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OPTIMUM DESIGN OF LUMPED FILTERS INCORPORATING IMPEDANCE MATCHING BY THE METHOD OF LEAST SQUARES

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Abstract—The method of least squares (MLS) is used to develop an algorithm for the optimum design of any type of filter under any design specifications for the realization of lowpass, bandpass, highpass and bandstop characteristics. The proposed filter design method can be used for any general filter network topology, which provides high flexibility for the selection of circuit configurations suitable for any desired application. The MLS filter design procedure also incorporates source and load impedance matching, which eventually leads to the simplicity of circuits. The proposed method of filter design may be used for lowpass prototype filters or directly for bandpass, highpass or bandpass filters. Several examples of MLS filter designs are given, which compare very well with the classical methods and indicate the advantages of the proposed method of filter design. The MLS filter design may realize any frequency response characteristics, such as spurious response elimination, multiband filter realization and enhancement of some desired behaviors.

1. INTRODUCTION

There are several methods available for the design of filter networks, such as image parameter, constant-k, m-derived, composite filters, insertion loss, Butterworth (maximally flat), Chebyshev (equal ripple) and elliptic filter methods. In a common classical procedure, a lowpass (LP) filter is first designed with the normalized source impedance and cutoff frequency, and then it is converted to the desired highpass (HP), bandpass (BP) and bandstop (BS) characteristics by appropriate filter transformations [1–3]. The filter specifications such as the ripple level

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in the pass band and attenuation in the stop band will determine the filter order. Then, the values of reactive elements are obtained. In such design procedueres, the filter topology is limited to some specific configurations and the filters are lossless. On the other hand, the source and load impedances are usually assumed real and equal. For some designs (such as Chebyshev filter with even order) these impedances are constrained to certain values, which require some appropriate impedace matching circuits in the input and output ports of the filters [4].

In this paper, we present a simple procedure for the design of filters based on the method of least squares, which is applicable to a general topology of two port networks, arbitrary source and load impedances, any type of lowpass, highpass, bandpass and bandstop filters and desired specifications on the filter frequency response in the pass band, transition and stop bands. The proposed procedure is also suitable for the realization of filter designs which suppress undesired spurious responses in any specified band and multiband filters.

A comprehensive study of the application of least mean square method to adaptive filters is given in [5].

However, in this paper, we are concerned with the design of lumped filters, which have direct applications for microwave distributed filters, which may be realized through the application of appropriate transformation formulas [6–9]. To the best of our knowledge, the method of least squares has not yet been applied to the design of lumped filters. The following points are significant improvement and extension to the Reference [10]:

i the filter design is combined by the impedance matching of the input and output impedances.

ii the input and output impedances may be complex (real, imaginary or complex).

iii design of all types of filters, such as lowpass, bandpass, highpass and bandpass filters.

iv specification of any frequency band.

v specification of any type of frequency response.

vi dual-band or multi-band frequency responses.

vii any selected cascaded circuit configuration.

viii design of impedance matching circuits for the input and output of a transistor amplifier.

2. METHOD OF FILTER DESIGN

The proposed method for filter design is based on the power loss ratio in its specified frequency response characteristics which for a two port network is (see Fig. 1);

$$P_{LR} = \frac{P_{avs}}{P_L} \tag{1}$$

where P_{avs} is the available power from the source and P_L is the power delivered to the load, which are, respectively [2];

$$P_{avs} = \frac{|E_S|^2}{8R_S} \tag{2}$$

$$P_L = \frac{1}{2} R_L |I_2|^2 \tag{3}$$

where the reflection coefficient at the source is assumed zero, $\Gamma_S = 0$ and various quantities are denoted in Fig. 1. The source and load impedances are $Z_S = R_S + jX_S$ and $Z_L = R_L + jX_L$, respectively. The source voltage is expressed in terms of the $ABCD$ parameters of the two port network.

$$E_s = Z_s I_1 + V_1 = \left[Z_s(CZ_L + D) + (AZ_L + B)\right] I_2 \tag{4}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad V_2 = Z_L I_2 \tag{5}$$

The scattering parameter S_{21} of the two port network with arbitrary source and load impedances is [11]:

$$S_{21} = \frac{2\sqrt{R_S R_L}}{AZ_L + B + CZ_S Z_L + DZ_S} \tag{6}$$

Then combining the above equations, we get

$$P_{LR} = \frac{1}{|S_{21}|^2} \tag{7}$$

The specifications of the filter designs are defined on the frequency response of $|S_{21}|$, which should lie inside the white areas in Fig. 2 for lowpass, bandpass, highpass and bandstop filter characterstics, where various parameters are denoted.

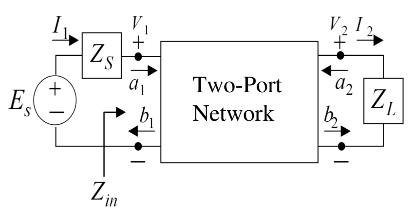

Figure 1. Schematic diagram of a two port network.

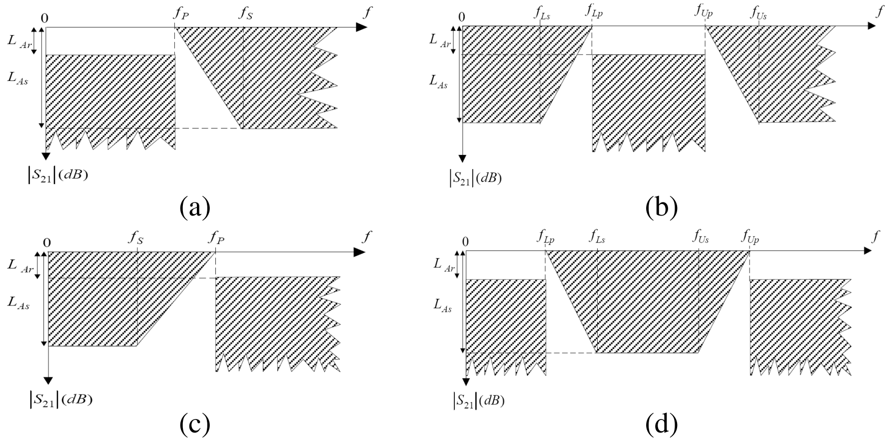

Figure 2. Diagrams of design specifications: (a) Lowpass, (b) bandpass, (c) highpass and (d) bandstop.

Table 1. Equivalent circuits for transformation of normalized lowpass filter to other filter types.

