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COMBINED STRATEGIES BASED ON MATRIX PENCIL METHOD AND TABU SEARCH ALGORITHM TO MINIMIZE ELEMENTS OF NON-UNIFORM ANTENNA ARRAY

S. Yang †

College of Electrical Engineering
Zhejiang University
Hangzhou 310027, China

Y. Liu

School of Electronic Engineering
University of Electronic Science and Technology of China
Chengdu, Sichuan 610054, China

Q. H. Liu

Department of Electrical and Computer Engineering
Duke University
Box 90291, 129 Hudson Hall, Durham, NC 27708-0291, USA

Abstract—The minimization of elements in a non-uniform antenna array is critical in some practical engineering applications such as satellite and mobile communications. However, due to the complexity in the synthesis of an antenna array, the available techniques are not equally successful for reducing the element number of a non-uniform antenna array with as few elements as possible with respect to both solution quality and solution efficiency. In this point of view, a combined strategy based on the matrix pencil method and tabu search algorithm is proposed with the goal of integrating the advantages of the high solution efficiency of the matrix pencil method and the strong global searching ability of the tabu search algorithm when solving an antenna array design problem. In the proposed strategies, the desired radiation pattern is firstly sampled to form a discrete pattern data set. The matrix pencil method is then employed to optimize the excitations

Corresponding author: Q. H. Liu (qhliu@ee.duke.edu).
† Also with Department of Electrical and Computer Engineering, Duke University, USA, as a visiting scholar while working on this paper.

and location distributions of the antenna array elements to reduce the element number. Finally, the excitation and location distributions of antenna array elements are (repeatedly) re-optimized by using a tabu search algorithm by starting from the solution of the matrix pencil method to efficiently find the global solution of the design problem. To make the tabu search algorithm suitable for solving antenna array designs, some innovative approaches such as the elimination of the tabu list, systematic diversification as well as intensification processes for neighborhood creations are made. Numerical examples have shown the effectiveness and advantages of the proposed combined strategies.

1. INTRODUCTION

Recently, the study on antenna arrays has re-flourished because of the significant potential of the antenna arrays in engineering applications. For example, in the age of wireless communications, an array of antennas mounted on vehicles, ships, aircrafts, satellites, and base stations is expected to play an important role in overcoming the problem of limited channel bandwidths, thereby satisfying an ever growing demand for a large number of mobiles on communications channels [1]. However, the premise of realizing communications to mobile devices is that the size of a portable communication device is as small as possible so that it is at an affordable cost. Therefore, the minimization of elements of antenna arrays is essential in the design of antenna arrays.

Synchronizing with the development of science and technology, originally, the main attentions of the study on antenna array designs have been along the line of the synthesis techniques of uniformly spaced arrays. In this regard, a huge amount of techniques are applicable [2]. However, a large number of elements are still required for a desired radiation pattern due to the limitation of uniform element spacing. Indeed, the application of a non-uniformly spaced antenna array will yield less antenna elements for the same desired radiation pattern. In this regard, recently, a large amount of efforts have been made to develop feasible techniques for the synthesis of non-uniform arrays. To name a few, these available techniques include optimization algorithms such as the simulated annealing method (SA) [3], analytical methods [4], and other synthesis techniques such as matrix pencil method [5]. Despite the success of the aforementioned synthesis techniques in reducing the number of elements in a completely nonuniform antenna array, the situation with respect to both solution quality and solution efficiency of these techniques is still unsatisfactory considering the following facts:

(1) The synthesis of a completely non-uniform antenna array is an inverse problem involving finding the solution of multimodal functions with a huge amount of local optima and unknowns (excitation amplitude, phase and position for each element). As well known, the techniques based on deterministic optimal algorithms and analytical methods can not guarantee that the final solution of the synthesis is the global one of the design problem. On the other hand, although in theory the approaches which are based on stochastic optimal algorithms, such as SA, genetic algorithm, differential evolutionary algorithm, and particle swarm optimization, are capable of finding the globally optimal solutions, the heavy computational burden required by these approaches is unaffordable in some practical applications. Moreover, our numerical experiences as reported below have shown that the exclusive usage of a stochastic algorithm in the synthesis of a completely non-uniform antenna array can always find the false solutions of an antenna array design problem.

(2) Most available techniques synthesize antenna arrays with prescribed lengths of arrays. They need to manually vary lengths of arrays to find all possible solutions with fewer array elements. Naturally, this is not a computationally effective and automatic way for synthesis of antenna arrays when the number of elements becomes extremely large.

