# IDEALLY HARD STRUTS TO ACHIEVE INVISIBILITY 

J. M. Fernández

Grupo de Radiación, Departamento de Señales
Sistemas y Radiocomunicaciones
Universidad Politécnica de Madrid
Ciudad Universitaria, Madrid 28040, Spain

## E. Rajo-Iglesias

Departamento de Teoría de la Señal y Comunicaciones Universidad Carlos III de Madrid
Avenida de la Universidad 30, Leganés, Madrid 28911, Spain

## M. Sierra-Castañer

Grupo de Radiación, Departamento de Señales
Sistemas y Radiocomunicaciones
Universidad Politécnica de Madrid
Ciudad Universitaria, Madrid 28040, Spain


#### Abstract

In this work, ideally hard struts with different cross sections are analyzed. Firstly, the characterization of the invisibility of a given object in terms of an equivalent blockage width is discussed. Then, the effect of the incidence angle on struts for reducing electromagnetic blockage using the same ideally hard cylinders is analyzed. It is shown that the variation of incidence angle in azimuth is very sensitive in terms of blockage for both polarizations. Finally, design charts for ideally hard struts which reduce blockage simultaneously for TE and TM cases are presented. This can be used to define some performance goals for final realized struts.


Corresponding author: J. M. Fernández (jmfdez@gr.ssr.upm.es).

## 1. INTRODUCTION

Achieving invisibility has been the subject of extensive studies in the physics and engineering communities for decades. The use of absorbing screens [1] and antireflection coatings [2] to diminish the backscattering from objects are common in several applications. For example, you can make an object invisible to a radar with good absorbers, or with a strong scattering in other directions, but this is not proper invisibility. The metamaterial community has been especially active within cloaking in the last years, proposing different solutions [3-5]. The cloaking can be seen as a reduction in the blockage caused by any object to the electromagnetic waves, which is achieved by covering the object to hide with an artificial surface. Obviously, all these contributions make use of one or another approach. Some of them are working (cloaking the object) only for one polarization, others work with the size of the object to hide [6] or even on the type of materials the object can be made of, for instance only dielectrics with a given permittivity as in [7]. Other more recently interesting and published works are $[8,9]$.

Invisibility means to reduce the field blockage caused by an object, i.e., the amplitude and phase of the incident waves are unperturbed after the object to reduce the total scattered field in general but mainly the forward scattering. The electromagnetic (EM) waves should be able to pass around or through the object without being perturbed, without reflections and absorptions but with a strong transmission and the wave phase front should be kept uniform after the object. This agrees with how a hard surface [10] behaves, as such surfaces enhance wave propagation along and around them (known as GO characteristics according to [11]). Related to this concept, the EM waves radiating from or being received by an antenna are often obstructed by some mechanical structures causing increased sidelobes and reduced antenna gain [12]. Consequently, this issue has been previously treated by the antenna community for problems such as the blockage caused by struts or masts supporting the feed in reflector antennas or printed reflectarrays.

Usually in antennas, the direction of the incident wave is known, so the struts can be designed to reduce the blockage for a given direction of incidence. A good example on how to reduce the blockage caused by struts was already presented in 1996 [13]. Typically, in such applications, the cross-section of the struts is electrically small (width $W$ much smaller than the wavelength $\lambda_{0}$ ). The field blockage caused by the struts can be reduced for any polarization by making use of an oblong cross-section and the concept of hard surfaces. The
hard condition for TE case needs a perfect electric conductor (PEC) or metal surface with an appropriate shape whilst for TM case requires a high surface impedance, i.e., a perfect magnetic conductor (PMC) or, in practice, an artificial magnetic conductor (AMC). This is valid for rather thin objects, although extensions for thicker objects are possible by letting the waves pass through the object in a controlled manner.

In this work, different oblong cross-sectional shapes are analyzed and compared in terms of performances over a large frequency band for blockage reduction of cylinders made of ideally hard surfaces. The effect of the incidence angle on the blockage of these ideal struts is also studied. These ideal struts have a surface consisting of ideally PEC and/or PMC hard cylinders, so that the strut works ideally as a hard surface for each one of the polarizations as well as for dual polarization.