Low-pass	High-pass	Band-pass	Band-stop
$\frac{L}{\omega_p}$	$\frac{1}{\omega_p L}$	$\frac{L}{\omega_0\Delta}$, $\frac{\Delta}{\omega_0 L}$	$\frac{1}{\omega_0 L\Delta}$, $\frac{L\Delta}{\omega_0}$
$\frac{C}{\omega_p}$	$\frac{1}{\omega_p C}$	$\frac{C}{\omega_0\Delta}$, $\frac{\Delta}{\omega_0 C}$	$\frac{1}{\omega_0 C\Delta}$, $\frac{C\Delta}{\omega_0}$
$where: \ \omega_p = 2\pi f_p, \ \Delta = \frac{\omega_{Up} - \omega_{Lp}}{\omega_0}, \ \omega_0 = \sqrt{\omega_{Up}\omega_{Lp}}$			

There are two procedures for filter design. In one procedure a LP filter is first designed and then the circuit transformations given in Table 1 are used to synthesize the desired HP, BP, and BS filters. In an alternative procedure the LP, HP, BP and BS filters are directly synthesized according to their specifications. The filter design is based on the realization of the desired frequency response specified

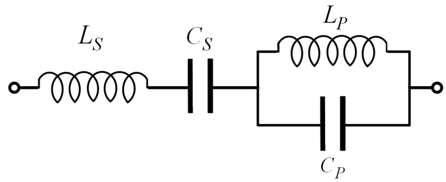

Figure 3. The schematic diagram of a circuit unit in a branch.

on the scattering parameter S_{21}, which should pass through the white areas in Fig. 2 for lowpass, bandpass, highpass and bandstop filters, respectively. Equivalently, a prototype lowpass filter may first be designed, and then by appropriate circuit transformations, other filter types may be derived.

We consider the filter topology composed of the cascade connection of consecutive series and parallel circuit units as shown in Fig. 3. The circuit unit next to the source or load may be selected as a series or shunt branch. Each series or shunt branch may be selected as a subset of the circuit unit shown in Fig. 3. After the selection of initial values for circuit elements (L and C), the computer programs seek to determine their optimum values under some specified constraints. In case, the value of inductance (L) of an inductor decreases towards small values (namely zero), then the inductor is removed and shorted out. In case, the value of capacitance (C) of a capacitor increases towards large values (namely infinity), then the capacitor is removed and shorted out. On the other hand, if the value of L increases towards high values and that of C decreases towards zero, they are open circuited. However, the shorting out and open circuiting of L and C should not cause on interruption in the filter circuit. As a result, the proposed algorithm may simplify the filter circuit configuration. Consequently, the filter designer can select a large variety of circuit topologies for the realization of the desired filter specifications and can obtain the configuration that best fits the specified characteristics (and circuit requirements and constraints), through some experimentation and trial and error on the computer programs.

The computer program for the optimum design of filters based on the method of least squares is realized by the flow chart in Fig. 4. Initially, the filter topology, namely the circuit configuration is selected, which consists of the required number of series and shunt circuits composed of a subset of the circuit unit in Fig. 2. In this chart, g shows the generation in the genetic algorithm (GA) or iteration in the conjugate gradient algorithm (CG) as FMINCON subroutine in the MATLAB toolbox. The error function ε_g is defined in the

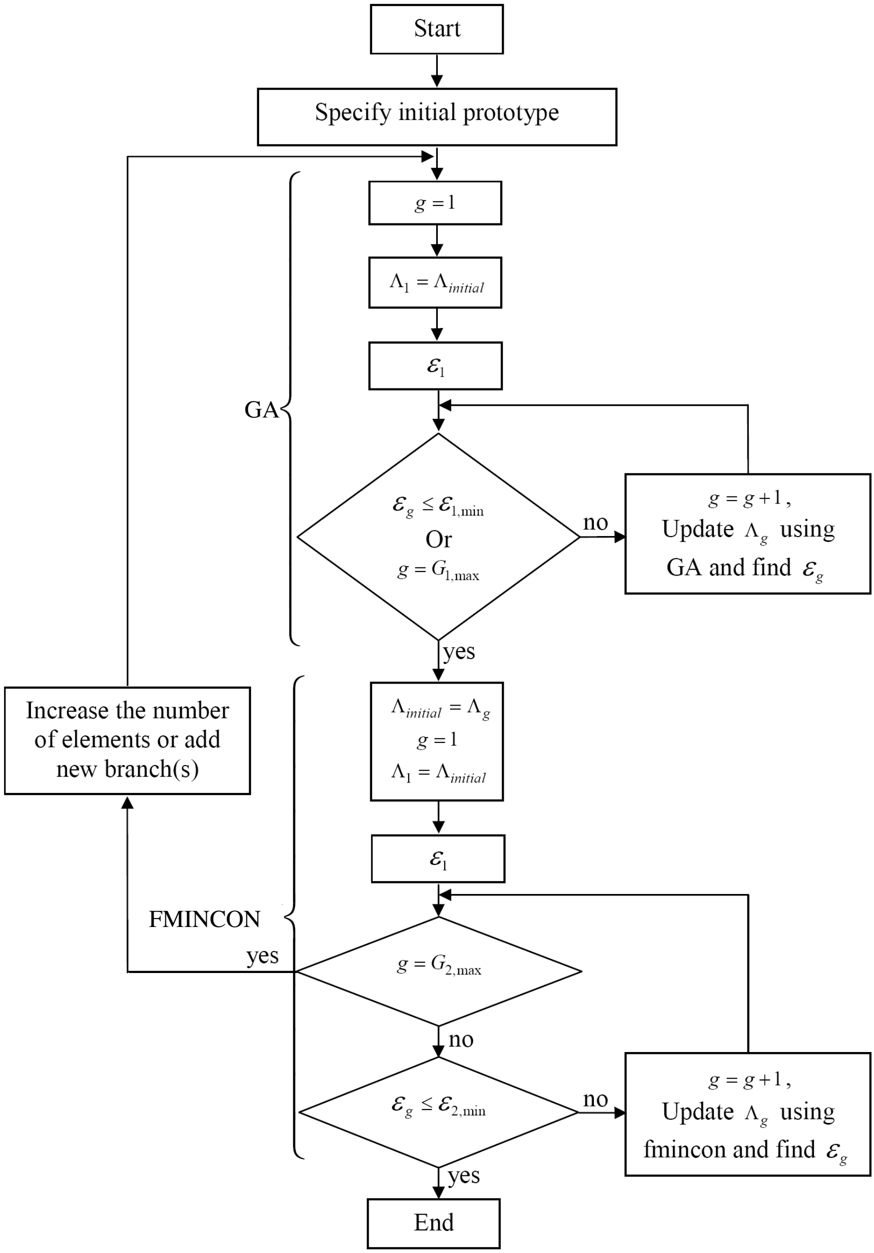

Figure 4. Flowchart of the MLS algorithm for the optimization of filter element values.

following relations. Several criteria are defined to stop the algorithm. If the value of the error function is larger than some initially specified criterion $\varepsilon_{1,\min}$ or $\varepsilon_{2,\min}$ in the GA or CG stages, respectively, then the computations are repeated by increasing the generation or iteration number g. However, if the error criterion is satisfied, namely $\varepsilon \leq \varepsilon_{1,\min}$, then the program proceeds to the following stage. Finally, the computer program stops upon the satisfaction of both error criteria $\varepsilon_{1,\min}$ and $\varepsilon_{2,\min}$. Furthermore, the computations stop in the GA and CG algorithms, whenever the numbers of iterations exceed $G_{1,\min}$ and $G_{2,\min}$, respectively.