To address the above two issues, an optimal strategy based on the matrix pencil method and tabu search algorithm is proposed with the goal of integrating the advantages of the high solution efficiency of the matrix pencil method and strong global searching ability of the tabu search algorithm to minimize the number of elements in a completely non-uniform antenna array. In the proposed strategies, the desired radiation pattern is first sampled to form a discrete pattern data set. The matrix pencil method is then used to reconstruct the excitation and location distributions of elements of a non-uniform antenna array to reduce the element number. Finally, the excitations and location distributions of the antenna array elements are repeatedly optimized by using a tabu search algorithm by starting from the solution of the matrix pencil method to efficiently find the global solution of the design problem by following an iterative routine which can symmetrically reduce the minimal number of array elements. To make the tabu search algorithm suitable for solving antenna array designs, some innovative approaches such as the elimination of the tabu list, systematic diversification as well as intensification processes for neighborhood creations are made.

2. MATHEMATICAL MODEL

Consider a liner array of M punctiform and omnidirectional elements placing along the z axis, as shown in Figure 1, the array factor is given by

$$F_M(\theta) = \sum_{i=1}^{M} R_i e^{jkd_i \cos\theta} \tag{1}$$

where, R_i is the complex excitation coefficient of the ith element located at $z = d_i$ along the linear array direction z, and $k = (2\pi/\lambda)$ is the spatial wavenumber.

In this paper, the attention is focused on finding a completely non-uniform antenna array with minimal number of elements to produce a satisfactory or acceptable field pattern which is close enough to the desired one. Consequently, the objective is formulated as:

$$\begin{cases} \text{Min}\,\{Q\} \\ \text{Const.}\left\{ \underset{\{R_i',d_i'\}_{i=1,\ldots,Q}}{\text{Min}} \left\| F_M(\theta) - \sum_{i=1}^{Q} R_i' e^{jkd_i'\cos\theta} \right\|_L \right\} \leq \varepsilon \end{cases} \tag{2}$$

where, R_i' and d_i' $(i = 1, \ldots, Q \leq M)$ are, respectively, the complex excitations and locations of the ith element for the Q antenna elements, and $L = 2$ if the least square error (LSE) is used.

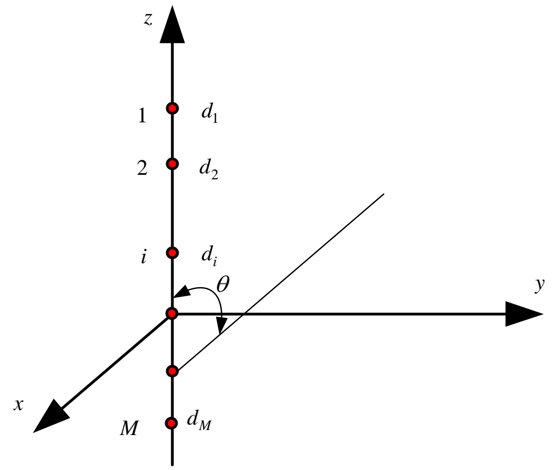

Figure 1. The schematic diagram of a liner antenna array of M punctiform and omnidirectional elements placing along the z axis.

In this study, the initially minimal number, say Q, and the initial values of excitations and locations of the antenna array are determined by using the matrix pencil method. The excitations and locations of each element of the Q element array are then optimized with the goal of minimizing the deviation between the desired radiation pattern and that produced by the completely non-uniform Q elements array obtained by using a tabu search algorithm. For this purpose, the mathematical model of the optimal problem is formulated as:

$$\min f = \sqrt{\frac{\sum_{i=1}^{N}\left[f_{desired}^{norm}(\theta_i) - f_{designed}^{norm}(\theta_i)\right]^2}{\sum_{i=1}^{N}\left[f_{desired}^{norm}(\theta_i)\right]^2}} \tag{3}$$

where, $f_{desired}^{norm}(\theta_i)$ is the value of the normalized desired radiation pattern at sampling point θ_i; $f_{designed}^{norm}(\theta_i)$ is the value of the radiation pattern produced by a designed array of Q elements; N is the number of total sampling points and set to be 2001 in this paper.

3. A COMBINED STRATEGY TO MINIMIZE ARRAY ELEMENTS

To minimize the number of elements in a completely non-uniform antenna array, a strategy combining the matrix pencil method and tabu search algorithm is proposed.

3.1. The Combined Optimal Strategy

Different from available techniques, the minimal number of elements in a non-uniform antenna array, and the excitations and locations of each element are optimized simultaneously in an automatically iterative way in the proposed strategy. This iterative procedure is described as:

Step 1: For a given error criterion ε_{given}, set $\varepsilon = \varepsilon_{given}$, as given in (2). The number of the elements in the antenna array, and the excitations and locations of the elements are estimated by using the matrix pencil method from solving (2).

