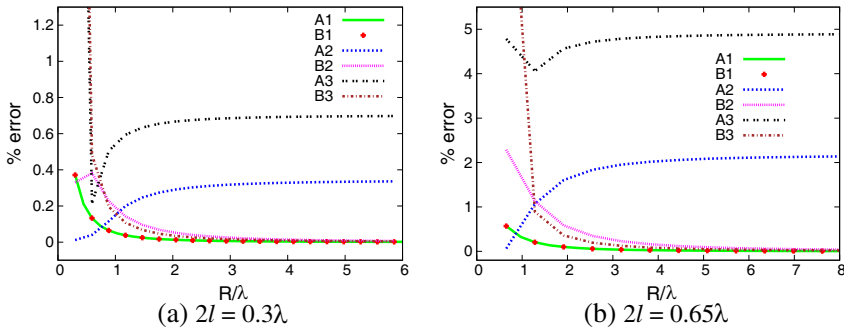
#### 3.1. Numerical Results

The foregoing analyses were evaluated and compared with numerical results for a number of slot lengths, azimuths and tilts using the single dipole, double dipole and point dipole approximations. The reference result is evaluated numerically from (4) by dividing each slot into cells and employing three point gaussian quadrature over each cell for the double integral. The error is calculated as the absolute value of difference between approximate and reference value and the percentage error with respect to the reference is plotted for comparison.

##### 3.1.1. Single Dipole Approximation

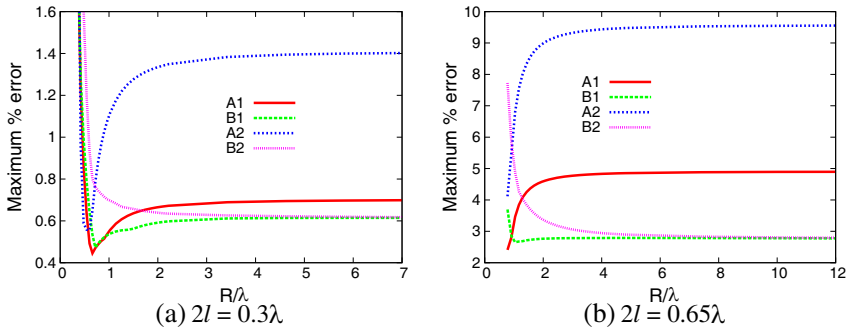
The error in the single dipole approximation with respect to numerical integration of (4) is plotted in Figs. 2(a) and 2(b) for parallel displaced slots for slot lengths of  $2l = 0.3\lambda$  and  $2l = 0.65\lambda$ , respectively.

It is seen that, for  $\Psi > 20^\circ$ , the error in single dipole formulation from [7] increases with slot separation  $R/\lambda$  before it flattens out at a relatively higher value compared to the approximation with  $\gamma_f$  proposed here, and the same is evident for  $\Psi = 45$  and  $\Psi = 90$  in Fig. 2. The error is particularly significant for slot length greater than  $0.5\lambda$  as the assumed distributions differ considerably. For collinear slots ( $\Psi = 90^\circ$ ), the error is more than 4% for  $2l = 0.65\lambda$  with  $\gamma_r$  factor. On the contrary, error for single dipole approximation with the correction factor  $\gamma_f$  proposed here, is seen to decrease with  $R/\lambda$  in this case for all azimuths  $\Psi$ , and gives much better results than  $\gamma_r$  for slot separation  $R > \lambda$ . The error is less than 1% for slots separated by  $2\lambda$  or more with  $\gamma_f$  factor. For  $\Psi = 0$ , as  $\gamma_f = \gamma_r$ , the two curves overlap.



**Figure 2.** % error with slot separation  $R/\lambda$  for parallel displaced slots. (A- Single dipole approximation with  $\gamma_r$ , B- single dipole approximation with  $\gamma_f$ . 1-  $\psi = 0$ , 2-  $\psi = 45$ , 3-  $\psi = 90$ .)





**Figure 3.** Maximum % error with slot separation  $R/\lambda$ . (A1- Single dipole approximation with  $\gamma_r$ , B1- single dipole approximation with  $\gamma_f$ . A2- Double dipole approximation with  $\gamma_r$ , B2- double dipole approximation with  $\gamma_f$ .)