The error functions ε_{LP}, ε_{BP}, ε_{HP} and ε_{BS} for the lowpass (LP), bandpass (BP), highpass (HP) and bandstop (BS) filters, respectively, are constructed as follows:

$$\begin{aligned}
\varepsilon_{LP} &= W_P \times ER_P + W_T \times ER_T + W_S \times ER_S && (8)\\
\varepsilon_{BP} &= W_{S1} \times ER_{S1} + W_{T1} \times ER_{T1} + W_P \times ER_P \\
&\quad + W_{T2} \times ER_{T2} + W_{S2} \times ER_{S2} && (9)\\
\varepsilon_{HP} &= W_S \times ER_S + W_T \times ER_T + W_P \times ER_P && (10)\\
\varepsilon_{BS} &= W_{P1} \times ER_{P1} + W_{T1} \times ER_{T1} + W_S \times ER_S \\
&\quad + W_{T2} \times ER_{T2} + W_{P2} \times ER_{P2} && (11)
\end{aligned}$$

where ER_P, ER_T and ER_S denote the error functions in the pass, transition and stop bands, respectively. There may be more than one type of error function for different types of filters. For example, the bandpass filter has two stop and transition bands (ER_S, ER_T). The selection of appropriate values for the weighting factors (W_P, W_T, W_S) may be selected by experience and some experimentation on the computer programs. If the emphasis of the filter frequency response is to be placed on the error term concerning the pass bands, transition bands or stop bands, then the values of W_P, W_T or W_S should be selected relatively quite high, respectively. The relative values of weighting factors affect the filter frequency response. The error functions corresponding to the pass, stop and transition bands are:

$$\begin{aligned}
ER_P &= \sum_{k \in pass\ band} \frac{1}{2}\left[1 - \mathrm{sign}\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{Ar}\right)\right]\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{Ar}\right)^2 \\
ER_S &= \sum_{k \in stop\ band} \frac{1}{2}\left[1 + \mathrm{sign}\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{As}\right)\right]\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{As}\right)^2 \\
ER_T &= \sum_{k \in transition\ band} \frac{1}{2}\left[1 + \mathrm{sign}\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{At}^k\right)\right]\left(\left|S_{21}^k\right|_{\mathrm{dB}} - L_{At}^k\right)^2 \\
L_{At}^k &= \frac{L_{As}}{\Delta f}\Delta f_k
\end{aligned} \qquad (12)$$

where,

$$\Delta f_k = \begin{cases} f_U - f_k & Positive\,slope \\ f_k - f_L & Negative\,slope \end{cases}$$

and subscripts L and U on f indicate the lower and upper frequencies of transition band. Furthermore, L_{Ar} and L_{As} are the ripple amplitude in the pass band and attenuation in the stop band, respectively. Also, Δf is the frequency interval of the transition band, which is delimited by f_S and f_P, as the frequency bounds of the stop and pass bands, respectively. The positive or negative slope of the transition band should also be taken into account. The *sign* function is defined as follows:

$$\text{sign}(x) = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases} \tag{13}$$

Due to the relation $|S_{11}|^2 + |S_{21}|^2 = 1$, it is only necessary to consider S_{11} or S_{21} in the construction of the error function.

Inclusion of both scattering parameters in the error function increases its complexity and the computation CPU time, without the improvement of the filter design. Computer experimentations have shown that their inclusion in the error function with different weighting factors actually confuses the error minimization.

The initial values of the inductors and capacitors, which are arrived at by experimentation and experience are collectively denoted by $\Lambda_{initial}$ and are taken as;

$$L = \frac{0.4(R_S + R_L)}{\omega'} \quad C = \frac{1}{(R_S + R_L)\omega'}$$

where $\omega' = \omega_P$ is the cutoff angular frequency of the lowpass and highpass filters and $\omega' = \omega_0$ is the center angular frequency of the bandpass and bandstop filters, where $\omega_0 = \sqrt{\omega_{Up}\omega_{Lp}}$.

3. NUMERICAL IMPLEMENTATIONS

In order to illustrate and verify the proposed MLS filter design procedure, we consider three examples from Reference [1] and one example from Reference [2]. We compare the procedure and preformance of the classical filter designs with those of the MLS designs presented in this paper.

Example 1. The circuit diagram of the lowpass Chebyshev filter of degree 8 (with 8 reactive elements) is shown in Fig. 5 which is taken from Reference [1, page 43 and Table 3.2]. The first element is a shunt capacitor.

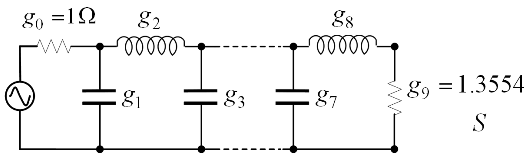

Figure 5. Circuit configuration of example 1.

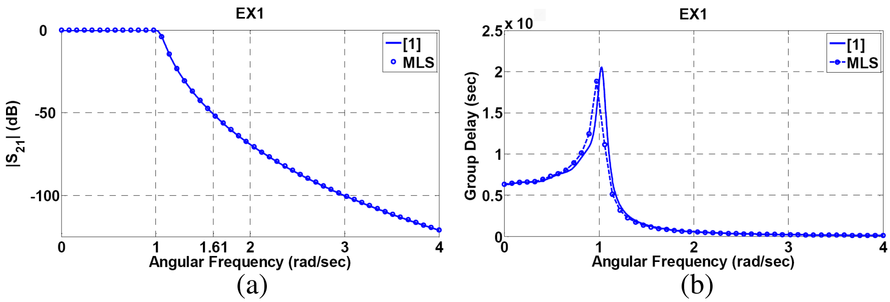

Figure 6. Example 1. (a) Transmission and (b) group delay versus angular frequency.

The source impedance is $g_0 = 1\Omega$ and the load conductance is $g_9 = 1.3554S$. The cutoff angular frequency is $\omega_P = 1\,(\text{rad/sec})$ and the upper angular frequency limit of the transition band is $\omega_S = 1.61\,(\text{rad/sec})$, as shown in Fig. 6(a).

The ripple magnitude in the pass band is $L_{Ar} = 0.1\,\text{dB}$, and the attenuation in the transition band (or equivalently in the stopband) is $L_{As} = 51\,\text{dB}$. The values of $g_1, g_2, \ldots, g_9$ are obtained from Reference [1] and written in the first line denoted by Ex1 in Table 2. This filter is also designed by the method of least squares and the optimum values of g's are written in the third line under Ex1 in Table 2.

In this network, the elements alternate between series and shunt connections and g_k have the definitions given in References [1] and [2].