Step 2: A tabu search algorithm is then used to optimize the excitations and locations of the array elements based on the objective function as defined in (3) by using the estimated values of array element number, locations, and excitations in Step 1 as the initial solution.

Step 3: Error test. If the error between the desired pattern and that produced by an optimized non-uniform array in *Step* 2 is smaller than ε_{given}, go to *Step* 4; otherwise, go to *Step* 5.

Step 4: Set $\varepsilon = \alpha\varepsilon$ and then go to *Step* 2.

Step 5: Output the final solution of the minimal number of the elements, and the excitations and the locations of the nonuniform antenna arrays.

It should be pointed out that the value of parameter α in *Step* 4 should be greater than 1.

3.2. The Matrix Pencil Method

The details about the matrix pencil method and its application to reduce the number of elements in a linear antenna array are referred to [5]. However, for the paper to be self contained, the following will give a brief description of this method.

In the matrix pencil method based approach to reduce the number of the elements of a linear antenna array, the desired radiation pattern is first sampled to form a discrete pattern data set. Then the discrete data set are organized in a form of Hankel matrix, and the singular value decomposition of the matrix is performed. By discarding the non-principal singular values, an optimal lower-rank approximation of the Hankel matrix is obtained. Essentially, the lower-rank matrix actually corresponds to fewer antenna elements. The matrix pencil method is then utilized to reconstruct the excitation and location distributions from the approximated matrix.

3.3. The Tabu Search Algorithm

The tabu search algorithm is a metaheuristic procedure that guides the local heuristic search procedure to explore the solution space to avoid local optimality. The structures used in literatures vary from very simple ones that include only certain basic components to those with very complicated strategies such as variable tabu lists, systematic diversification and intensification processes for neighborhood creations as well as short and long term memories, resulting in different performances and complexities in programming.

The proposed tabu search algorithm is based on that developed by Hu [6]. However, some improvements are proposed to enhance its global search ability and make it more robust in a general manner. For the space limitations, only the improvements made in this paper are explained, and the details about the tabu search method are referred to [6, 7].

3.3.1. Intensification Phase

To reinforce the moves that incorporate the attributes of good solutions founded in the previous search process, an intensification searching phase is designed within the proposed tabu search algorithm. The first N_B best solutions searched so far are memorized in P_{best} and used in this phase to try to find better solutions by following the procedures below:

Step 1: For each of the N_B best solutions, a new candidate solution is generated by using

$$x_d^i(new) = r_1 x_d^i(old) + (1 - r_1)\left[g_d - x_d^i(old)\right] \quad (4)$$

where, $x_d^i(new)/x_d^i(old)$ is the dth component of the ith candidate/current best solution; g_d is the dth component of the best solution so far searched; r_1 is a random parameter which is chosen uniformly from [0 1].

Step 2: Update P_{best}.

Step 3: Termination Test: If the test is passed, terminate the intensification phase, and go to the diversification strategy; otherwise, go to *Step* 1 for the next cycle of iterations.

3.3.2. Diversification Phase

To drive the search into unexplored space uniformly to escape from local optima, a diversification phase is included in the proposed tabu search algorithm. In this regard, a new formula is proposed for new move generations. This formula differs from the most common ones used in the study of computational electromagnetics. Mathematically, for a current state, $x(old)$, its random move, $x(new)$, in its h_l neighbor is generated by using

$$x_d(new) = x_d(old) + r_2\delta_d \quad (5)$$

where, $x_d(new)/x_d(old)$ is the dth component of $x(new)/x(old)$; r_2 is a random parameter which is chosen uniformly from $[(N_l)_d,\ (N_u)_d]$; δ_d is a precision parameter of the decision variables in the dth coordinate direction, and

$$(N_l)_d = \text{integer}\left\{\max\left(\frac{a_d - (x_{old})_d}{\delta_d}, \frac{-h_l}{\delta_d}\right)\right\} \quad (6)$$

$$(N_u)_d = \text{integer}\left\{\min\left(\frac{b_d - (x_{old})_d}{\delta_d}, \frac{h_l}{\delta_d}\right)\right\} \quad (7)$$

where a_d and b_d are, respectively, the lower and upper bounds of the decision variables in the dth coordinate direction.

3.3.3. Transition between Inten- and Diver-sification Phases

The proposed tabu search algorithm will dynamically determine when to change phases, i.e., when to start from the intensification phase; if the objective function has not improved significantly in the last consecutive k_D iterations, the algorithms will switch to the diversification phase; the algorithm will continue in the diversification phase until the objective function begins to improve or if a maximum number of iterations in diversification searches is reached.