### 3.1.2. Double Dipole Approximation

In [7], the error was calculated with parallel displaced slots. We have also evaluated the maximum error over all the azimuths  $\Psi$  for each slot tilt  $\theta$ , and the results are plotted against slot separation  $R/\lambda$  in Figs. 3(a) and 3(b) for slot lengths  $2l = 0.3\lambda$  and  $2l = 0.65\lambda$  respectively. The error with double dipole approximation is seen to be greater than that with the corresponding single dipole approximation. However, the error for double dipole formulation with  $\gamma_f$  is seen to be lesser than that due to  $\gamma_r$  for separation greater than  $0.8\lambda$  and  $0.9\lambda$ , respectively for slots shorter or longer than  $0.5\lambda$ . The error with  $\gamma_r$  is better below this separation. Thus, for  $0.3\lambda \leq 2l \leq 0.5\lambda$ , the above distance of  $0.8\lambda$  can be taken as the crossover separation and by choosing the factor  $\gamma_r$  or  $\gamma_f$  appropriately, the maximum error over all tilts and azimuth angles can be kept less than 2.5% for a slot separation of  $1.2 \times 2l$  or more. For slots longer than  $0.5\lambda$ , the error increases as the difference in cosinusoidal and sinusoidal distributions becomes sizable, e.g., for  $2l = 0.65\lambda$ , the corresponding error at  $1.2 \times 2l$  is 4.2% with  $\gamma_r$ . The error with  $\gamma_r$  increases to about 6% for slots separated by  $0.9\lambda$  and the factor  $\gamma_f$  gives much better results after this. The maximum error with  $\gamma_f$  decreases with slot spacing while that with  $\gamma_r$  increases and then remains constant around 10% for collinear slots as shown in Fig. 3.

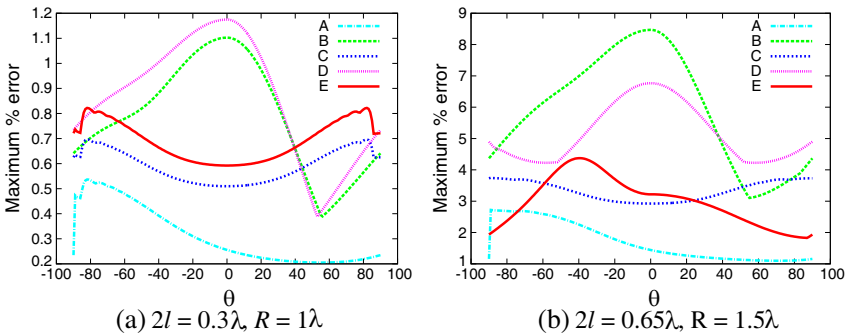
The approximate expressions for slot separation giving an error of 2.5% or less and for the maximum error expected at a particular separation for double dipole approximation are summarised in Table 1. The entry with  $\gamma_r$  factor shows the maximum expected % error at those slot spacings above which the  $\gamma_r$  factor gives useful results. The error

with  $\gamma_f$  is given at crossover separation, beyond which the  $\gamma_f$  factor is more accurate. The error for slot separation between two tabulated values is in between the corresponding error values listed. Farther than crossover slot separation of  $0.8\lambda$  or  $0.9\lambda$ , the error is less than the corresponding value tabulated for  $\gamma_f$ .

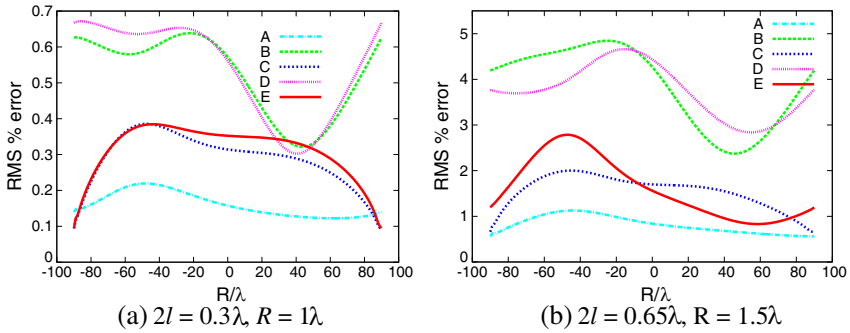
The maximum error in mutual coupling gives an idea about the upper limit for error to be expected. However, the maximum error often occurs when the coupling itself is very less. Hence, the rms error taken over all the azimuths  $\Psi$  for a particular slot tilt  $\theta$ , along with the maximum error provides a good idea about the usefulness of the above approximations. The maximum and rms errors for double and single dipole expressions are plotted against slot tilt at a distance of

