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SCATTERING BY JACKET STRUCTURES ANALYSIS VIA THE EXTENDED METHOD OF AUXILIARY SOURCES EMAS

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Abstract—This paper establishes the extension of the method of auxiliary sources EMAS for the purpose of modeling the electromagnetic scattering response by jacket cylindrical structures constituted by a finite number of dielectric eccentric cylindrical inclusions embedded in a host dielectric one. Appropriate boundary conditions mixed with judicious decomposed domains leads to the prediction of the backscattering cross section. The algorithm also integrates the global electromagnetic coupling between the inclusions. The EMAS is validated by varying the inner cylinders repartitions and fine-tuning the electric permittivity according to different geometries. The EMAS level of accuracy compared with the indirect matching mode method IMM reveals a good agreement between the numerical computation results.

1. INTRODUCTION

The purpose of this paper is to develop an efficient technique in order to gauge the electromagnetic scattered fields from a dielectric cylinder with N parallel eccentric cylindrical dielectric inclusions; such a structure has been studied widely [1, 2]. An accurate addition was published [3], and a faithful problem solution was formulated in [4]. Various applications are linked to this problem, such as recognition of fabrication imperfections in multifibre optical cables, quality check of cotton, etc. [4].

The method of auxiliary sources MAS has the following advantages: being meshless, not needing a complicated discretisation of the domain, not being integrated over the boundary, being

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straightforward to implement [3] and broadly being used to model scattering problems (photonics, metamaterials, arrays, etc.) [4, 5].

The distinctive highlight of the auxiliary sources method MAS compared to mesh ones (MoM, FEM, FDTD, etc.) is to swap the differential equation and boundary conditions [6], inducing the singularity removal in the singular integral equation by swinging the auxiliary sources contour relative to integration one [7], entraining the non vanishing distance separating sources and observation points. Thus, the singularity Green's function problem is eliminated. Proceeding from this, the electromagnetic fields are directly estimated by expanding them over the basis generated by the particular solutions of Helmholtz equation [8]. Moreover, the method of auxiliary sources is more accurate, efficient, and versatile; it also reveals important computational advantages over the method of moments [19]. Thus, the MAS is surely preferable to the MoM for a large class of electromagnetic problems [20] and particularly for the structure mentioned before.

According to the method of auxiliary sources and for a dielectric cylinder without inclusion illuminated by a monochromatic plane wave, we have to set up two bases of auxiliary sources around the boundary. The interior one operates outside the cylinder and the exterior one inside [9], knowing that the auxiliary sources are the particular solutions of Helmholtz equation [10].

The scattering matrix method SMM cannot deal with structures in which the set of cylinders is embedded inside a jacket consisting of a dielectric medium that is itself surrounded by another external medium [5], but the combination of SMM and fictitious sources methods MFS permits the efficient investigation of finite dielectric body riddled by galleries problems. The method illustrated in this paper is fundamentally based on the MAS essence with acclimatized boundary conditions and passing around hybrid numerical methods complexity [5, 15].

The foremost contribution here is to subdivide the inhomogeneous structure in finite, homogeneous, recovered and strongly coupled mediums [11–13]. For each inclusion boundary, the sum of the radiated electromagnetic fields by the overall inner different auxiliary sources bases inclusions and the outer host cylinder ones satisfies the tangential continuity condition leading to a linear system. The method established here can deal with sets of objects having different electromagnetic parameters and shapes. Nevertheless, we will limit our theoretical analysis to circular cylindrical forms.

The extended method of auxiliary sources EMAS is applied to model coupling between different, infinite, and parallel dielectric cylindrical inclusions, illuminated by a TM monochromatic plane wave. The global coupling is modeled through the overall mutual fulfilment of the boundary conditions just on the collocation points of every cylinder.

These boundary conditions lead to a linear system having as solution the amplitude and phase of scattered fields [3].

Commonly, each part of an array interacts with the rest of the array in the aim to model precise or global coupling between different parts which conduct to a wholly filled matrix entraining a difficult computation, particularly for a large number of inclusions.

Numerical results acquired by the EMAS algorithm code agree very well with the indirect matching mode references [4] for the global coupling.

The time factor is assumed to be $e^{j\omega t}$ and suppressed throughout this paper.