3.3.4. Elimination of the Tabu List

In accordance with the original tabu search algorithm for combinatorial optimal problems, the most available tabu search algorithms applied to computational electromagnetics still inherit the usage of a tabu list. However, the following arguments show that it is not wise to use a tabu list in optimal problems involving continuous variables. To explain clearly, let x be the current state, y be the best solution of feasible moves generated in the neighbors of x. Now y is accepted as a new current one, the corresponding step h_l is memorized in the tabu list to prevent the iterative procedure to move back to x; the algorithm begins a new cycle of iterations; one should keep in mind the facts:

(1) Not only the transfer form y to x (denoted using x_{ik} in what follows) is forbidden, but also to all states in the h_l neighbor of state y.

(2) The moving back from y to x_{ik} can not be effectively avoided if x_{ik} is located in some other neighbor h_j of y;

(3) With the increment of the elements in the tabu list, the number of feasible moves generated in the current state reduces, thus the information used to guide searches is also reduced; correspondingly the possibility for the algorithm to be trapped in local optima increases;

(4) The cycle or stagnation occurs in the combinatorial optimization for tabu search methods can be avoided effectively in the case of the optimization of multimodal functions with continuous variables by moves of different size neighbors.

So the tabu list is eliminated in the proposed algorithm.

3.3.5. The Power of the Proposed Tabu Search Algorithm for Global Optimizations

A well designed mathematical function having 10^5 local optima, which is categorized as a hard function for an optimizer to find the global

Table 1. The averaged performances of 100 runs of the proposed tabu search algorithm for solving the mathematical function.

No. of iterations	Success rate	Error between the searched and the exact solution
1128	100/100	6.75×10^{-8}

solution, is solved by using the proposed tabu search algorithm to demonstrate its searching power for global optimal solutions of complex multimodal functions. Mathematically, the function is defined as

$$\begin{aligned} \text{minimize } f(x) = & \frac{\pi}{n}\left\{10\sin^2(\pi x_1) + (x_n-1)^2 \right. \\ & \left. + \sum_{i=1}^{n-1}\left[(x_i-1)^2(1+10\sin^2(\pi x_{i+1}))\right]\right. \\ \text{subject to } & -10 \le x_i \le 10 \quad (i = 1, 2, \ldots, 5) \end{aligned} \tag{8}$$

For this function, the proposed tabu search algorithm independently runs 100 times, and the averaged performances of these 100 runs are given in Table 1. Here, a success run or "to find the exact global solution" means that the tolerance between the searched and exact global solutions is 10^{-7} in absolute value. Obviously, the global search ability of the proposed tabu search algorithm is powerful and robust since its success rate to find the exact global optima of this complex multimodal function having nearly infinite local optima is 100% for the 100 independent runs.

4. NUMERICAL EXAMPLES

To show the advantages of the proposed optimal strategy over other available techniques in reducing the elements of a completely nonuniform antenna array, extensive computer simulations are conducted, and only three of these case studies are reported.

4.1. Case Study One

In this case study, the desired radiation pattern is produced by using a uniformly spaced Woodward array of 18 elements with the main beam starting and ending at angles of 70° and 110°, respectively.

In the numerical experiment, the proposed optimal strategy is used to optimize only the excitations and locations of the nonuniform antenna array elements, with the number of elements already determined by the matrix pencil method in [5]. For the purpose of performance comparison, the proposed tabu search algorithm (Tabu-Direct) is also run directly on the optimal problem as formulated in (3) by starting randomly under the same element number of arrays. In addition, the matrix pencil method (MPM) [5] is solely employed to study the same problem. In the numerical implementation, each strategy (algorithm) is run 100 times to evaluate its robustness.

For the desired radiation pattern produced by an ***18*** uniformly spaced Woodward array, the number of elements of the non-uniform arrays optimized by using the matrix pencil method is ***9***. The averaged performance parameters of the aforementioned three optimal (algorithms) strategies of the 100 runs are tabulated in Table 2. It should be pointed out that the values of the objective function in this table are normalized by using that of the optimized results of the matrix pencil method as the base value. The location, amplitude (Ampl_R) and phase (Phas_R, in degree) of R_i for each element of the optimized arrays of a typical run of the three optimal (algorithms) strategies are presented in Table 3, and the corresponding radiation

Table 2. Averaged performances of different (algorithms) strategies of 100 runs on minimizing elements of a nonuniform antenna array.

Strategy	Number of averaged iterations	Averaged value of objective function (pu)
MPM	/	1
Proposed Strategy	44050	0.5782
Tabu-Direct	124055	0.7330

Table 3. Positions, amplitudes and phases of the optimized nonuniformly spaced array by using different methods.