Thereby, the aim of this study is to analyze ideally hard cylinders with different cross sections and evaluate the influence that the variation of the incidence angle $\varphi$ in the azimuth plane has in terms of blockage for TE and TM polarizations. The paper is divided in the next sections. Section 2 discusses how to characterize and quantify the blockage reduction or invisibility of a given strut. In Section 3, some of the results of different ideally hard cross sectional shapes and the incidence angle variation on it are presented and finally the conclusions of this work are drawn in Section 4.

## 2. CHARACTERIZATION OF INVISIBILITY

An important issue is how to characterize and quantify the invisibility of a given object. The forward scattered field is traditionally characterized in terms of an induced field ratio (IFR) [14, 15]. It is even


Figure 1. Plane wave scattering (2D case).
better to characterize blockage in terms of an equivalent blockage width $W_{e q}$ that is proportional to the product of the IFR and the physical width $W$. The study in this work is limited to a plane wave incident on an infinitely long scatterer which is a 2D scattering problem as shown in Fig. 1. When an infinite cylinder is immersed in an incident plane wave, its IFR is defined as the ratio of the forward-scattered field to the hypothetical field radiated in the forward direction by the plane wave in the reference aperture of width equal to the shadow of the geometrical cross section of the object (i.e., the scatterer) on the incident wavefront as (1):

$$
\begin{equation*}
\operatorname{IFR}=-\frac{\vec{E}_{s}\left(\varphi=0^{\circ}\right)}{\vec{E}_{r e f}\left(\varphi=0^{\circ}\right)} \tag{1}
\end{equation*}
$$

where $E_{s}\left(\varphi=0^{\circ}\right)$ is the forward scattered electric field of the object and $E_{r e f}\left(\varphi=0^{\circ}\right)$ is the forward scattered electric field of a reference object. The equivalent blockage width $W_{e q}$ is by definition a complex value, where both real part and the absolute value are representative for characterizing invisibility. Different information can be obtained from the separate analysis of its real part and absolute value as explained in $[12,13]$.

$$
\begin{equation*}
W_{e q}=-\mathrm{IFR} \cdot W \tag{2}
\end{equation*}
$$

where IFR is the induced field ratio and $W$ is the physical width of the object. The total scattered power integrated over all directions is proportional to the real part of the equivalent blockage width $W_{e q}[16]$. The blockage loss due to support struts in a reflector is proportional to the real part of $W_{e q}$, i.e., the reduction in dB of the directivity of the antennas due to the blockage. The high sidelobe level due to the struts appearing near the main lobe is proportional to the absolute value of $W_{e q}$ as explained in [12]. From this, it is evident that it is more important to reduce the real part of the equivalent blockage width than its absolute value, as the real part determines the directivity reduction and represents the scattered power averaged over all directions around the cylinder. This parameter is a good measure of the blockage or shadow of a given object when it is illuminated by a plane wave. When the equivalent blockage width $W_{e q}$ of the object is much smaller than the physical width $W\left(W_{e q} \ll W\right)$, the blockage can be considered small and therefore we have invisibility of the object. On the contrary, when the equivalent blockage width $W_{e q}$ becomes equal to the physical width $W\left(W_{e q}=W\right)$, a strong blockage appears and the object is no more invisible. Such is the case for opaque objects.

## 3. IDEALLY HARD STRUTS

The hard surface is ideally a perfect electric conductor (PEC) for TE case (E-field orthogonal to the cylinder axis) and a perfect magnetic conductor (PMC) for TM case (H-field orthogonal to the cylinder axis) [17]. Thereby, metal struts are ideally hard for TE case. As explained in [18], as far as the hard or GO condition is obtained, the waves would be guided around the cylinder surface. In this sense, it is also known that the shape of the cross section plays an important role in blockage reduction, and it has been stated from antenna applications that oblong shapes are required to minimize that blockage.

