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WAVE ANALYSIS FOR INDUCTIVELY MATCHED MILLIMETER WAVE AMPLIFIER DESIGN

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Abstract—A new design approach based on wave analysis has been implemented in order to derive voltage gain, center frequency and bandwidth of millimeter wave amplifier using parameters of transmission lines (TL). The derived formula allow one to design high frequency amplifier with predetermined bandwidth and centner frequency. It has been shown that in the case of lossy TL or at high frequency, circuit theory cannot predict the amplifier gain behavior while presented wave theory can accurately predict the frequency response of the amplifier in both low and high frequency ranges.

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1. INTRODUCTION

Millimeter-wave is of great interest in conventional and emerging applications. Short range communication, radiometry, radio astronomy, remote sensing and high resolution imaging are some of recently reported examples of millimeter-wave systems [1–6].

With recent advancement of transistor technology by aggressive scaling of the gate length, transistor with cut off frequency of few hundred GHz is available [3, 6]. These technologies allow one to design very high frequency amplifiers up to the sub-millimeter wave band.

The input impedance of these transistors is capacitive due to gate to source capacitance. Therefore, in order to match the input impedance of transistor to the source and resonating out this capacitor, inductive input matching circuit have been widely used [1–4].

Figure 1(a) shows a single gain cell with input and output matching inductors, L_g and L_d. At higher frequency band of millimeter wave, the inductors are usually realized by a piece of transmission line as the lumped inductors are difficult to implement in these frequencies. Also in this frequency range, the lumped element model cannot precisely predict the frequency characteristics of the gain cell due to presence of many parasitics.

Figure 1(b) is Fig. 1(a) in which the inductors, L_g and L_d are replaced by piece of transmission line with length of l_g and l_d respectively. In this paper, in order to analyze the circuit shown in

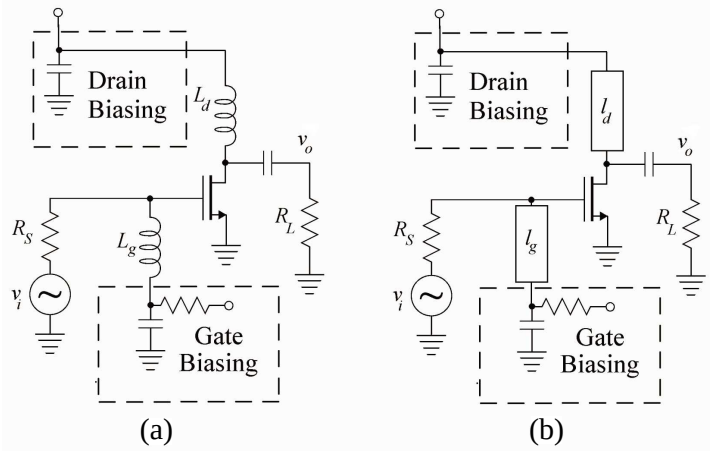

Figure 1. The single gain cell with input/output matching using: (a) inductors, (b) transmission line.

Fig. 1(b), wave theory has been implemented for accurate prediction of the circuit behavior at high frequencies.

Based on the wave equations, the voltage gain of the gain cell has been derived and compared with that obtain by lumped element model. It is shown that at the low frequency, the circuit theory and wave theory predict the similar frequency characteristics for the gain cell. However, at high frequency, the gain function obtained by circuit theory for lumped element model is no longer accurate. In addition, using the gain function based on wave equation, the design formula for the center frequency and bandwidth of the gain cell have been derived. Using two terms expansion of this gain function, the simplified design formula have been obtained which can be used in high frequency design of the amplifier.

2. ANALYSIS OF MILLIMETER WAVE GAIN CELL

Figure 2 shows small signal equivalent model of Fig. 1.