MPM			Proposed Strategy			Tabu-Direct		
d_i/λ	Ampl_R	Phas_R	d_i/λ	Ampl_R	Phas_R	d_i/λ	Ampl_R	Phas_R
4.251	0.6179	0.0	4.352	0.5945	−0.1	4.309	0.2518	−0.0
3.540	1.5236	0.0	3.629	1.4586	−0.1	3.617	0.5225	−0.0
2.242	2.0544	−180.0	2.196	2.0308	180.0	2.218	0.6430	180.0
0.727	5.6793	−0.0	0.734	5.6822	0.0	0.874	1.3536	0.0
0.000	8.6778	−0.0	−0.000	8.7099	0.0	0.347	1.8531	0.0
−0.727	5.6793	0.0	−0.735	5.6768	0.0	−0.187	2.4727	0.0
−2.242	2.0544	180.0	−2.197	2.0300	180.0	−0.843	1.6419	0.0
−3.540	1.5236	−0.0	−3.630	1.4578	−0.0	−2.257	0.6316	180.0
−4.251	0.6179	0.0	−4.353	0.5929	0.1	−3.663	0.4717	0.

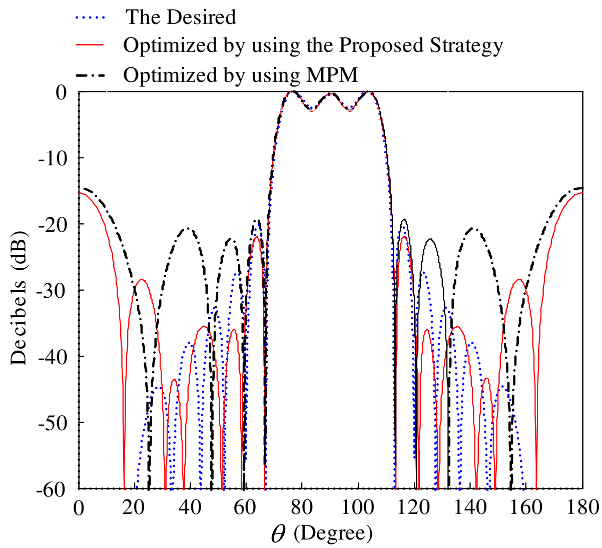

Figure 2. Comparison of the field pattern of the optimized non-uniform arrays by using MPM and the proposed Strategy.

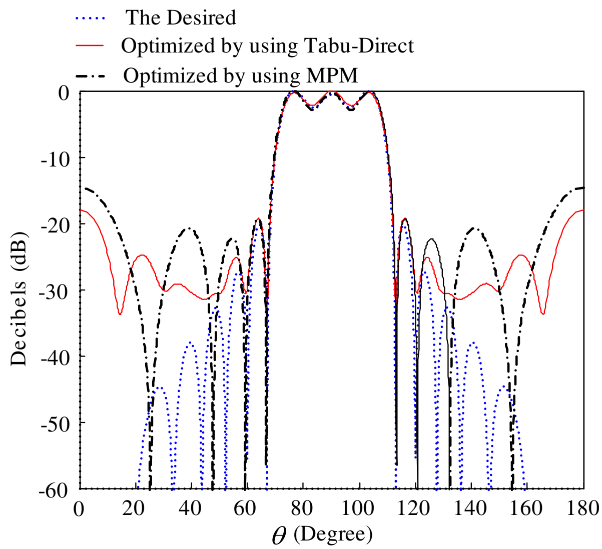

Figure 3. Comparison of the field pattern of the optimized nonuniform arrays by using MPM and the tabu search algorithm directly.

patterns of the optimized antenna arrays and the desired one are depicted in Figures 2–3.

From these numerical results, it is obvious that:

(1) Due to the complexity in the optimal design of a non-uniform antenna array, the powerful tabu search algorithm having strong global searching ability as already validated in Section 3.3.5 can not find an "acceptable" solution compared with that of the proposed combined strategy.

(2) The proposed strategy finds the "far best" solution of this case study. One uses the "far best" rather than the "global optimal" here to define the final solution obtained by the proposed strategy because one can not guarantee that this solution is the global one although it is "far better" than those of both MPM and Tabu-Direct.

(3) Although the matrix pencil method is computationally efficient, its final solution is tremendously worse than that of the Tabu-Direct, to say nothing of that of the proposed strategy for this shaped beam pattern.

(4) The number of averaged iterations of the proposed is about one-third of that of the Tabu-Direct one.

(5) In summary, the proposed strategy is superior to the Tabu-Direct approach in view of both the solution efficiency and solution quality.