2. EXTENDED METHOD OF AUXILIARY SOURCES FORMULATION

Let us consider an infinite z-axis cylindrical structure composed of L cylindrical dielectric inclusions in the host dielectric one. Each part of the space is typified by the wavenumber $k_{(m)}$, and to the index m, are allocated the integers $0, 1, 2, \ldots, L$. Free space is assigned for the exterior region of the host cylinder (region 0). We assume that a plane wave impinging from a direction with polar angle φ_{inc} illuminates the structure. The incident wave polarization is presumed to be transverse magnetic TM with respect to z-axis, where the electric field has only a z-component [17, 18]. We symbolize the permittivity of the jth dielectric cylindrical inclusion by ε_j. The geometry is revealed in Fig. 1.

The incident transverse electromagnetic wave has electric and magnetic fields [19]:

$$\begin{aligned} E_z^{inc}(x,y) &= E_0 \exp\{j(k_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\}\hat{z} \qquad (1) \\ H^{inc}(x,y) &= -\frac{E_0}{Z_0}(\hat{x}\sin\varphi_{inc} - \hat{y}\cos\varphi_{inc}) \\ &\qquad \exp\{j(k_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\} \qquad (2) \end{aligned}$$

Here, k_0 is the vacuum wavenumber, and $\hat{z}$ denotes the unit vector in the z-direction. Since the incident electric field is z-directed and independent of z, we deduce that the scattered field is z-directed too, downgrading the scattering problem to a bidimensional one.

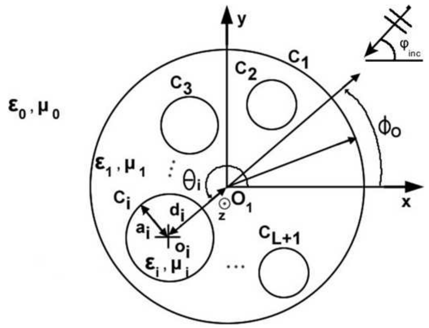

Figure 1. Geometry of the problem.

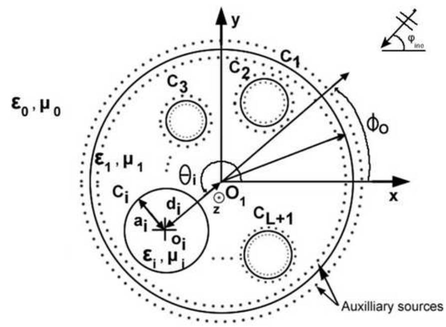

Figure 2. Bases of auxiliary sources repartition around the physical contours.

For every boundary, two auxiliary sources are regularly distributed outside (out) and inside (in) the boundary contour on which are sited the collocation points (Fig. 2).

The continuity of the tangential total electric field [14] on every boundary collocation point leads to:

$$\text{For the boundary } 1: \qquad E_{(1)}^{1,out} = \sum_{l=1}^{L+1} E_{(1)}^{l,in} + E_{(1)}^{inc} \tag{3}$$

$$\text{For the boundary } 2: \qquad E_{(2)}^{2,out} = \sum_{l=2}^{L+1} E_{(2)}^{l,in} + E_{(2)}^{1,out} \tag{4}$$

$$\text{For the boundary } 3: \qquad E_{(3)}^{3,out} = \sum_{l=2}^{L+1} E_{(3)}^{l,in} + E_{(3)}^{1,out} \tag{5}$$

$$\text{For the boundary } j: \qquad E_{(j)}^{j,out} = \sum_{l=2}^{L+1} E_{(j)}^{l,in} + E_{(j)}^{1,out} \tag{6}$$

$$\text{For the boundary } L+1: \qquad E_{(L+1)}^{L+1,out} = \sum_{l=2}^{L+1} E_{(L+1)}^{l,in} + E_{(L+1)}^{1,out} \tag{7}$$

$E_{(k)}^{l,out}$ is the total electric field radiated by all the outer auxiliary sources of the boundary l and acting on the boundary k.

$E_{(k)}^{l,in}$ is the total electric field radiated by all the inner auxiliary sources of the boundary l and acting on the boundary k.

We suppose that the number N of auxiliary sources per basis is equal to the number M of collocation points for each boundary. The above $(L+1)\cdot N$ equations have $2(L+1)\cdot N$ unknowns.