**Table 1.** Maximum % error for double dipole approximation at different slot separations with  $\gamma_r$  and  $\gamma_f$  factors.

Sr. No	Slot length $2l$	Factor used	Spacing $d$	error %
1	$0.3\lambda \leq 2l \leq 0.5\lambda$	$\gamma_r$	$1.2 \times 2l$	2.5
	$0.1\lambda \leq 2l \leq 0.3\lambda$		$0.8^{(\frac{2l+0.05}{0.15})} \times 4l$	
	$0.01\lambda \leq 2l \leq 0.1\lambda$		$\sqrt{\frac{0.064}{2l}} \times 1.1^{[2 \log(\frac{0.1}{2l})]} \times 4l$	
2	$0.01\lambda \leq 2l \leq 0.5\lambda$	$\gamma_f$	$0.8\lambda$	1
3	$0.5\lambda \leq 2l \leq 0.65\lambda$	$\gamma_r$	$1.2 \times 2l$	$4.2 \times (\frac{2l-0.5}{0.15})^{1.5}$
4	$0.5\lambda \leq 2l \leq 0.65\lambda$	$\gamma_f$	$0.9\lambda$	$6 \times (\frac{2l-0.5}{0.15})^{1.5}$



**Figure 4.** Maximum % error in mutual coupling with slot tilt. (A- Single dipole approximation with  $\gamma_f$ , B- double dipole approximation with  $\gamma_r$ , C- double dipole approximation with  $\gamma_f$ , D- point dipole approximation with  $\gamma_r$ , E- point dipole approximation with  $\gamma_f$ .)



**Figure 5.** RMS % error in mutual coupling with slot tilt. (A- Single dipole approximation with  $\gamma_f$ , B- double dipole approximation with  $\gamma_r$ , C- double dipole approximation with  $\gamma_f$ , D- point dipole approximation with  $\gamma_r$ , E- point dipole approximation with  $\gamma_f$ .)

$R = 1\lambda$  for slot length  $2l = 0.3\lambda$  and at a distance of  $R = 1.5\lambda$  for  $2l = 0.65\lambda$  in Fig. 4 and Fig. 5 respectively. The  $\gamma_f$  factor is seen to be more accurate than  $\gamma_r$  factor from these figures.

### 3.1.3. Point Dipole Approximation

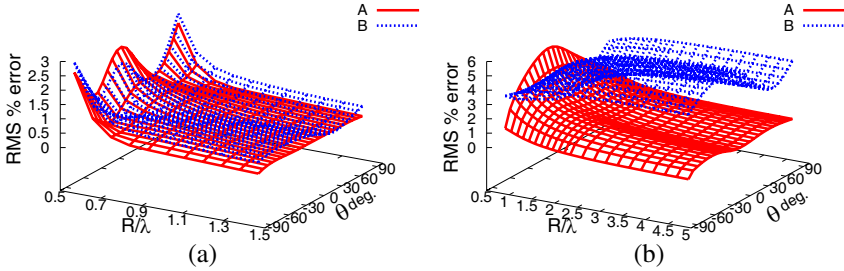
The point dipole approximation leads to simpler of the two expressions. The observations for point dipole approximation are summarised in Table 2 along with the recommended correction factor to be used for some important slot separations. For in between separations, the same comments as given earlier for Table 1 hold good. The error is seen to be about 2.5% for slot separation of  $1.5 \times 2l$  or more for slots shorter than  $0.5\lambda$  and about 6.5% for a separation of  $1.7 \times 2l$  for  $2l = 0.65\lambda$ . The formulation with  $\gamma_f$  gives less error than that with  $\gamma_r$  for slot separation greater than  $0.85\lambda$  for  $2l < 0.5\lambda$  and greater than  $1.3\lambda$  for  $2l > 0.5\lambda$ . The point dipole approximation using  $\gamma_f$  is compared with that employing  $\gamma_r$  in Fig. 4 and Fig. 5 for slot length  $2l = 0.3\lambda$ ,  $R = 1\lambda$  and  $2l = 0.65\lambda$ ,  $R = 1.5\lambda$  and the  $\gamma_f$  factor proposed here is seen to be more accurate than the  $\gamma_r$  factor.