Therefore, it is the purpose of the present paper to analyze different ideally hard cross sectional shapes under oblique incidence namely, metal struts for TE case, ideal PMC struts for TM case and ideally PEC/PMC hard strip loaded struts for dual polarization. The equivalent blockage width is readily computed by considering a plane wave incident on an infinitely long strut. Note that for all the results in the paper the incidence angle of the wave is varying in the azimuthal plane. The obtained results have been computed using the FITD (Finite Integration Time Domain) software CST Microwave Studio with periodic boundary conditions as explained in [18]. The studied blocking objects have physical widths $W$ comparable or sensitively larger than the wavelength $\lambda_{0}$. In the simulations, we use physical cross sections of all cylinders of $W=54.2 \mathrm{~mm}\left(f_{0}=8.5 \mathrm{GHz} \Rightarrow \lambda_{0}=\right.$ $35.3 \mathrm{~mm} \Rightarrow W / \lambda_{0}=1.53$ ), and we compute the equivalent blockage widths in the frequency range 0.1 to 20 GHz , i.e., when $W$ is between $0.018 \lambda_{0}-3.6 \lambda_{0}$.

### 3.1. Different Cross Sectional Shapes

Initially, we have computed the equivalent blockage width of ideally hard cylinders of different cross sectional (cylinder and rectangle) shapes with the same physical width. The real part of the equivalent blockage width $W_{e q}$ of cylinders with circular, rhombic, cross-shaped and thin rectangular cross sections are shown in Fig. 2. The thickness of the rectangular cross section is 1 mm , and this is the same for both the transverse and longitudinal part of the cross-shaped section.

The results show that the rhombic cross section of the strut has the smallest real part of $W_{e q}$ when the physical width is comparable to the wavelength, whereas the cross-shaped and thin rectangular cross section are the best at low frequency (when the physical cross section $W$ is narrower than approximately $0.2 \lambda_{0}$ ). We also show that the oblong cross section is the best at high frequency (when the physical width is larger than $0.2 \lambda_{0}$ ), whereas for narrower cross sections (at


Figure 2. Equivalent blockage width for TE case of ideally hard cylinders with different cross-sectional shapes (basic-shaped cross sections) under normal incidence ( $\varphi=0^{\circ}$ ).


Figure 3. Equivalent blockage width of ideally hard cross-shaped sections (width $W$, length $L=2 W$ ) under normal incidence ( $\varphi=0^{\circ}$ ).
low frequency) the transverse thin rectangular cross section performs better. The latter can be strengthened by a cross-shaped section, without significant change in the blockage width. It is observed and validated that the real part of $W_{e q}$ becomes at high frequency equal to the physical width $W$ as mentioned in Section 2.

In reference [19], an optimization algorithm was used to numerically optimize the metallic cross section shape with the purpose of reducing its blockage width for TE polarization. The optimization
was performed for a frequency range corresponding to the fixed physical width being between 8 and 17 GHz . The optimized oblong shape is explained by a smooth transition of the waves passing the cylinder, which is facilitated by the GO characteristics of the hard surfaces. The thin rectangular cross section is explained as a quasi-static solution. The cross section is so narrow in terms of wavelength that transverse currents cannot be induced. The cross-shaped section works like the transverse rectangular cross section, because the orthogonal rectangular part making up the two other arms of the cross are invisible to the wave because of their small thickness. The optimized final cross section is represented in one of the insets of Fig. 3 and has a cross shape. In the same figure, a comparison with the equivalent blockage width of three cross sections that are identical in terms of physical width $W=54.2 \mathrm{~mm}$ and length $L=108.4 \mathrm{~mm}$ are included.

When comparing to Fig. 2, we can observe that the strut with the rhombic shape yields an equivalent blockage width that is larger compared to the optimized shape when the physical width is smaller than $0.2 \lambda_{0}$ (at low frequency), despite the fact that the physical width and length of the two cross sections are identical. On the contrary, when the physical width is comparable to the wavelength, the blockage is smaller with the rhombic strut than with the optimized strut.

### 3.2. Ideally PEC and PMC Hard Rhombic Cross-sections

As previously mentioned, the hard condition for TM polarization can also be easily achieved using an ideal PMC material. In this subsection, we analyze normal incidence on ideal PMC rhombic cross section of physical width $W=54.2 \mathrm{~mm}$ for TM case to be compared to ideal


Figure 4. Equivalent blockage width of ideally PEC and PMC rhombic cross-section under normal incidence $\left(\varphi=0^{\circ}\right)$.