The continuity of the tangential total magnetic field on every boundary collocation point leads to:

$$\text{For the boundary 1}: \qquad H_{t,(1)}^{1,out} = \sum_{l=1}^{L+1} H_{t,(1)}^{l,in} + H_{t,(1)}^{inc} \tag{8}$$

$$\text{For the boundary 2}: \qquad H_{t,(2)}^{2,out} = \sum_{l=2}^{L+1} H_{t,(2)}^{l,in} + H_{t,(2)}^{1,out} \tag{9}$$

$$\text{For the boundary 3}: \qquad H_{t,(3)}^{3,out} = \sum_{l=2}^{L+1} H_{t,(3)}^{l,in} + H_{t,(3)}^{1,out} \tag{10}$$

$$\text{For the boundary } j: \qquad H_{t,(j)}^{j,out} = \sum_{l=2}^{L+1} H_{t,(j)}^{l,in} + H_{t,(j)}^{1,out} \tag{11}$$

$$\text{For the boundary } L+1: \qquad H_{t,(L+1)}^{L+1,out} = \sum_{l=2}^{L+1} H_{t,(L+1)}^{l,in} + H_{t,(L+1)}^{1,out} \tag{12}$$

The index t denotes the tangential component of the magnetic field. We obtain conclusively a linear system with $2(L+1)\cdot N$ equations and unknowns. In the object to itemize the system coefficients, some details in the future calculations have to be clarified.

If we consider the boundary j, for example, its outer auxiliary sources will radiate electromagnetic fields in the interior region bounded by this contour and filled with the medium j. Likewise, the auxiliary sources placed inside the boundary j will radiate outside the region filled with the medium 1 bounded by the host cylindrical contour.

Consequently, the cylindrical dielectric inclusions radiate electromagnetic fields collectively in the space filled with the medium one, due to their distinct inner auxiliary sources basis. However, each exterior auxiliary basis will radiate EM fields individually in the inner cylindrical inclusion area. Explicitly, the exterior boundary 1 auxiliary sources breeding fields in the host cylinder operates as the incident excitation according to the inclusions.

Hence, the aforementioned boundary conditions incorporating the domain decompositions reciprocally couples all the dielectric cylinders.

Following this reasoning and considering the boundary j, the outer and inner regions fields are:

$$E_{(l)}^{j,in} = \sum_{i=1}^{N} a_{(j)}^{i,in} H_0^{(2)} \left[K_{(1)} \left| r_{Cm}^{l} - r_{Si}^{j,in} \right| \right] \tag{13}$$

$$E_{(j)}^{j,out} = \sum_{i=1}^{N} a_{(j)}^{i,out} H_0^{(2)} \left[K_{(j)} \left| r_{Cm}^{j} - r_{Si}^{j,out} \right| \right] \tag{14}$$

Where, $H_0^{(2)}$ is the Hankel function of the second kind of zero order, $K_{(j)}$ the wavenumber in the medium j, r_{Cm}^{l} the space vector of the collocation point m on the lth boundary, $r_{Si}^{j,out}$ the space vector of the outer auxiliary source i on the boundary j and $a_{(j)}^{l,out}$ the unknown complex currents on the boundary l.

We define the functions:

$$\begin{aligned} \delta(i-j) &= 1 \text{ if } i=j; 0 \text{ elsewhere} \\ \xi(i-j) &= 0 \text{ if } i=j; 1 \text{ elsewhere} \\ \text{And} \qquad \tau(j) &= 1 \text{ if } j=1; 2 \text{ elsewhere} \end{aligned}$$

Then, the electric field continuity condition for the jth boundary is expressed as:

$$E_{(j)}^{j,out} = \sum_{l=\tau(j)}^{L+1} E_{(j)}^{l,in} + \xi(j-1) E_{(j)}^{1,out} + \delta(j-1) E_{(j)}^{inc} \tag{15}$$

After replacing the electric fields by their expressions, for the collocation point m on the boundary j:

$$\begin{aligned} &\sum_{i=1}^{N} a_{(j)}^{i,out} H_0^{(2)} \left[K_{(j)} \left| r_{Cm}^{j} - r_{Si}^{j,out} \right| \right] \\ &= \sum_{l=\tau(j)}^{L+1} \sum_{i=1}^{N} a_{(l)}^{i,in} H_0^{(2)} \left[K_{(1)} \left| r_{Cm}^{j} - r_{Si}^{l,in} \right| \right] + \xi(j-1) E_{(j)}^{1,out} \delta(j-1) E_{(j)}^{inc} \end{aligned} \tag{16}$$

By varying, $1 \ll m \ll N$ and $1 \ll j \ll L+1$, we get $(L+1) \cdot N$ equations.