The frequency responses of these two filters as $|S_{21}|$ versus ω and the group delays as $\frac{\partial \varphi_{21}}{\partial \omega} = \frac{\partial \angle S_{21}}{\partial \omega}$ versus ω as obtained by Reference [1] and MLS designs are drawn in Figs. 6(a) and (b), respectively, for comparison.

Example 2. The circuit diagram of a lowpass elliptic filter of degree 7 is drawn in Fig. 7, which is taken from Reference [1, page 45, Table 3.3]. The design specifications are: $L_{Ar} = 0.1\,\text{dB}$, $L_{As} = 47.57\,\text{dB}$, $\omega_P = 1\,\text{rad/sec}$, $\omega_S = 1.61\,\text{rad/sec}$, the source and load impedances as $g_0 = g_8 = 1\,\Omega$.

Table 2. Three examples for the validation of the method of least squares for filter design.

		L_{As}	g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	*error*
Ex.1	**Ref. [1]**	51 dB	1	1.1898	1.4346	2.1199	1.601	2.17	1.5641	1.9445	0.8778	1.3554	---	---
	$\Lambda_{initiall}$		1	0.5	0.8	0.5	0.8	0.5	0.8	0.5	0.8	1.3554	---	---
	MLS		1	1.205	1.4300	2.08	1.6599	2.075	1.6037	1.975	0.8424	1.3554	---	1.57E-11
Ex.2	**Ref. [1]**	47.5 dB	1	1.0481	0.1244	1.2416	1.6843	0.3540	1.0031	0.8692	1	---	---	---
	$\Lambda_{initiall}$		1	0.8	0.8	0.5	0.8	0.8	0.5	0.8	1	---	---	---
	MLS		1	1.0385	0.1237	1.2450	1.6903	0.3558	1.0000	0.8721	1	---	---	0
Ex.3	**Ref. [1]**	77.9 dB	1	1.1272	1.3506	0.0647	1.8985	1.3485	0.1903	1.7235	1.0417	0.1903	0.8913	---
	$\Lambda_{initiall}$		1	0.5	0.8	0.5	0.5	0.8	0.5	0.5	0.8	0.5	0.5	---
	MLS		1	0.89	1.0966	0.34	1.795	1.5128	0.06	1.95	1.1249	0.23	0.985	0

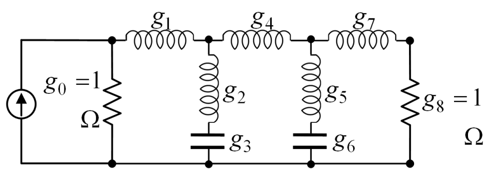

Figure 7. Circuit configuration of example 2.

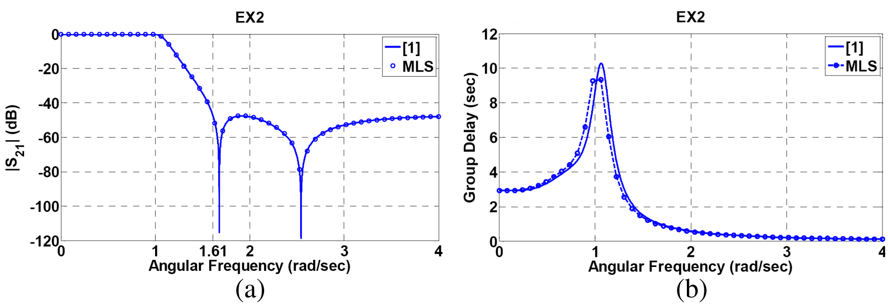

Figure 8. Example 2. (a) Transmission and (b) group delay versus angular frequency.

The values of $g_1, g_2, \ldots, g_7$ obtained from Reference [1] and the corresponding values of g's are also determined by the method of least squares are written in Table 2, together with the initial values of g's. The frequency responses of these two filters as $|S_{21}|$ versus ω and group delay versus ω are drawn in Figs. 8(a) and (b), respectively for comparison.

Example 3. The circuit diagram of a lowpass elliptic filter of degree 10 is drawn in Fig. 9, which is taken from Reference [1, page 45, Table 3.3]. The design specifications are: $L_{Ar} = 0.1\,\mathrm{dB}$,

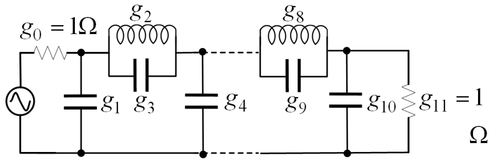

Figure 9. Circuit configuration of example 3.

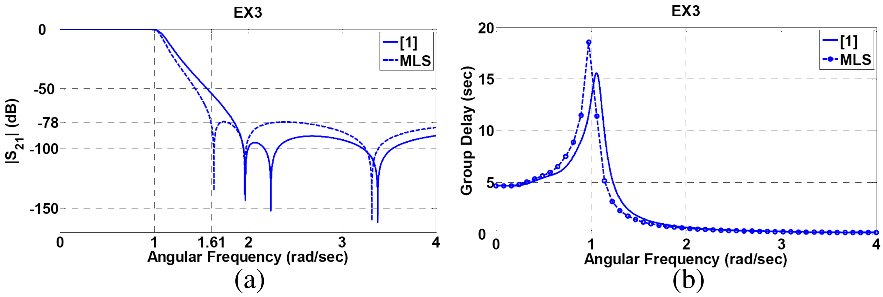

Figure 10. Example 3. (a) Transmission and (b) group delay versus angular frequency.

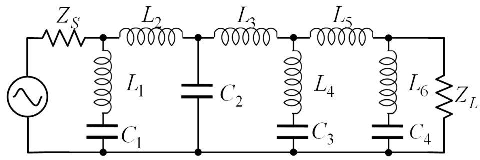

Figure 11. Circuit diagram of filter in [2].

$L_{As} = 77.94\,\text{dB}$, $\omega_P = 1\,\text{rad/sec}$, $\omega_S = 1.61\,\text{rad/sec}$ and source and load impedances as $g_0 = g_1 = 1\,\Omega$. The values of g's are obtained from [1] and determined by the proposed filter design method and written in Table 2. The frequency response of these two filters as $|S_{21}|$ versus ω and group delay versus ω are drawn in Fig. 10 for comparison.

Examples 4. The cicuit diagram of a lowpass composite filter is drawn in Fig. 11, which is taken from Reference [2, page 443]. This filter is designed by the constant-k and m-derived methods. The design specifications are: $L_{Ar} = 0.08\,\text{dB}$, $L_{As} = 40\,\text{dB}$, $f_P = 1.95\,\text{MHz}$, $f_S = 2.05\,\text{MHz}$ and $Z_S = Z_L = 75\,\Omega$.

The values of reactive elements as obtained in Reference [2] and determined by the MLS filter design are listed in Table 3.

The frequency response of this filter as $|S_{21}|$ versus f is drawn in Fig. 12 for comparison.