From Table 1 and Table 2 and from Figs. 4 and 5, it can be seen that the point dipole approximation gives error almost comparable to that of double dipole approximation while at the same time, it retains the simplicity of formulation.

The rms error variation for point dipole and double dipole approximation is plotted with slot separation for each tilt angle  $\theta$ , as a 3D plot in Fig. 6. Slot length  $2l = 0.45\lambda$  is used for point dipole approximation in Fig. 6(a) and  $2l = 0.65\lambda$  is used for double dipole

**Table 2.** Maximum % error for point dipole approximation at different slot separations with  $\gamma_r$  and  $\gamma_f$  factors.

Sr. No	Slot length $2l$	Factor used	Spacing $d$	error %
1	$0.3\lambda \leq 2l \leq 0.5\lambda$	$\gamma_r$	$1.5 \times 2l$	2.5
	$0.075\lambda \leq 2l \leq 0.3\lambda$		$1.5 \times 2l \times 1.23^{(\frac{0.3-2l}{0.2})}$	
	$0.01\lambda \leq 2l \leq 0.075\lambda$		$1.9 \times 2l \times 2^{[2 \log(\frac{0.075}{2l})]}$	
2	$0.01\lambda \leq 2l \leq 0.5\lambda$	$\gamma_f$	$0.85\lambda$	1.6
3	$0.5\lambda \leq 2l \leq 0.65\lambda$	$\gamma_r$	$1.7 \times 2l$	$1.6 \times 2^{(\frac{2l-0.5}{0.075})}$
4	$0.5\lambda \leq 2l \leq 0.65\lambda$	$\gamma_f$	$1.3\lambda$	$1.85^{(\frac{2l-0.5}{0.05})}$



**Figure 6.** RMS % error in mutual coupling with slot separation and tilt  $\theta$ . (a) Point dipole approximation,  $2l = 0.45\lambda$ . (b) Double dipole approximation,  $2l = 0.65\lambda$ . (A- With  $\gamma_f$ , B- with  $\gamma_r$ .)

approximation in Fig. 6(b). It is interesting to see how the error varies with tilt angle as the slot separation increases. The maximum rms error decreases with distance for  $\gamma_f$ , while for  $\gamma_r$  it increases after a certain separation and later remains more or less constant at a value much higher than that obtained with  $\gamma_f$  approximation in both the cases.

### 3.2. Discussion

Several interesting observations can be made from these figures. The maximum and rms errors are the least for single dipole approximation with  $\gamma_f$ . For slot lengths lesser than  $0.5\lambda$ , the double dipole approximation with  $\gamma_f$  gives error comparable to that of single dipole approximation with  $\gamma_r$  and the error is better than point dipole approximation. For slot lengths greater than  $0.5\lambda$ , the error with  $\gamma_r$  increases more rapidly than that with  $\gamma_f$ , such that even the point

dipole approximation gives error that is almost equal to that of single dipole approximation with  $\gamma_r$  for  $2l = 0.65\lambda$ . Interestingly, for the point dipole approximation, the maximum error and worst rms error taken over all the azimuths for a particular tilt, for slot lengths between  $0.3\lambda$  to  $0.65\lambda$  and for slot separation  $R = 1.5\lambda$ , is around 4.25% and 2.75%, respectively; almost the same as that calculated from the formulation given in [7] using numerical evaluation of single integral as seen from Fig. 3. Further, the maximum error for point dipole approximation with  $\gamma_f$  decreases for  $R > 0.9\lambda$ , while that for [7] increases. The maximum error for the point dipole approximation for slot lengths  $2l \leq 0.5\lambda$  is less than 1.35% for  $R > 1\lambda$ .