The continuity of the total tangential magnetic field on jth boundary collocation m permits:

$$\begin{aligned} &n_{(j)m} \Lambda H_{(j)}^{j,out} \\ &= \sum_{l=\tau(j)}^{L+1} n_{(j)m} \Lambda H_{(j)}^{l,in} + \xi(j-1) n_{(j)m} \Lambda H_{(j)}^{1,out} + \delta(j-1) n_{(j)m} \Lambda H_{(j)}^{inc} \end{aligned} \tag{17}$$

Where, $n_{(n)m}$ is the unit vector perpendicular to the nth boundary just on the collocation point m.

According to the plane wave scattering by a take apart dielectric cylinder [Appendix A], we get:

$$a_{(l)}^{i,in} = -\left(k_{(1)} Z_{(1)}/4\right) I_{(l)}^{i,in} \tag{18}$$

$$a_{(l)}^{i,out} = -\left(k_{(l)} Z_{(l)}/4\right) I_{(l)}^{i,out} \tag{19}$$

In addition, for the host dielectric cylinder [Appendix A]:

$$a_{(1)}^{i,in} = -\left(k_{(0)} Z_{(0)}/4\right) I_{(1)}^{i,in} \tag{20}$$

$$a_{(1)}^{i,out} = -\left(k_{(1)} Z_{(1)}/4\right) I_{(1)}^{i,out} \tag{21}$$

Where, $Z_{(l)}$ is the medium impedance and $I_{(l)}^{i,in}$ the inner auxiliary source complex current.

Therefore, the magnetic field components for the lth boundary are expressed as:

$$H_y^{l,in} = \sum_{i=1}^{i=N} \left(k_{(1)} \left(x - x_{(l)i}^{i,in}\right)/4jR_{(l)}^{i,in}\right) H_1^{(2)} \left[k_{(l)} R_{(l)}^{i,in}\right] I_{(l)}^{i,in} \tag{22}$$

$$H_x^{l,in} = \sum_{i=1}^{i=N} \left(k_{(1)} \left(y_{(l)}^{i,in} - y\right)/4jR_{(l)}^{i,in}\right) H_1^{(2)} \left[k_{(l)} R_{(l)}^{i,in}\right] I_{(l)}^{i,in} \tag{23}$$

$$H_y^{l,out} = \sum_{i=1}^{i=N} \left(k_{(l)} \left(x - x_{(l)}^{i,out}\right)/4jR_{(l)}^{i,out}\right) H_1^{(2)} \left[k_{(l)} R_{(l)i}^{i,out}\right] I_{(l)}^{i,out} \tag{24}$$

$$H_x^{l,out} = \sum_{i=1}^{i=N} \left(k_{(l)} \left(y_{(l)}^{i,out} - y\right)/4jR_{(l)}^{i,out}\right) H_1^{(2)} \left[k_{(l)} R_{(l)}^{out}\right] I_{(l)}^{i,out} \tag{25}$$

Here, $x_{(n)i}^{in}$ denotes the abscissa of the inner auxiliary source i on the lth boundary and $R_{(l)}^{i,in}$ the distance between the auxiliary source $n^\circ i$ and the point with (x, y) coordinates.

The final linear system will have this expression:

$$\begin{aligned}
&\sum_{i=1}^{N} a_{(j)}^{i,out} H_0^{(2)}\left[K_{(j)}\left|r_{Cm}^{j} - r_{Si}^{j,out}\right|\right] - \sum_{l=\tau(j)}^{L+1} \sum_{i=1}^{N} a_{(l)}^{i,in} H_0^{(2)}\left[K_{(1)}\left|r_{Cm}^{j} - r_{Si}^{l,in}\right|\right] \\
&-\xi(j-1) \sum_{i=1}^{N} a_{(1)}^{i,out} H_0^{(2)}\left[K_{(1)}\left|r_{Cm}^{j} - r_{Si}^{1,out}\right|\right] = \delta(j-1) E_{(j)}^{inc}
\end{aligned} \tag{26}$$

$$\begin{aligned}&\left(n_{x(j)m}H_{y(j)}^{j,out}-n_{y(j)m}H_{x(j)}^{j,out}\right)-\left(\sum_{l=\tau(j)}^{L+1}\left(n_{x(j)m}H_{y(j)}^{l,in}-n_{y(j)m}H_{x(j)}^{l,in}\right)\right)\\&-\xi(j-1)\left(n_{x(j)m}H_{y(j)}^{1,out}-n_{y(j)m}H_{x(j)}^{1,out}\right)\\&=\delta(j-1)\left(n_{x(j)m}H_{y(j)}^{inc}-n_{y(j)m}H_{x(j)}^{inc}\right)\end{aligned} \tag{27}$$