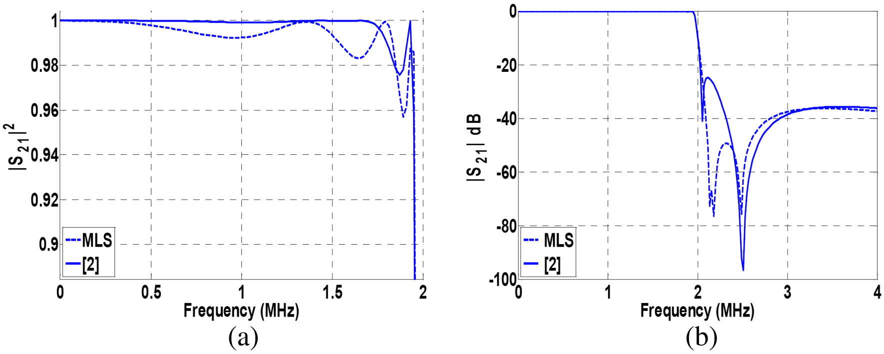

Figure 12. Example 4. (a) $|S_{21}|^2$ versus frequency from 0 to 2 MHz. (b) $|S_{21}|$ dB versus frequency from 0 to 4 MHz.

Table 3. Values of the elements of circuit in Fig. 11.

$f_P = 1.95\,\text{MHz}$, $f_S = 2.05\,\text{MHz}$, $L_{Ar} = 0.08\,\text{dB}$, $L_{As} = 40\,\text{dB}$										
	L_1 **(µH)**	C_1 **(pF)**	L_2 **(µH)**	C_2 **(pF)**	L_3 **(µH)**	L_4 **(µH)**	C_3 **(pF)**	L_5 **(µH)**	L_6 **(µH)**	C_4 **(pF)**
Ref. [2]	6.368	636.5	9.552	2122	7.28	12.94	465.8	4.892	6.368	636.5
MLS	5.667	726.4	9.112	2212	7.76	7.06	783.5	5.063	11.39	465.2

Example 5. The number of elements of a band stop filter is usually twice that of a lowpass or highpass filter having comparable frequency responses.

Consequently, it is perferable to design a lowpass filter initially and then obtain the desired bandpass or bandstop filter according to the transformations given in Table 1. Therefore, in order to speed up the proposed filter design algorithm based on the method of least squares, the lowpass filter is first designed and then the required filter transformations are made, which may actually give the the initial values for the reactive elements of the desired bandpass and bandstop filters. Then, the MLS algorithm may proceed to optimize the filter design.

Consider the schematic circuit of a bandpass filter shown in Fig. 13. The design specifications are: $f_{LS} = 6\,\text{GHz}$, $f_{LP} = 8\,\text{GHz}$, $f_{HP} = 10\,\text{GHz}$, $f_{HS} = 12\,\text{GHz}$, $L_{Ar} = 0.1\,\text{dB}$, $L_{As} = 40\,\text{dB}$, $Z_S = Z_L = 50\,\Omega$. The frequency response of filter as $|S_{21}|$ versus f and group delay versus f are drawn in Fig. 14. One curve is the performance of the bandpass filter as obtained from the transformation of the optimized lowpass filter design, and the other is the further optimization of the resulting BP filter. The relevant data are given in Table 4.

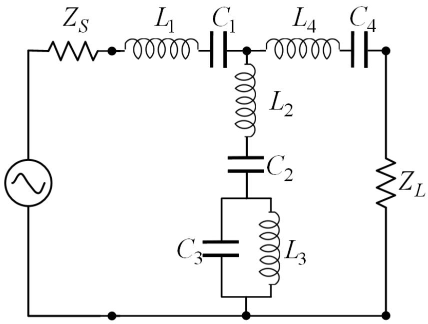

Figure 13. Circuit diagram of the BP Filter in Example 5.

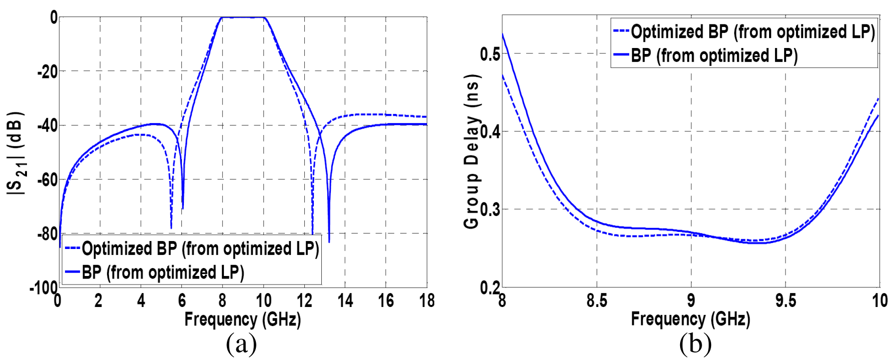

Figure 14. Comparison of performance of BP filter as derived from the optimized LP filter and further optimization of BP filter from optimized lowpass case to bandpass; (a) $|S_{21}|$ dB. (b) Group delay.

Table 4. Values of the elements of circuit in Fig. 13.

$f_{Ls} = 6$ GHz, $f_{Lp} = 8$ GHz, $f_{Hp} = 10$ GHz, $f_{Hs} = 12$ GHz, $L_{Ar} = 0.1$ dB, $L_{As} = 40$ dB								
	L_1 **(nH)**	C_1 **(pF)**	L_2 **(nH)**	C_2 **(pF)**	L_3 **(nH)**	C_3 **(pF)**	L_4 **(nH)**	C_4 **(pF)**
LP design	4.8085	0.2891	0.0658	1.0950	0.1846	1.7154	4.5116	0.0702
BP design	4.7084	0.0658	0.3248	1.3709	0.1811	1.7132	4.3342	0.0723

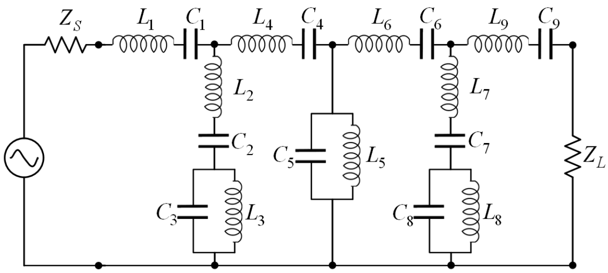

Figure 15. Circuit diagram of the BP filter in example 6.