4.2. Case Study Two

From the results of Figures 2–3, it is obvious that the proposed strategy does significantly improve the synthesis results of available techniques. However, the synthesized sidelobe level (SLL) is not acceptable in certain senses. After extensively investigating on the numerical result for case study one, one finds that the result is due to the significant difference in the order of the absolute value of the field pattern in the main beam and sidelobe level. To address this problem, a new objective function is designed and used in this case study. Mathematically, this new objective function is the aggregation of two different ones and can be formulated as

$$\min f_{new} = w_1 f + w_2 f_2 \tag{9}$$

where, f is given in (3); w_1 and w_2 are two weighting factors; f_2 is defined as

$$f_2 = \sqrt{\frac{\sum_{i=1}^{N}\left[\frac{f_{desired}^{norm}(\theta_i)-f_{designed}^{norm}(\theta_i)}{f_{designed}^{norm}(\theta_i)}\right]^2}{\sum_{i=1}^{N}\left[f_{desired}^{norm}(\theta_i)\right]^2}} \tag{10}$$

Obviously, due to the difference in the order of the absolute value of the field pattern in the main beam and sidelobe level, the objective function f favors optimizing the field pattern in the main beam, and the objective function f_2 gives equal emphasis on the whole pattern. Consequently, one would expect an equal success in significantly improving the synthesis results in both the main beam pattern and sidelobe level and shapes if the weighting factors w_1 and w_2 are well compromised. Moreover, after some numerical experiments, one selects $w_1 = 0.999999$ and $w_2 = 1 \times 10^{-6}$ in this paper.

In this case study, the proposed strategy is used to minimize the elements of a non-uniform antenna array once the desired pattern is given with a predefined error criterion. This study will show the advantage of the proposed optimal strategy to further reduce the element number of an antenna array that has been obtained by another available synthesis technique. The desired pattern is the same as that used in case study one. To begin with, the procedure starts to use the matrix pencil method to solve (1) and to estimate the maximum number of the elements in the non-uniform arrays, and then to use the proposed tabu search algorithm to optimize the locations and excitations by starting from the values of these corresponding parameters obtained by using the matrix pencil method. From the numerical results of the previous section, it is clear that the error between the desired pattern and that produced by the just optimized array by means of the tabu search algorithm in this turn will be smaller than the predefined error criterion. Consequently, as described previously, the procedure will symmetrically increase the error limit for the matrix pencil method and then solve the optimal problem of (9) repeatedly until the error between the desired pattern and that produced by the optimized array of the tabu search algorithm is equal to or larger than the predefined error criterion. For this case study, it is found that ***4*** iterative cycles are needed for the proposed strategy to converge to the final solution. In view of the number of the antenna array, the number required for a non-uniform antenna array searched by using the proposed strategy to produce an acceptable field pattern to the desired one within the tolerable error is ***6***, which is compared to ***9*** when a matrix pencil method is solely used for the same error criterion.

Table 4. Performance comparison of the proposed strategy and the matrix pencil method under the same error criterion condition.

Strategy	No. iterations	No. Elements	Objective function (pu)
MPM	/	9	1
Proposed Strategy	42521	6	0.549

Table 5. The final solutions of the 6 elements array obtained by using the proposed strategy under the same error condition.

d_i/λ	Ampl_R	Phas_R
3.583840	1.264891	13.416970
2.074514	1.930540	−178.880800
0.6745915	5.702659	−2.213324
−0.003548275	8.725947	0.3489524
−0.6948128	5.473213	1.533463
−2.131860	2.160556	173.760800

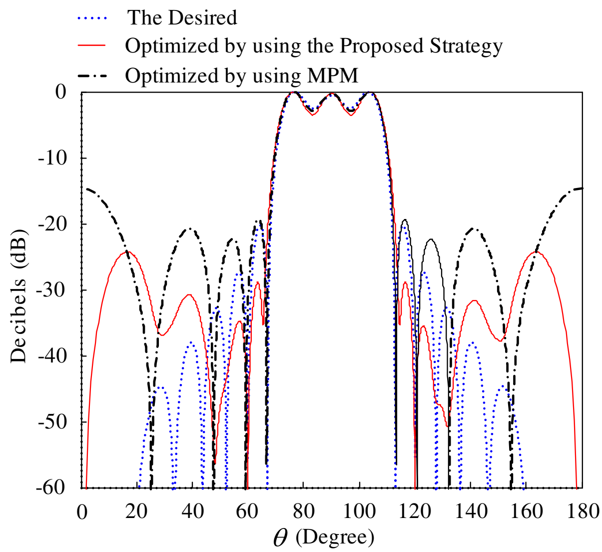

Figure 4. Comparison of the field pattern of the optimized non-uniform 9 element array by using the proposed strategy in the first cycle of the iterations and that of the optimized non-uniform 9 element array by using MPM.