Each TL is characterized by three parameters, Z_0, α and β which are characteristics impedance, attenuation constant and phase constant respectively. The voltage and current in a piece of transmission line short circuited (as in Fig. 2(b)) at one end at distance

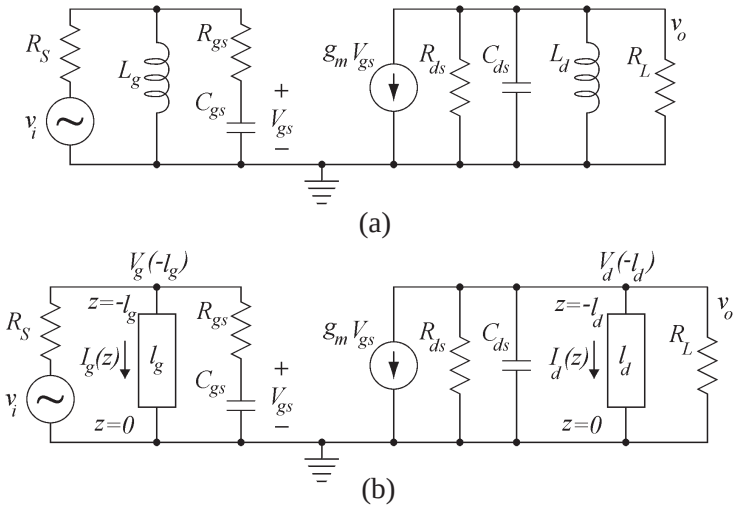

Figure 2. Small signal model of gain cell with: (a) inductors, (b) transmission line.

z can be written as:

$$V(z) = V^+(e^{-\gamma z} - e^{\gamma z}); \quad I(z) = \frac{V^+}{Z_0}(e^{-\gamma z} + e^{\gamma z}) \tag{1}$$

where $\gamma = \alpha + j\beta$. Using equation Eq. (1) for Fig. 1(b), the voltage and current on the input of the gate and drain lines are expressed as:

$$V_g(-l_g) = V_g^+(e^{\gamma_g l_g} - e^{-\gamma_g l_g}); \quad I_g(-l_g) = \frac{V_g^+}{Z_g}(e^{\gamma_g l_g} + e^{-\gamma_g l_g}) \tag{2}$$

$$V_d(-l_d) = V_d^+(e^{\gamma_d l_d} - e^{-\gamma_d l_d}); \quad I_d(-l_d) = \frac{V_d^+}{Z_d}(e^{\gamma_d l_d} + e^{-\gamma_d l_d}) \tag{3}$$

For Fig. 1(b) and for the gate circuit:

$$\frac{V_i - V_g(-l_g)}{R_S} = I_g(-l_g) + sC_{gs}V_{gs}; \quad \frac{V_g(-l_g) - V_{gs}}{R_{gs}} = sC_{gs}V_{gs} \tag{4}$$

By substituting Eq. (2) in Eq. (4), the gate gain function can be calculated as:

$$H_1(s) = \frac{V_{gs}}{V_i} = \frac{1}{\left[1 + \frac{R_S}{Z_g}\coth(\gamma_g l_g)\right](1 + sC_{gs}R_{gs}) + sC_{gs}R_S} \tag{5}$$

using the same procedure as above in the drain circuit one can obtain:

$$\frac{V_o}{V_{gs}} = -g_m R_L H_2(s) \tag{6}$$

where:

$$H_2(s) = \frac{1}{\left[\frac{R_L}{Z_d}\coth(\gamma_d l_d)\right] + sC_{ds}R_L + \frac{R_{ds}+R_L}{R_{ds}}} \tag{7}$$

Therefore the voltage gain of the gain cell can be expressed as:

$$H(s) = \frac{V_o}{V_i}(wave\ theory) = -g_m R_L H_1(s) H_2(s) \tag{8}$$

Using the conventional circuit theory for Fig. 1(a), the voltage gain of the gain cell can be easily calculated as:

$$G(s) = \frac{V_o}{V_i}(circuit\ theory) = -g_m R_L G_1(s) G_2(s) \tag{9}$$

where:

$$G_1(s) = \frac{sL_g}{C_{gs}L_g(R_{gs} + R_S)s^2 + (L_g + R_{gs}C_{gs}R_S)s + R_S}$$

$$G_2(s) = \frac{sR_{ds}L_d}{L_d R_L R_{ds} C_{ds} s^2 + L_d(R_L + R_{ds})s + R_L R_{ds}}$$