PEC rhombic cross section for TE case for $L=4 W=216.8 \mathrm{~mm}$. It is interesting to compute ideal cases of struts because this will provide more general results than studying a specific realization. These general results will be useful in determining fundamental physical limitations. The ideally PMC solid rhombic cross section is modeled in CST Microwave Studio 2009 by using a magnetic material with $\varepsilon_{r}=1$ and $\mu_{r}>1000$ (approximation of the ideal PMC material).

We can observe in Fig. 4 that the obtained results for an ideally PMC solid rhombic cross section for TM polarization corroborates the results obtained for a PEC solid rhombic cross section for TE polarization. Besides, the analysis concerning the effect of varying the incidence angle in the azimuth plane on the PEC rhombic objects for $L=4 W=216.8 \mathrm{~mm}$ in terms of equivalent blockage width is shown in Fig. 5.


Figure 5. Equivalent blockage width of PEC rhombus under variation of $\varphi$ in the azimuth plane for TE polarization.

It is observed that the PEC rhombic cross section is very sensitive to the incidence angle $\varphi$ as expected. This means that oblong cross section is the best in terms of blockage width reduction for normal incidence $\left(\varphi=0^{\circ}\right)$ but probably one of the worse options for the variation of incidence angle $\varphi$ in the azimuth plane.

Now, we study the same effect of azimuthal angular variation but for a practical implementation of hard condition for TM case. It is known from [18] that the dielectric coating of metal strut is a simple way to implement TM case. The results of this study are presented in Fig. 6 where the effect of the variation of incidence angle $\varphi$ for TM polarization in a metallic rhombus $(W=54.2 \mathrm{~mm}$ and $L=216.8 \mathrm{~mm}$ ) with dielectric coating $\varepsilon_{r}=2.2$ and thickness $d=\lambda_{0} / 4 \sqrt{\varepsilon_{r}-1}=8.05 \mathrm{~mm}$ at 8.5 GHz . This design is narrow band.


Figure 6. Equivalent blockage width of PEC rhombus with dielectric coating $\varepsilon_{r}=2.2$ under variation of $\varphi$ in the azimuth plane for TM polarization.

It is observed that oblong metallic rhombus with a dielectric coating is also very sensitive for the TM polarization when the incidence of the plane wave is varying with $\varphi$ angle. When $\varphi>15^{\circ}$, the performance in terms of equivalent blockage width $W_{e q}$ or invisibility is destroyed because $W_{e q}>W$.

The next analyzed case is a rhombic cross section with a hard surface covering realized by dielectric coating with narrow metallic strips on its surface (Fig. 7) to obtain low blockage for dual polarization. Fig. 8 and Fig. 9 show TE and TM performances of this design assuming the same dielectric coating as in the previous example (the period and the width of the strips are detailed in the caption of the figures and in Fig. 7). This structure allows simultaneously blockage reduction for TE and TM polarizations but in a narrow frequency band


Figure 7. Cross section detail of strip-loaded dielectric coated metal strut: strip period $p$ and strip width $s$.
limited by the TM polarization.
Here again, we can see that both TE and TM cases are very sensitive to the variation of the incidence angle $\varphi$ in the azimuth plane and the blockage reduction is bad for oblique incidence. The best case is for normal incidence $\left(\varphi=0^{\circ}\right)$ where the invisibility is quite good at 8.5 GHz in a narrow band for the TE and TM polarization simultaneously.

Finally, the dielectric coating is replaced by an ideal PMC keeping


Figure 8. Equivalent blockage cross section of strip-loaded dielectric coated metal strut when the strip period $p=6 \mathrm{~mm}$ and the strip width $s=3 \mathrm{~mm}$ under variation of $\varphi$ in the azimuth plane for TE polarization.


Figure 9. Equivalent blockage cross section of strip-loaded dielectric coated metal strut when the strip period $p=6 \mathrm{~mm}$ and the strip width $s=3 \mathrm{~mm}$ under variation of $\varphi$ in the azimuth plane for TM polarization.


Figure 10. Equivalent blockage width of ideally PMC hard strut with narrow metalic strips ( $p=6 \mathrm{~mm}$ and $s=3 \mathrm{~mm}$ ) under normal incidence ( $\varphi=0^{\circ}$ ).