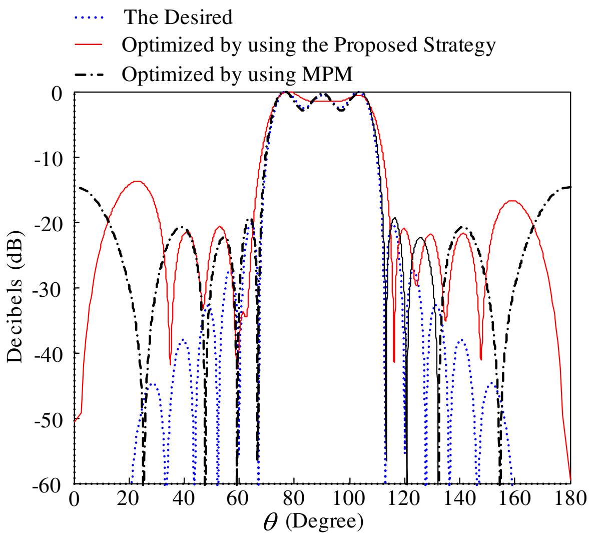

Figure 5. Comparison of the field pattern of the optimized non-uniform 6 element array by using the proposed strategy in the third cycle of the iterations and that of the optimized non-uniform 9 element array by using MPM.

Table 4 summarizes, in details, the performance comparison results of the proposed optimal strategy and matrix pencil method, and Table 5 gives the final optimal results of the optimized 6 elements non-uniform antenna array of the proposed optimal strategy. Figure 4 and Figure 5 depict, respectively, the corresponding field patterns of the optimized 9 and 6 element non-uniform antenna arrays searched in the first and third cycles of the proposed combined optimal strategy, as well as that produced by the optimized 9 elements non-uniform antenna array of the matrix pencil method and the desired one for comparison purpose. To give an intuitive impression of the convergence trajectory of the proposed optimal strategy, the number of the elements in the antenna array and the objective function values for each cycle in the iteration process of the proposed strategy are shown in Table 6. Also, the values of the objective function are normalized by using that of the optimized 9 elements non-uniform array of the matrix pencil method as the base value.

From these numerical results it is natural to draw that:

(1) If an appropriate optimal model is used, the proposed strategy can significantly improve the synthesis results of available techniques such as the matrix pencil method in both the main beam and sidelobe level;

Table 6. The numbers of the elements of the optimized antenna array and the objective function values for each cycle of the proposed strategy under the same error condition.

No. cycle	No. optimized antenna array	Objective function value (pu)
1	9	0.2325
2	8	0.2988
3	6	0.5490
4	5	1.02454

(2) If a pertinent optimal model is used, the proposed strategy can also produce an acceptable beam shape for the desired radiation pattern;

(3) In view of reducing the number of elements of a non-uniform antenna array, the proposed optimal strategy is more effective compared with the available techniques such as matrix pencil method under the same error tolerance;

(4) In terms of solution speed, the proposed strategy is computationally expensive compared with the available techniques MPM.

In summary, it is argued and should be emphasized that the number of the elements in the completely non-uniform antenna array optimized by available techniques can be further reduced, such as from ***9*** to ***6*** for this case study, by using the proposed strategy without sacrificing the solution quality or accuracy. Moreover, the proposed optimal strategy is applicable to improve synthesis results of available techniques in both main beam and sidelobe level, and beam shape if an appropriate mathematical model (objective function) is designed and used.

4.3. Case Study Three

As the last case study, a shaped beam with a cosecant variation is reconstructed by using non-uniform antenna array. The desired pattern is defined as [8]: the field will vary following a cosecant function in interval $\cos\theta \in [0.1\ 0.5]$ and having maximum SideLobe Level (MSLL) less than -25 decibels in the residual intervals. Mathematically, the desired pattern is defined as:

$$F_D(\cos\theta) = \begin{cases} \text{cosecant}(\cos\theta) & (0.1 \leq \cos\theta \leq 0.5) \\ 0 & (\text{elsewhere}) \end{cases} \qquad (11)$$
$$MSLL_D \leq -25\,\text{dB}$$