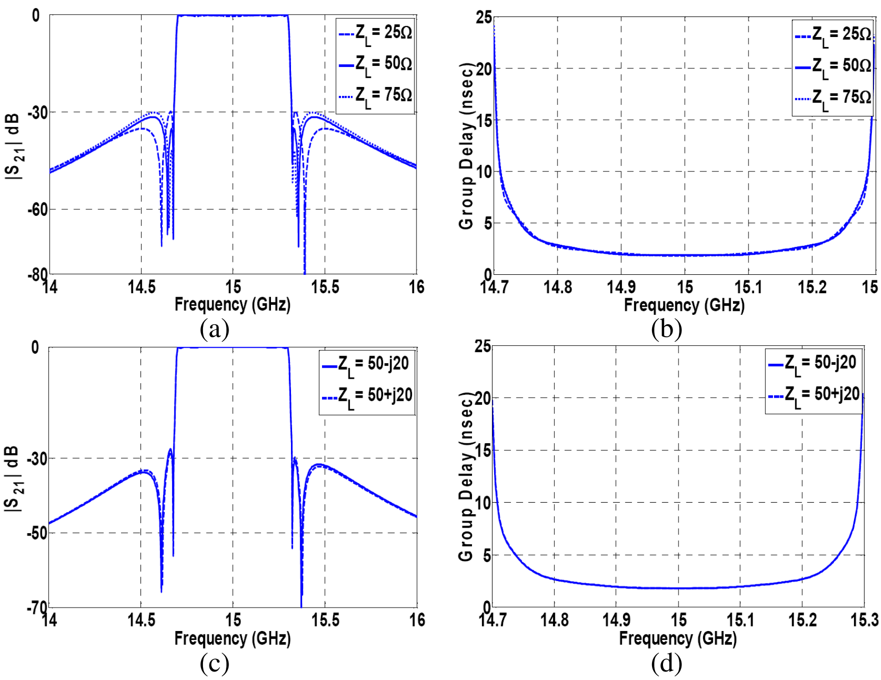

Figure 16. Example 6. (a) Frequency and (b) group delay response of bandpass filters with real load impedances. (c) Frequency and (d) group delay response of BP filters with complex load impedances.

Example 6. Consider the circuit diagram of a bandpass filter as shown in Fig. 15. The filter design specifications are: $f_{LS} = 6\,\text{GHz}$, $f_{LP} = 8\,\text{GHz}$, $f_{HP} = 10\,\text{GHz}$, $f_{HS} = 12\,\text{GHz}$, $L_{Ar} = 0.1\,\text{dB}$, $L_{As} = 40\,\text{dB}$. Five filters are designed for various values of source and load impedances, namely:

$$\begin{aligned}
Z_S &= 50\,\Omega, \quad Z_L = 50\,\Omega\\
Z_S &= 50\,\Omega, \quad Z_L = 75\,\Omega\\
Z_S &= 50\,\Omega, \quad Z_L = 25\,\Omega\\
Z_S &= 50\,\Omega, \quad Z_L = 50 + j20\\
Z_S &= 50\,\Omega, \quad Z_L = 50 - j20
\end{aligned}$$

The frequency response of $|S_{21}|$ and group delay for various source and load impedances are drawn in Fig. 16. The relevant data for filters are given in Table 5.

Eample 7. Consider the circuit diagram of a bandstop filter shown in Fig. 17. The design specifications of four different band stop filters are given in Table 6. The performance of these filter designs are given in Fig. 18.

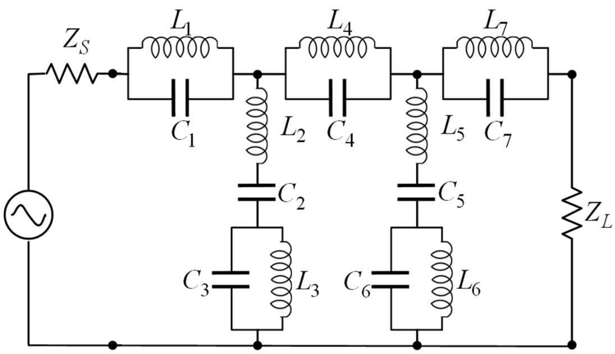

Figure 17. Prototype for band stop design.

Example 8. The method of least squares may be readily applied for the design of multiband filters. For example, the error function for a dual bandpass filter is:

$$\begin{aligned}
\varepsilon_{BP-2band} = & ER_{S1} + ER_{T11} + ER_{P1} + ER_{T12} + ER_{S2} + ER_{T21}\\
& + ER_{P2} + ER_{T22} + ER_{S3}
\end{aligned}$$

Its design specifications are selected as $L_{Ar} = 0.2\,\text{dB}$, $L_{As} = 40\,\text{dB}$ for both bands.

The first band has a center frequency 3 GHz, band width 500 MHz and transition band 70 MHz. the second band has a center frequency 4 GHz, band width 300 MHz and transition band 50 MHz. Its circuit diagram and frequency response are shown in Figs. 19 and 20 respectively.

Table 5. Bandpass filter design for different load impedances.

$f_{Ls}=14.68$GHz, $f_{Lp}=14.7$GHz, $f_{Hp}=15.3$GHz, $f_{Hs}=18.32$GHz, $\omega_0=94.2289$GHz									
$Z_S=Z_L=50\Omega$									
L_1(nH)	5.1533	L_3(nH)	58.1621	L_5(nH)	13.5217	L_7(nH)	16.1498	L_9(nH)	9.1010
C_1(pF)	0.0218	C_3(pF)	1.9364	C_5(pF)	8.3291	C_7(pF)	0.0070	C_9(pF)	0.0124
L_2(nH)	31.1560	L_4(nH)	21.7859	L_6(nH)	24.6672	L_8(nH)	36.6020		
C_2(pF)	0.0036	C_4(pF)	0.0052	C_6(pF)	0.0045	C_8(pF)	3.0770		
$Z_S=50\Omega$, $Z_L=75\Omega$									
L_1(nH)	6.5733	L_3(nH)	54.9648	L_5(nH)	15.4207	L_7(nH)	25.5662	L_9(nH)	10.1158
C_1(pF)	0.0171	C_3(pF)	2.0490	C_5(pF)	7.3034	C_7(pF)	0.0044	C_9(pF)	0.0111
L_2(nH)	29.0567	L_4(nH)	23.5571	L_6(nH)	30.0415	L_8(nH)	54.8284		
C_2(pF)	0.0039	C_4(pF)	0.0048	C_6(pF)	0.0037	C_8(pF)	2.0541		
$Z_S=50\Omega$, $Z_L=25\Omega$									
L_1(nH)	5.0034	L_3(nH)	18.9877	L_5(nH)	6.3335	L_7(nH)	13.8920	L_9(nH)	2.4491
C_1(pF)	0.0225	C_3(pF)	5.9314	C_5(pF)	17.7823	C_7(pF)	0.0081	C_9(pF)	0.0460
L_2(nH)	6.9619	L_4(nH)	13.7659	L_6(nH)	9.7324	L_8(nH)	25.9075		
C_2(pF)	0.0162	C_4(pF)	0.0082	C_6(pF)	0.0116	C_8(pF)	4.3472		
$Z_S=50\Omega$, $Z_L=50+j20$									
L_1(nH)	7.6391	L_3(nH)	27.8755	L_5(nH)	11.3074	L_7(nH)	29.1793	L_9(nH)	2.4421
C_1(pF)	0.0147	C_3(pF)	4.0334	C_5(pF)	9.9602	C_7(pF)	0.0039	C_9(pF)	0.0428
L_2(nH)	10.9118	L_4(nH)	21.6093	L_6(nH)	17.9796	L_8(nH)	54.3160		
C_2(pF)	0.0103	C_4(pF)	0.0052	C_6(pF)	0.0062	C_8(pF)	2.0748		
$Z_S=50\Omega$, $Z_L=50-j20$									
L_1(nH)	7.6431	L_3(nH)	27.8044	L_5(nH)	11.3219	L_7(nH)	29.1811	L_9(nH)	2.5784
C_1(pF)	0.0147	C_3(pF)	4.0397	C_5(pF)	9.9518	C_7(pF)	0.0039	C_9(pF)	0.0478
L_2(nH)	10.9124	L_4(nH)	21.6120	L_6(nH)	17.9785	L_8(nH)	54.3352		
C_2(pF)	0.0104	C_4(pF)	0.0052	C_6(pF)	0.0063	C_8(pF)	2.0737		