The preceding system is written under the form: $Z \cdot I = V$, where Z the square matrix representing physically the entire shared coupling connecting the different mediums.

$$\left\|\begin{array}{cc}\left|\begin{array}{cccc}Z_{1,1}^{e,in} & Z_{1,2}^{e,in} & \cdots & Z_{1,L+1}^{e,in}\\ 0 & Z_{2,2}^{e,in} & \cdots & \vdots\\ \vdots & \vdots & \cdots & \vdots\\ 0 & Z_{L+1,2}^{e,in} & \cdots & Z_{L+1,L+1}^{e,in}\end{array}\right| & \left|\begin{array}{cccc}Z_{1,1}^{e,out} & 0 & 0 & 0\\ Z_{2,1}^{e,out} & Z_{2,2}^{e,out} & 0 & 0\\ \vdots & 0 & \ddots & 0\\ Z_{L+1,1}^{e,out} & 0 & 0 & Z_{L+1,L+1}^{e,out}\end{array}\right|\\ \left|\begin{array}{cccc}Z_{1,1}^{h,in} & Z_{1,2}^{h,in} & \cdots & Z_{1,L+1}^{h,in}\\ 0 & Z_{2,2}^{h,in} & \cdots & \vdots\\ \vdots & \vdots & \cdots & \vdots\\ 0 & Z_{L+1,2}^{h,in} & \cdots & Z_{L+1,L+1}^{h,in}\end{array}\right| & \left|\begin{array}{cccc}Z_{1,1}^{h,out} & 0 & 0 & 0\\ Z_{2,1}^{h,out} & Z_{2,2}^{h,out} & 0 & 0\\ \vdots & 0 & \ddots & 0\\ Z_{L+1,1}^{h,out} & 0 & 0 & Z_{L+1,L+1}^{h,out}\end{array}\right|\end{array}\right\|$$

Every element of Z is a $M \cdot N$ square matrix; $Z_{L,2}^{h,in}$ stands for the magnetic effect of the inner auxiliary sources of boundary 2 on the boundary L over the collocation points.

For far cylinders inclusions, the mutual coupling can be neglected, hence the corresponding matrix is enforced to zero. We only have to implement the coupling between neighboring cylinders; in that case, the computational cost will diminish significantly.

3. NUMERICAL RESULTS

Some numerical results achieved by computer EMAS execution code to corroborate the validity and accuracy of the aforementioned numerical model are attached in this part. In the subsequent examples, the permeability is assumed of free space ubiquitously except the jacket structure.

A cross section describes the spatial distribution of scattered power. This fictitious area is the radar cross section and for 2D structures is the scattering width defined as:

$$SW = \lim_{\rho\to\infty}\left[2\pi\rho\frac{|E^{sc}|^2}{|E^{inc}|^2}\right] \tag{28}$$

For specified auxiliary surfaces, the convergence scale and the accuracy of the method are only reliant on the number of auxiliary sources M. According to MAS, the approximate solution of the boundary problem will tend to exact solution as $M \to \infty$. Therefore, the convergence is ensured [8].

The boundary condition error for the boundary 1 is defined by the ratio of the absolute difference between the tangential electric fields intensity around the considered boundary to the maximum magnitude of the corresponding incident field:

$$\Delta E_{bc} = \frac{\left\| \sum_{l=1}^{L+1} E_{(1)}^{l,in} + E_{(1)}^{inc} - E_{(1)}^{l,out} \right\|}{\max \left\| E_{(1)}^{inc} \right\|} * 100 \tag{29}$$

The accuracy scrutinizing involves an optimization between at least two parameters: the auxiliary distance sandwiched between the auxiliary and physical contours and the number of auxiliary sources per basis for one cylinder. To clarify the point, we consider the geometry depicted in Fig. 4, where four different auxiliary distances and eight bases are considered for estimating the scattered field beside the jacket, thus the convergence will be attained when the boundary condition error reach the predesigned accuracy by varying the bases dimensions.