As a practical application, for an RLSA like that in [3], having 1866 slots of arbitrary polarisation with respect to any given slot, the analysis given herein can be efficiently applied. In this case, assuming dielectric filled waveguide and slot lengths  $2l \leq \lambda/2$ , around 97% of the slots lie outside the  $1\lambda$  circle. Then, as seen from Table 2, if simple point dipole approximation with  $\gamma_f$  is used for these slot pairs, mutual coupling for 97% of the slot pairs can be evaluated very efficiently, with an error of less than 1.35%. The slots outside a circle of  $1.25\lambda$  have an error of 1% or less, as compared to a radius of  $2.4\lambda$  for similar error from the formulation in [3].

The approximations proposed here are compared in Table 3 with some of the results in [8] for  $2l = 0.45\lambda$  and parallel displaced slots ( $\theta = 0$ ). The results were found to be generally more accurate than those

**Table 3.** Comparison of results from methods discussed in this work with those from Ref. [8] and Ref. [7] for parallel displaced slots.  $2l/\lambda = 0.45$ ,  $\theta = 0$ .

Sr. No	$y/\lambda$	$z/\lambda$	error in Mutual Coupling $Y_{21}$ (dB)						
			error [8]	single dipole		double dipole		point dipole	
				approx. with		approx. with		approx. with	
				$\gamma_f$	$\gamma_r$ [7]	$\gamma_f$	$\gamma_r$	$\gamma_f$	$\gamma_r$
1	0.5	0	-43	-59	-59	-53	-53	-44	-44
2	3	0	-59	-87	-87	-81	-81	-71	-71
3	0	1	-53	-58	-48	-52	-42	-53	-38
4	0.5	1	-59	-55	-53	-49	-46	-43	-59
5	3	1	-55	-109	-66	-103	-60	-72	-63
6	3	5	-59	-80	-49	-73	-43	-71	-43

reported earlier in [7, 8]. The results from approximations given here are found to be more accurate than those from [9–11], as well. Thus, for slot lengths normally used in practice, it can be seen that the double dipole and point dipole approximations proposed here are simple to use and at the same time, more accurate than other formulations given in the literature. The errors listed in Table 1 and Table 2 are maximum errors in mutual coupling with respect to numerical results at the corresponding minimum separation indicated. These errors are seen to be quite small and the proposed approximations give quite acceptable results for most of the applications in practice. The actual error could be even lesser, depending upon the actual slot length, orientation and separation, thus making the approximations more accurate. Also, for the RLSA example discussed earlier, the formulation employing  $\gamma_f$  factor will give more accurate results than other closed form approximations, for 97% of the cases, leading to better overall results.

The mutual coupling for slots with higher order basis functions can be evaluated using the above approximations and superposition. Mutual coupling for non-planar slots or magnetic sources can be estimated by suitably extending the approximations developed here using similar results for non-planar dipoles, e.g., [15].

#### 4. CONCLUSION

Closed form approximations like the double dipole approximation and the point dipole approximation are developed from the known results for PWS aperture distribution, using the new correction factor  $\gamma_f$  for evaluating the mutual coupling between slots. These approximations are more accurate than other closed form approximations used earlier in the literature and quite useful results are obtained for slot separation of  $1.2 \times \text{slot length}$  or more, depending upon the slot length and the approximation chosen. Elaborate tables are given to aid the choice of approximation to be used. Both the approximations are relatively simple to use as no numerical integration is involved. Also, the maximum error for a particular tilt angle, taken over all azimuth angles between slots, was found to decrease with slot separation. The double dipole approximation gives more accurate and useful overall results, whereas the point dipole approximation with  $\gamma_f$  results in quite simple yet accurate expressions for mutual coupling in practical cases. Using these methods with  $\gamma_f$ , the maximum error can be kept less than 1.6% for slots shorter than  $0.5\lambda$  and separated by  $0.85\lambda$  or more. The methods proposed here are useful for efficient evaluation of mutual coupling in large arrays of slots or for EMC analysis.

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