Example 9. Consider the equivalent circuit of a transistor shown inside dashed lines in Fig. 21, which should be matched between its input and output circuits. In order to demonstrate the capability of the proposed MLS procedure for the optimum design of filters combined with impedance matching, we design the amplifier circuit composed of the transistor equivalent circuit together with its input and output circuits, as given in [4]. This transistor has a 5 dB/octave power gain roll-off from 4 GHz according to the relation $X(\text{dB}) = -\frac{5}{\log 2}\log\frac{f(\text{GHz})}{4}$.

Table 6. BS filter designs for different load impedances.

$f_{Lp} = 6.97$GHz, $f_{Ls} = 6.975$GHz, $f_{Hs} = 7.025$GHz, $f_{Hp} = 7.03$GHz, $\omega_0 = 43.3982$GHz									
$Z_S = Z_L = 50\Omega$									
L_1(nH)	6.8348	C_2(pF)	0.0024	L_4(nH)	13.8793	C_5(pF)	0.0034	L_7(nH)	8.8016
C_1(pF)	75.6355	L_3(nH)	10.3077	C_4(pF)	37.2465	L_6(nH)	2.9556	C_7(pF)	58.7340
L_2(nH)	213.3601	C_3(pF)	50.1522	L_5(nH)	121.3056	C_6(pF)	174.9041		
$Z_S = 50$, $Z_L = 75\Omega$									
L_1(nH)	8.7526	C_2(pF)	0.0020	L_4(nH)	17.4908	C_5(pF)	0.0032	L_7(nH)	10.6959
C_1(pF)	59.0627	L_3(nH)	12.0750	C_4(pF)	29.5558	L_6(nH)	3.9337	C_7(pF)	48.3321
L_2(nH)	248.3479	C_3(pF)	42.8118	L_5(nH)	160.7507	C_6(pF)	131.4160		
$Z_S = 50\Omega$, $Z_L = 25\Omega$									
L_1(nH)	6.4061	C_2(pF)	0.0038	L_4(nH)	10.1752	C_5(pF)	0.0058	L_7(nH)	6.5878
C_1(pF)	80.6862	L_3(nH)	6.2903	C_4(pF)	50.8053	L_6(nH)	2.8048	C_7(pF)	78.4717
L_2(nH)	137.5312	C_3(pF)	82.1828	L_5(nH)	89.0763	C_6(pF)	184.3121		
$Z_S = 50\Omega$, $Z_L = 50 + j15$									
L_1(nH)	7.0360	C_2(pF)	0.0023	L_4(nH)	14.5713	C_5(pF)	0.0042	L_7(nH)	9.5026
C_1(pF)	73.4730	L_3(nH)	10.6755	C_4(pF)	35.4777	L_6(nH)	3.0347	C_7(pF)	54.4012
L_2(nH)	220.8296	C_3(pF)	48.4246	L_5(nH)	122.6443	C_6(pF)	170.3486		

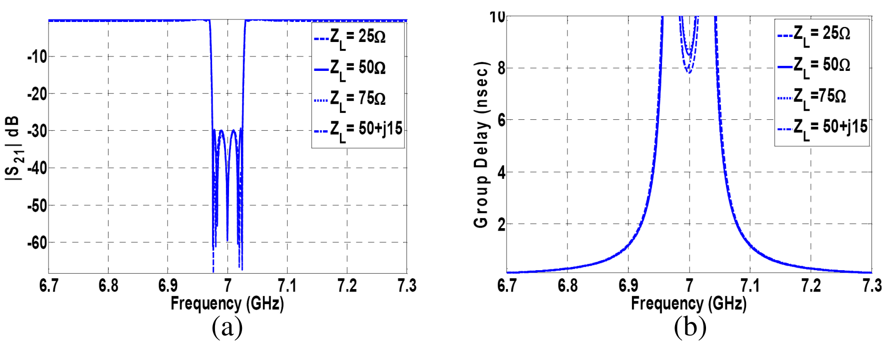

Figure 18. BS filter design for (a) different load impedances and related (b) group delays of the circuit in Fig. 17.

This value of gain roll-off will be compensated by including its effect in the scattering parameter S_{21} computed for the transistor input circuit by modifying it according to

$$S_{21}(\text{dB}) \leftarrow S_{21}(\text{dB})\big|_{\substack{input \\ circuit}} + X(\text{dB})$$

which will be inserted in the expression of error function.

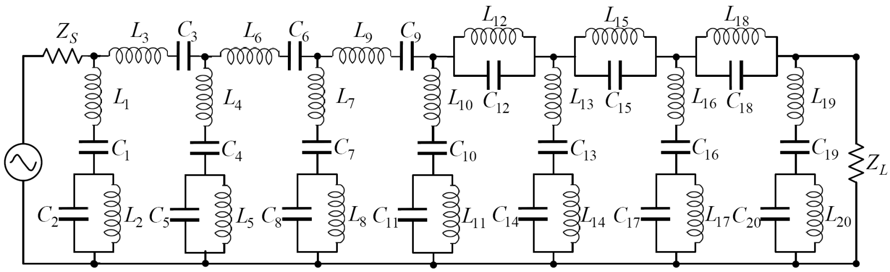

Figure 19. Prototype for dual-band.

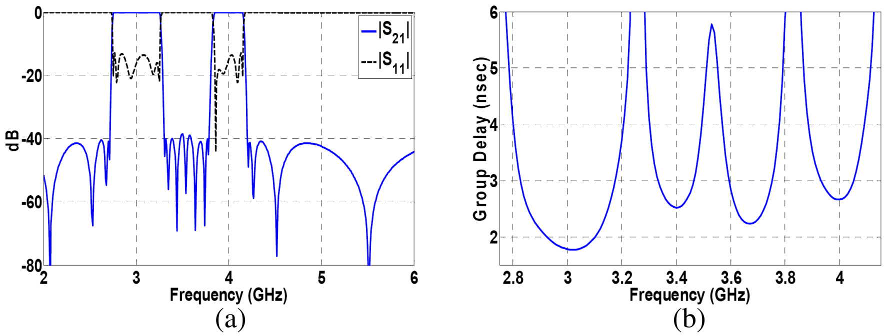

Figure 20. Dual-band filter design. (a) Frequency and (b) group delay response.