To reach the maximal conditionality of the obtained algebraic matrix, the collocation method is utilized and implemented in the

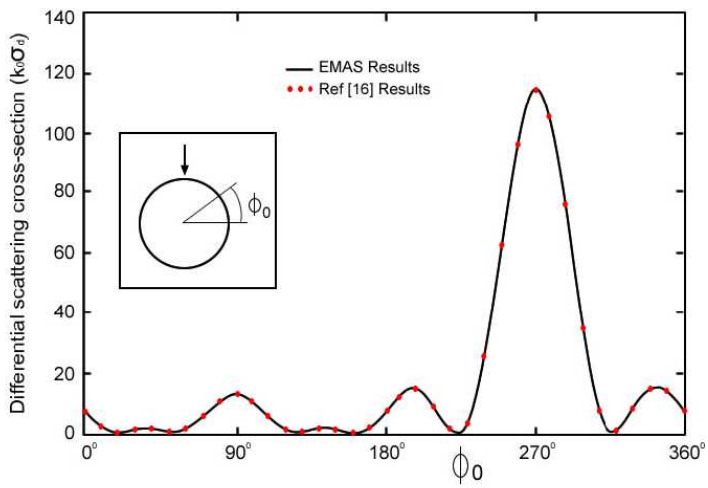

Figure 3. Differential scattering cross-section of a dielectric cylinder as functions of observation angle.

algorithm code.

The incident wave frequency is set to 300 MHz, and the number of auxiliary sources per base is equal to the number of collocation points for every cylinder.

Initially, the particular case is considered. Fig. 3 illustrates the differential scattering cross-section $k_0\sigma_d$ of a dielectric circular cylinder versus the observation angle Φ_0, where $k_0a_1 = 2.5$, $n_1 = 2.0$, and $\Phi^{inc} = 270°$. The number of auxiliary sources $M = 100$ and $d_{aux} = 0.015\lambda$, where d_{aux} is the distance between the auxiliary contour and the boundary. The highest boundary condition error is around 0.10%. The numerical results accordance is noticeable in favor of EMAS.

Figure 4 reveals the differential scattering cross-section $k_0\sigma_d$ of dielectric circular cylinder with three inclusions of air holes as functions of observation angle Φ_0, where $k_0a_2 = k_0a_3 = k_0a_4 = 0.3$, $n_2 = n_3 = n_4 = 1.0$, $k_0d_2 = 0.35$, $k_0d_3 = 1.0$, $k_0d_4 = 1.65$, $\theta_2 = 90°$, $\theta_3 = 210°$, $\theta_4 = 330°$, $k_0a_1 = 2.5$, $n_1 = 2.0$, and $\Phi^{inc} = 270°$. The number of auxiliary sources $M = 150$ and $d_{aux} = 0.012\lambda$, the error is around 0.15%. Numerical results divulged by EMAS and those by [16] are identical.

Figures 5–7 show the backscattering cross-section of a dielectric circular cylinder with respectively one, two and three inclusions under TM plane wave incidence versus the refractive index step; all the

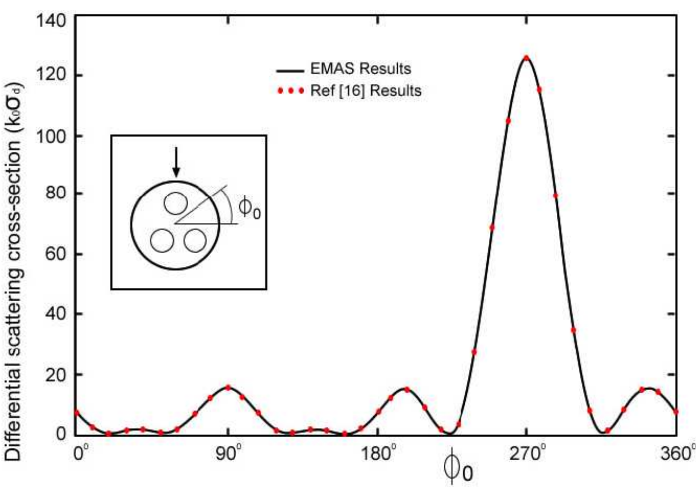

Figure 4. Differential scattering cross-section of cylinder with three inclusions of air holes as functions of observation angle.