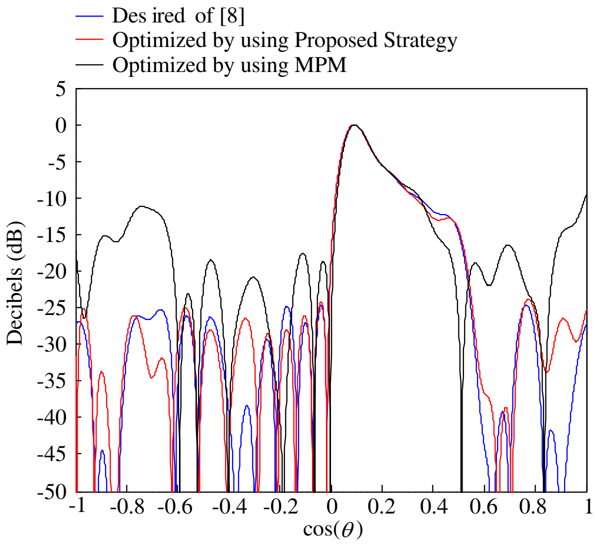

Figure 6. Comparison of the field patterns of the optimized non-uniform 19 element array by using the proposed optimal strategy and the MPM, as well as the desired one of a uniform 30 element array.

To optimize the non-uniform antenna array to produce an acceptable pattern to the desired one with as few elements as possible, the proposed combined optimal strategy and matrix pencil method are used. In this case study, the objective function as defined in (9) is used, with other things being equal to case study one. For the desired radiation pattern produced by a ***30*** uniformly spaced antenna array optimized by using a modified tabu search algorithm [8], the number of elements of the non-uniform arrays searched by using the matrix pencil method and proposed combined strategy is ***19***. Figure 6 shows the field patterns of the optimized non-uniform antenna arrays as well as the result given by [8]. Table 7 presents the final optimal results of these 19 element arrays. The values of the objective function for the reconstructed non-uniform antenna arrays of the proposed strategy and the MPM are, respectively, 6.10743×10^{-2} and 4.27919×10^{-1} in absolute value. From these numerical results, it is evident that, in view of optimizing an antenna array to producing an acceptable shaped beam,

(1) the proposed optimal strategy is superior to the MPM method with respect to the entire reconstructed pattern;

(2) the proposed algorithm outperforms significantly an available

Table 7. Positions, amplitudes and phases of the optimized nonuniformly spaced array by using different methods for case study three.

MPM			Proposed Strategy		
d_i/λ	Ampl_R	Phas_R	d_i/λ	Ampl_R	Phas_R
7.233222	0.012237	12.61126	7.070958	0.01.084696	18.69492
6.601439	0.006196	23.91222	6.310844	0.00948048	26.40717
5.818535	0.018473	21.16231	5.711176	0.01649258	11.08393
5.090666	0.031071	18.17439	5.023398	0.02354693	19.04838
4.108876	0.036996	46.75605	4.150241	0.02859240	55.56984
3.356791	0.047933	39.47542	3.267579	0.04488219	53.81874
2.377290	0.048210	76.42467	2.401060	0.05638363	84.47192
1.408240	0.072186	88.72939	1.488790	0.07814772	95.87332
0.831110	0.113386	131.24110	0.7070541	0.1213742	128.51710
0.000000	0.151119	180.00000	0.000579453	0.1434754	−179.99950
−0.831110	0.113386	−131.24110	−0.7057374	1.214775	−128.58450
−1.408240	0.072186	−88.72939	−1.487224	0.07825839	−95.92277
−2.377290	0.048210	−76.42467	−2.399645	0.05636728	−84.47823
−3.356791	0.047933	−39.47542	−3.266262	0.04489603	−53.86277
−4.108876	0.036996	−46.75605	−4.148553	0.02864174	−55.55625
−5.090666	0.031071	−18.17439	−5.021846	0.02361438	−19.03081
−5.818535	0.018473	−21.16231	−5.711895	0.01661726	−11.15831
−6.601439	0.006196	−23.91222	−6.315044	0.00940931	−26.44066
−7.233222	0.012237	−12.61126	−7.072688	0.01082110	−18.78617

technique which is based on the tabu search algorithm as developed in [8] in sense of the number of elements used in the antenna array to produce the same shaped pattern. Concluded in one word, the proposed strategy is also applicable to improve synthesis results of available techniques on producing an acceptable shaped beam.

5. CONCLUSION

A combined strategy based on the matrix pencil method and tabu search algorithm is proposed for optimizing a completely non-uniform antenna array. Compared with the available techniques, the salient advantage of the proposed strategy is that it can further improve the final solution of the available techniques. The error between the desired pattern and that reconstructed by an available technique for a non-uniform antenna array can be further reduced, if the number of the antenna elements is kept constant. The exclusive merit of the proposed strategy is that the minimal number of the elements of the optimized non-uniform antenna array can be further decreased, if the error tolerance is kept constant. Consequently, the proposed works provide a feasible and efficient alternative for non-uniform antenna array designs. Moreover, the numerical results have also revealed that the proposed strategy would be equally successful in the shaped beam pattern synthesis problems if a suitable optimal model (objective function) is designed and used, and this is the direction for future work in antenna array design.

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