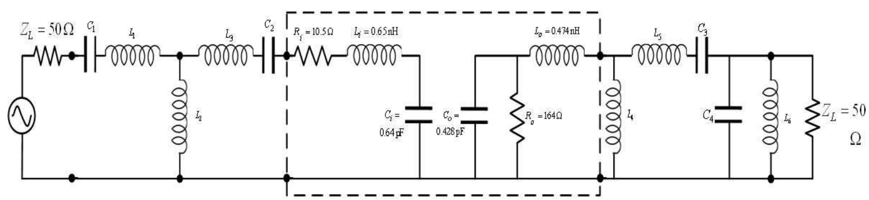

Figure 21. Prototype of input and output highpass filter for matching in pass band.

We would like to make the S_{21} frequency response of the combined input circuit and transistor equivalent input circuit flat. Then, we need to add the gain roll-off X to S_{21} parameter, since the input circuit is passive.

We omit the term on the transition band in the error function (ER_T). Now we specify the ripple of resultant S_{21} equal to 0.5 dB.

In order to adapt this specification to the proposed MLS procedure as given in error function ($\varepsilon_{MA} = W_P \times ER_P + W_S \times ER_S$), we specify $L_{Ar} = -5.8$ dB, $L_{As} = -5.3$ dB (which gives ripples equal to $L_{As} - L_{Ar} = 0.5$ dB). The minimization of error function gives the element values (L_1, L_2, L_3 and C_1) to match the source impedance to the transistor input equivalent circuit.

The values of S_{21} for only the input circuit and the combined equivalent circuit are drawn in Fig. 22(a). These curves are compared with those given in [4]. It is seen that S_{21} of the combined circuit is quite flat in the band width 4–8 GHz, and it is under the -5 dB line.

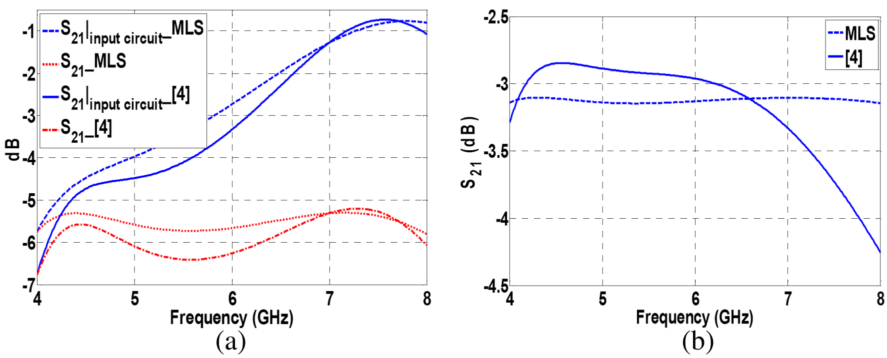

Figure 22. Design of impedance matching circuits of a transistor equivalent circuit. (a) Input BP filter response, (b) output BP filter response.

Table 7. Dual-band filter designs.

$f_{Ls1} = 2.65$GHz, $f_{Lp1} = 2.75$GHz, $f_{Hp1} = 3.25$GHz, $f_{Hs1} = 3.35$GHz, $\omega_{01} = 18.77$ GHz $f_{Ls2} = 3.78$GHz, $f_{Lp2} = 3.85$GHz, $f_{Hp2} = 4.15$GHz, $f_{Hs2} = 4.22$GHz, $\omega_{02} = 25.10$GHz $L_{Ar1} = L_{Ar2} = 0.2$dB, $L_{As1} = L_{As2} = L_{As3} = 40$dB									
L_1(nH)	9.2820	L_3(nH)	4.2601	L_5(nH)	1.6516	L_7(nH)	2.0204	L_9(nH)	7.1011
C_1(pF)	0.23912	C_3(pF)	0.52100	C_5(pF)	1.3441	C_7(pF)	1.0985	C_9(pF)	0.3126
L_2(nH)	2.0550	L_4(nH)	8.4476	L_6(nH)	4.9980	L_8(nH)	0.7021	L_{10}(nH)	0.9573
C_2(pF)	1.0800	C_4(pF)	0.2627	C_6(pF)	0.4441	C_8(pF)	3.1612	C_{10}(pF)	2.3185
L_{11}(nH)	0.9962	L_{13}(nH)	12.7357	L_{15}(nH)	0.4428	L_{17}(nH)	0.2628	L_{19}(nH)	15.1182
C_{11}(pF)	2.2280	C_{13}(pF)	0.1589	C_{15}(pF)	4.5718	C_{17}(pF)	7.7042	C_{19}(pF)	0.1339
L_{12}(nH)	0.2659	L_{14}(nH)	0.1567	L_{16}(nH)	14.3673	L_{18}(nH)	0.4138	L_{20}(nH)	0.0482
C_{12}(pF)	7.6113	C_{14}(pF)	12.9145	C_{16}(pF)	0.1409	C_{18}(pF)	4.8924	C_{20}(pF)	41.9926

Table 8. Element values of the circuit in Fig. 21.

	C_1(pF)	L_1(nH)	L_2(nH)	L_3(nH)	C_2(pF)	L_4(nH)	L_5(nH)	C_3(pF)	C_4(pF)	L_6(nH)
Ref. [5]	0.37	1.67	1.62	0.347	0.5	1.35	1.843	0.358	1.77	0.447
MLS	0.5117	1.2084	1.5114	0.0281	0.7726	1.3656	0.9475	0.358	2.5969	0.2061

For the design of the bandpass filter in the transistor output circuit, we need to obtain the values of elements (L_4, L_5, C_3, C_4 and L_6) to match the transistor to the load impedance $Z_L = 50\,\Omega$. Again, the S_{21} curve ripple is selected as 0.045 dB, which is made compatible to the error function by taking $L_{Ar} = -3.345\,\text{dB}$, $L_{As} = -3.3\,\text{dB}$. The frequency responses obtained by MLS and provided in [4] are drawn in Fig. 22(b). The S_{21} curve is under the -3 dB line. All the values of elements of the input and output filters are given in Table 8.

4. CONCLUSION

A filter design procedure is developed based on the method of least squares, which incorporates source and load impedance matching leading to the drastic reduction of circuit size and complexity. General network topologies and filter characteristics may be considered for the filter design. Quite general desired design specifications and frequency responses may be realized by the design algorithm, such as sperious response elimination, multiband filter characteristics, realization of any frequency response, applications in general electronic circuits, and enhancement of some special effects. The filter design method may be applied for microwave circuits too. The method of least squares may potentially be used for the determination of optimum topology of filter configurations, which could be the subject of further investigations.

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