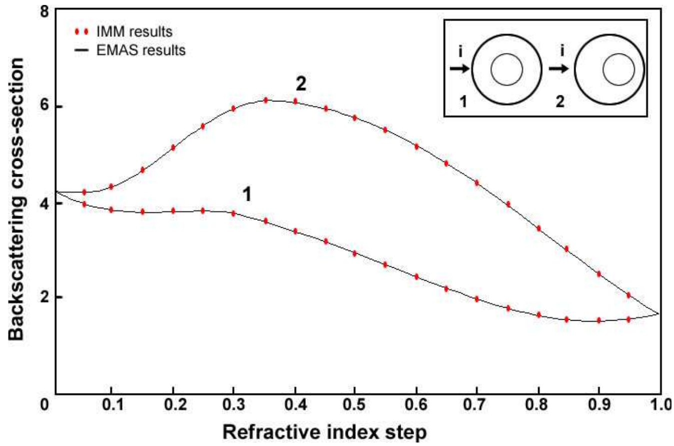

Figure 5. Backscattering cross-section of a dielectric cylinder with one inclusion as functions of refractive index step.

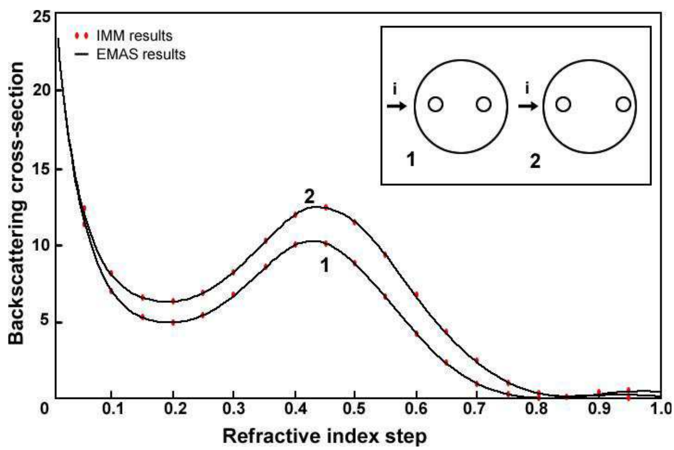

Figure 6. Backscattering cross-section of a dielectric cylinder with two inclusions as functions of refractive index step.

parameters are identical with those in [4]. The number of auxiliary sources $M = 100$ and d_{aux} is fluctuating around $0.07a_1$. The EMAS results are compared with those given by the indirect matching method IMM, and the agreement between the results is obvious.

Figures 8–10 show the backscattering cross-section of a dielectric circular cylinder with respectively one, two and three inclusions under TM plane wave incidence versus the eccentricity index step; all the

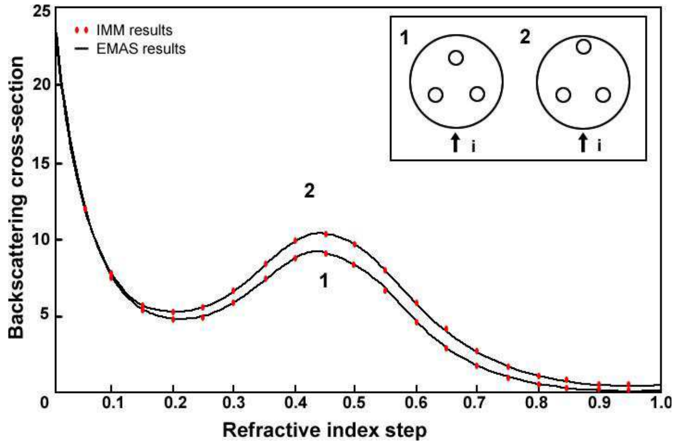

Figure 7. Backscattering cross-section of a dielectric cylinder with three inclusions as functions of refractive index step.

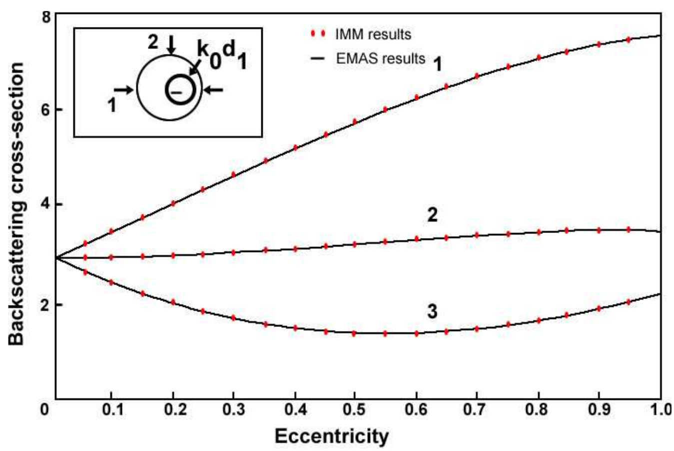

Figure 8. Backscattering cross-section of a dielectric cylinder with one inclusion as functions of eccentricity.

parameters are identical with those in [4]. The incident wave frequency is set to 300 MHz. The number of auxiliary sources $M = 100$ and d_{aux} is fluctuating around 0.03λ. The EMAS results are compared with those given by the indirect matching method IMM, and the agreement between the results is obvious.