Figure 11. Equivalent blockage width of a ideally PMC hard strut with narrow metallic strips ( $p=6 \mathrm{~mm}$ and $s=3 \mathrm{~mm}$ ) under variation of $\varphi$ for TE case.
the narrow metallic strips on its surface. Fig. 10 shows the results for the normal incidence ( $\varphi=0^{\circ}$ ) for dual polarization in terms of equivalent blockage width. As the rhombic cross section is an ideal PMC material, we can observe that for TM case, the equivalent blockage width is not narrow band as it happened with dielectric coating (Fig. 9), but it has the same behavior as the TE case. Fig. 10 shows that for TE and TM polarization under normal incidence, this strut is almost invisible in a large frequency band. The strut can be considered invisible when $W_{e q} \ll W$.

To conclude this numerical study, Fig. 11 and Fig. 12 present the
effect of the incidence angle in the azimuth plane on these ideal PMC struts with narrow metallic strips. They show similar behavior and that the structure is very sensitive to the variation of incidence angle $\varphi$, being this sensitivity higher for TE case. At $\varphi=20^{\circ}$, this structure has small blockage for a narrow bandwidth at low frequencies. These struts have a surface consisting of parallel metallic strips on ideal PEC or PMC struts, so that the strut works ideally as a hard surface for dual polarization when the direction of the strips is parallel to the plane of incidence of the plane wave on the strut. The analysis in this paper is done for the case where the plane of incidence is aligned with the


Figure 12. Equivalent blockage width of a ideally PMC hard strut with narrow metallic strips ( $p=6 \mathrm{~mm}$ and $s=3 \mathrm{~mm}$ ) under variation of $\varphi$ for TM case.


Figure 13. Equivalent blockage width for simulated model prototype: ideal PEC rhombic cross section strut of $W=54.2 \mathrm{~mm}$ and $L=216.8 \mathrm{~mm}$ under normal incidence ( $\varphi=0^{\circ}$ ).
strip direction. In a future work, an analysis when it deviates from it, will be realized to determine the sensitivity of the equivalent blockage width as a function of the direction of incidence $\theta$ (oblique incidence in the elevation plane).

Finally, Fig. 13 illustrates the ideal PEC rhombic cross section strut prototype manufactured in order to verify the previous results for normal incidence. It is noticed that the measurements have a lot of ripples but they follow the trend of the simulation results. These ripples are probably due to the mutual interactions, multiple reflections between the measurement element setup and because of the location of the scatterer at the limit of the farfield condition of the horns in the anechoic chamber. Different possible improvements will be considered in the experiment setup.

Figure 14 shows the measurement setup used to measure the blockage reduction of the PEC rhombic cross section strut. The measurements were done with the measurement setup described in [15]. They were realized with two different transmit horn antennas, one working between $5-10 \mathrm{GHz}$ and the other one between $12-18 \mathrm{GHz}$. The transmit and receive horn antennas should be directive enough to concentrate all its illumination in the center of the scatterer. This


Figure 14. Measurement setup for measuring blockage width of scatterers in the anechoic chamber at Universidad Politécnica de Madrid.
experimental setup was done in the anechoic chamber of Universidad Politénica de Madrid.

## 4. CONCLUSION

Different ideally PEC and PMC hard struts have been analyzed in terms of equivalent blockage width under variation of the incidence angle $\varphi$. The performance for TE and TM case depends respectively on the realization of the PEC and PMC surfaces. For TM case, the bandwidth is normally narrow and the blockage is very sensitive to the variation of the incidence angle $\varphi$ in the azimuth plane. Nevertheless, the ideal PMC solid rhombic cross section allows wider bandwidth similar to the ideal PEC structures. The bandwidth is always limited by TM case, since TE case has wide band in most cases. Considering ideal PMC hard rhombic cross section with narrow metallic strips, the results show wide band behavior in terms of equivalent blockage width but again, it is very sensitive to the variation of incidence angle $\varphi$ in the azimuth plane. Some simulations have been compared with measurements to validate our results.

It is interesting to compute ideal cases of struts as this provides more general results than studying a specific realization, which will be useful in determining fundamental physical limitations. The obtained results of ideal struts allow getting design chart that gives some performance goals for a final realized strut. Both factors, shape and realization of the hard surface for the struts are fundamental to achieve invisibility.

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