As can be seen from Eq. (9), the gain function of a single gain cell is cascade of two band pass gain functions whose center frequencies are f_{01} and f_{02}. These center frequencies for each of these gain functions are found by solving $\mathrm{Im}[G_1(j2\pi f_{01})] = 0$ and $\mathrm{Im}[G_2(j2\pi f_{02})] = 0$. The results are as follows:

$$f_{01} = \frac{1}{2\pi}\sqrt{\frac{R_S}{C_{gs}L_g(R_S + R_{gs})}}, \quad f_{02} = \frac{1}{2\pi}\sqrt{\frac{1}{C_{ds}L_d}} \tag{10}$$

The bandwidth of the gain cell is $|f_{02}-f_{01}|$ and its center frequency is $f_0 = (f_{01} + f_{02})/2$.

For lossless TL, $\gamma = j\beta$. Then:

$$\coth(\gamma_g l_g) = \coth(j\beta_g l_g) = \coth\left(j\frac{\omega}{v_g}l_g\right) = \coth(s\tau_g) \tag{11}$$

Using Taylor expansion for $\coth(s\tau_g)$ which is:

$$\coth(s\tau_g) = \frac{1}{s\tau_g} + \frac{s\tau_g}{3} + \ldots \tag{12}$$

and considering the low frequency approximation of Eq. (12) which is $\coth(s\tau_g) \simeq 1/s\tau_g$:

$$\frac{\coth(s\tau_g)}{Z_g} \simeq \frac{1}{s\tau_g Z_g} = \frac{v_g}{sl_g Z_g} = \frac{1/\sqrt{\bar{L}_g\bar{C}_g}}{sl_g\sqrt{\bar{L}_g/\bar{C}_g}} = \frac{1}{sL_g} \tag{13}$$

where $L_g = \bar{L}_g l_g$ and $\bar{L}_g$ and $\bar{C}_g$ are inductor and capacitor per unit length of gate TL respectively. Substituting $\coth(s\tau_g)/Z_g \simeq 1/sL_g$ into Eq. (5) one can easily prove that $H_1(s) = G_1(s)$. Using the same procedure as above, one can show that $H_2(s) = G_2(s)$.

It means that at low frequency and for the lossless case, the wave analysis and circuit theory based on lumped element model predict similar frequency response for the amplifier.

The above approximation is valid if $1/s\tau_g \gg s\tau_g/3$ i.e., $1/s\tau_g > 20s\tau_g$ which means $l < 0.06\lambda$.

Now, considering $\coth(s\tau_g) \simeq 1/s\tau_g + s\tau_g/3$ and substituting into equation Eq. (5), the second order approximation of $H_1(s)$ can be obtained. Using the same approach as above, the center frequency of this gain function, f_{01}, at which $\mathrm{Im}[H_1(j2\pi f_{01})] = 0$ can be calculated as follow:

$$f_{01} = \frac{Z_g}{2\pi}\sqrt{\frac{3R_S}{L_g[L_g R_S + 3Z_g^2 C_{gs}(R_S + R_{gs})]}} \tag{14}$$

With the same procedure, one can calculate the second order approximation of $H_2(s)$ and the center frequency of the gain function, f_{02}, as follow:

$$f_{02} = \frac{Z_d}{2\pi}\sqrt{\frac{3}{L_d(L_d + 3Z_d^2 C_{ds})}} \tag{15}$$

As an example, for a gain cell with center frequency of $f_0 = 124.5\,\text{GHz}$ and bandwidth of 25 GHz, L_g and L_d can be derived from Eq. (14) and Eq. (15) for $f_{01} = 112\,\text{GHz}$ and $f_{02} = 137\,\text{GHz}$ respectively.