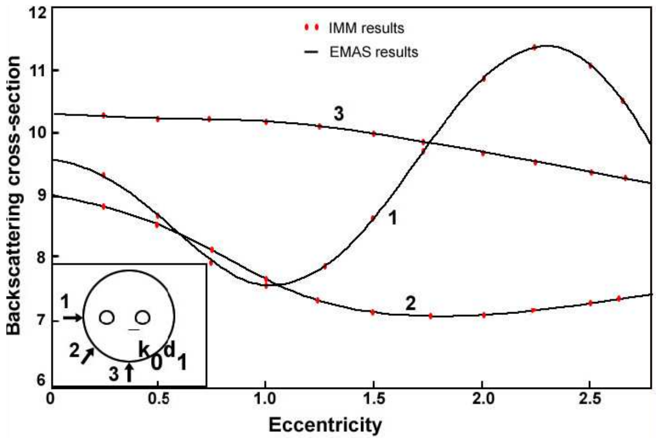

Figure 9. Backscattering cross-section of a dielectric cylinder with two inclusions as functions of eccentricity.

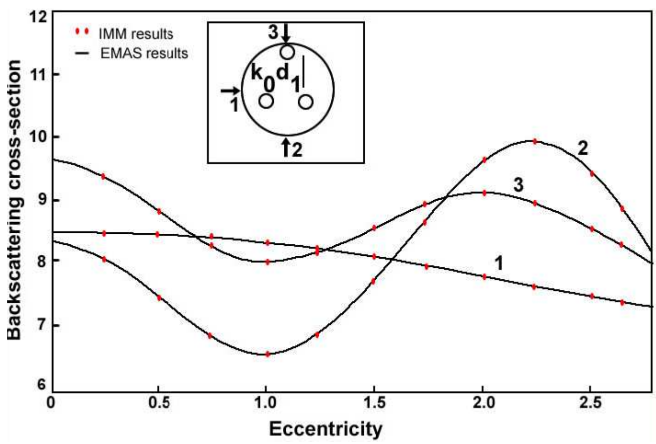

Figure 10. Backscattering cross-section of a dielectric cylinder with three inclusions as functions of eccentricity.

4. CONCLUSION

In this paper, we have developed a numerical method EMAS for the scattering problems by finite number of dielectric eccentric cylindrical inclusions embedded in a host dielectric one. The global electromagnetic coupling between different cylindrical inclusions is taken into account, and the numerical results validate the inner field distributions for the refractive index and eccentricity variations.

The extended method of auxiliary sources authorizes large standpoints, especially, the modelisation for arrays with dielectric eccentric cylindrical inclusions with coupled elements, leading to tuning the backscattering cross section.

APPENDIX A.

If a TM plane wave illuminates one dielectric cylinder. The inner and outer regions are respectively denoted by II and I. The fields radiated in the region I by one auxiliary source are:

$$E^I_{iz} = -(k_I Z_I/4) H_0^{(2)}[k_0 R^I_i] I^I_i \tag{A1}$$

$$H^I_{ix} = (k_I(y^I_i - y)/4jR^I_i) H_1^{(2)}[k_0 R^I_i] I^I_i \tag{A2}$$

$$H^I_{iy} = (k_I(x - x^I_i)/4jR^I_i) H_1^{(2)}[k_0 R^I_i] I^I_i \tag{A3}$$

Moreover, those radiated in the region II by an outside auxiliary source are:

$$E^{II}_{iz} = -(kZ/4) H_0^{(2)}[k_0 R^{II}_i] I^{II}_i \tag{A4}$$

$$H^{II}_{ix} = (k(y^{II}_i - y)/4jR^{II}_i) H_1^{(2)}[k_0 R^{II}_i] I^{II}_i \tag{A5}$$

$$H^{II}_{iy} = (k(x - x^{II}_i)/4jR^{II}_i) H_1^{(2)}[k_0 R^{II}_i] I^{II}_i \tag{A6}$$

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