3. CIRCUIT THEORY AND WAVE ANALYSIS COMPARISON

Figure 3 compare the gain functions obtained by wave theory and those obtained by circuit theory using the parameters for a typical millimeter wave gain stage for a single gain cell. In Fig. 4, we also compared the power gain obtained by wave and circuit theories for a cascade of 2 gain cells.

In this gain stage, 210 µm transmission lines in a substrate with $\epsilon_{reff} = 4.1$ has been used to realize 125 pH inductors for both gate and drain lines. As can be seen from Fig. 4, at low frequency where the loss is negligible and $l < 0.06\lambda$, the gain obtained by lumped element model and that obtained by wave theory are close. However, at high frequency where $l < 0.06\lambda$ is no longer valid and the loss is significant, there is a considerable difference between the gain calculated by each model and for amplifier design at high frequency the circuit theory is no longer valid and wave theory should be implemented.

The first row in Table 1 shows the center frequencies derived by numerically solving $\text{Im}[H_1(j2\pi f_{01})] = 0$ and $\text{Im}[H_2(j2\pi f_{02})] = 0$ from Eq. (5) and Eq. (7) respectively. In this table, we also compared these center frequencies with those obtained by first and second order approximation of the same equations.

This table clearly shows that the first order approximation is not accurate as for the frequencies above 100 GHz where $\lambda_{eff} < 1470\,\mu\text{m}$ and the assumption of $l < 0.06\lambda_{eff}$ for $l = 210\,\mu\text{m}$ is no longer valid. However, the center frequencies obtained by second order approximation are quite closed to those obtained by full wave analysis.

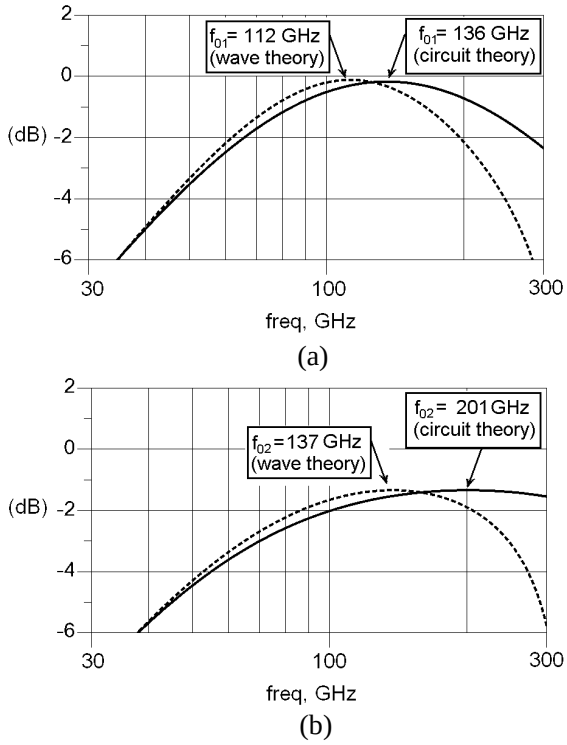

Figure 3. Comparison of gain functions for: (a) G_1, H_1, (b) G_2, H_2 for single gain cell.

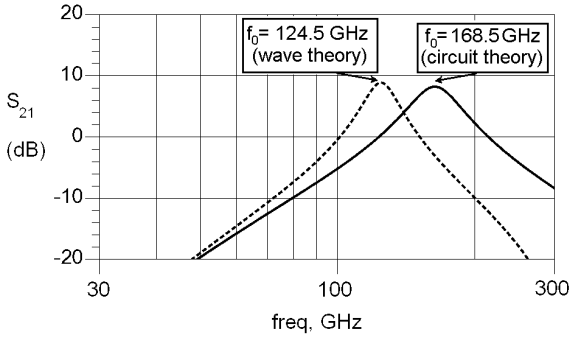

Figure 4. Comparison of power gains for 2-cascaded gain cell using wave theory (dashed line) and circuit theory (solid line).

Table 1. Comparison results.

Calculation Method	f_{01} (GHz)	f_{02} (GHz)	f_0 (GHz)
Full-wave analysis from Eqs. (5) and (7)	112	137	124.5
Second order approx. using Eqs. (14), (15)	111	139	125
Circuit theory using Eq. (10)	136	201	168.5

The gain cell parameters are: $L_g = L_d = 125\,\text{pF}$, $C_{gs} = 10\,\text{fF}$, $C_{ds} = 5\,\text{fF}$, $R_{gs} = 5\,\Omega$, $R_{ds} = 300\,\Omega$, $R_S = R_L = 50\,\Omega$ and $Z_g = Z_d = 86\,\Omega$.

4. MEASUREMENT RESULT AND DISCUSSION

To verify the validity of the proposed design method, an amplifier with 4 cascaded gain cell shown in Fig. 1(b) using IBM-CMOS technology has been fabricated to operate in 60 GHz band.

To obtain optimum performance, a transistor with gate width of 54.8 µm using 40 gate finger is selected from a library of devices in the design kit. The transistor is biased to draw 9 mA from a 1.5 V supply. The intrinsic parameters of this transistor for the model shown in Fig. 2(b) are extracted as $C_{gs} = 94.5\,\text{fF}$, $C_{ds} = 71.7\,\text{fF}$, $R_{gs} = 3.9\,\Omega$, $R_{ds} = 102\,\Omega$.

In this technology, stripline which offers higher inductance per length and lower parasitics elements is selected to implement both gate and drain lines. The stripline uses top metal layer with 4 µm thickness for signal path and the lowest thin metal layer which is slotted for returned path. This configuration prevents the EM-filed from penetration into lossy silicon substrate. The geometry is optimized to provide minimum loss and maximum characteristics impedance. Using the measured S-parameters of this line the inductance and capacitance per unit length are obtained as $\bar{L} = 0.675\,\text{pF}/\mu\text{m}$ and $\bar{C} = 0.105\,\text{fF}/\mu\text{m}$ which results $Z_g = Z_d = 80\,\Omega$. The Q of as high as 23 is achieved for this line.

Having the above parameters and based on Eq. (14) and Eq. (15), $L_g = 69\,\text{pH}$ and $L_d = 76.8\,\text{pH}$ are obtained for a gain cell to operate from $f_{01} = 59\,\text{GHz}$ to $f_{02} = 66\,\text{GHz}$. These values correspond to $l_g = 102\,\mu\text{m}$ and $l_d = 113\,\mu\text{m}$ respectively.

Figure 5 compares the measured gain of the fabricated amplifier and that obtained by Eq. (8) based on wave analysis.

As can be seen from Fig. 5, due to the many loss mechanisms in the fabrication of the amplifier, the measured gain is lower than that predicted by Eq. (8). However, the bandwidth and the center frequency of the fabricated amplifier are close to those obtained using

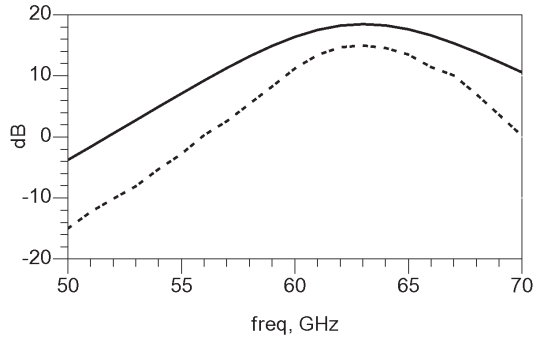

Figure 5. Comparison of measured gain (dashed line) and wave analysis (solid line).

wave analysis. Therefore, the proposed method can be applied to design of millimeter-wave amplifier with pre-determined bandwidth and centner frequency.

5. CONCLUSION

The design formula for gain, center frequency and bandwidth were derived for a gain cell whose inductors are realized by lossy transmission lines. It is shown that the results obtained by the wave analysis and circuit theory are in excellent agreement at low frequencies while the circuit theory is no longer valid at high frequencies and the wave theory should be implemented. To show the capability of the proposed method, a 4-stage cascaded amplifier with shunt inductor matching has been designed and fabricated. The measure result verifies the ability of the method for high frequency amplifier design.

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